welcome to today's lecture again let us recall what we learnt in the last class we learnt about heat engine and refrigerator we shall quickly recapitulate these two machines before we proceed to the discussion of second law at some length

So heat engine or refrigerator the important point is they work in a complete cycle okay and whatever quantities i will refer to heat absorbed heat released walk done they correspond to a complete cycle of operation and they come back to the same point which means same state with same set of thermodynamic variables hence the change in internal energy over a closed loop is equal to 0 because internal energy is a state function and depends on the state the thermodynamic variables

So let us recall heat engine heat engine let me pictorially tell you what a heat engine is at heat engine works in a complete cycle between two reservoirs one is hot t1 other is called t2 this is my working substance whatever it may be will choose ideal gas for some reason and this is the heat released q2 to the cold resolver

So there exist two reserve wires one is hot other is cold and the working substance operate in a closed cycle between these two reservoirs absorbing heat q one from the hot reservoir releasing heat q two to the cold disorder in the process it does some walk w ok conservation of energy tells us that q one is equal to q two plus w ok and we define the efficiency of the engine that is eta which is w by q 1 that is work done by the heat absorbed from the hot reservoir and this can be written as q one minus q two by q one or one minus q two by q one now we ask the question is it possible i set q two is equal to zero this implies that efficiency of the engine will be one is this a possibility in that case i do not need any cold reservoir my machine will extract heat from the hot reservoir and convert it completely to the work question is is it possible i say it answer is no second law forbids this

So according to second law this is not possible though remember conservation of energy which is the first law that is satisfied then we talked about refrigerator again i should recall that engine and refrigerator which we are talking about they are all reversible at the moment you can have irreversible things which i will discuss at some point but at the moment discussion is entirely on reversible engine and refrigerator which means that if i have an engine operate in a reverse cycle i get a refrigerator

So what is a refrigerator refrigerator works in a reverse order

So i have again a hot reservoir t one cold reservoir t2 and working substance which is working in a closed cycle between these two reservoirs but the difference is it absorbs heat from the cold reservoir remind you t one is greater than t two

So it absorbs q two amount of heat from the reservoir t two and then dumps q one amount of it to the reservoir t one

So it extracts heat from the cold reservoir and dumps it to the hot reservoir but to make this happen we have to do some work w on the refrigerator ok again conservation tells me one is equal to q two plus w remember this is how the air conditioners work if you ever try standing close to the vent of an air condition you will find out it is releasing very hot year this happens because it is extracting heat from the room and dumping more amount of heat to the outside world which is my universe ok now we define the coefficient of performance phi which is nothing but q two by q one minus q two ok now we ask the question is it possible that i have q one minus q two which is equal to w is equal to zero is it possible if that is possible then my refrigerator will extract heat from the cold reservoir and continuously dumping it in the hot reservoir and i don't have to do any work on the refrigerator that is possible or not again this is not possible second law forbids

So you see second law takes us far beyond energy conservation energy conservation is always satisfied but still i cannot have an engine with efficiency one or a refrigerator with efficiency or coefficient of performance infinity okay now with this let us proceed to the proper formal definition of second law before that i told you that there is a possibility of having two type of machines ok first one called perpetual motion of the first kind historically physicists philosophers ask these questions one is perpetual motion of the first kind what does it mean it means can i have a machine which produces work without the input of energy i will not provide any heat energy but still i will keep on extracting work from the machine and it will be a perpetual motion in a closed loop is it possible it is not possible because first law already tells us there should be conservation of energy i cannot generate energy in an isolated system if an isolated system i have in mind in that isolated system how can i generate energy

So question is first law tells us that perpetual motion of the first kind is not possible

now come to the second law it is related to the question whether i can have a perpetual machine of the second kind what do i mean by that how it is different from the first kind it is asking the question can we convert the entire heat energy extracted from the hot reservoir to work that means i do not at all meet the cold reservoir at temperature t2 i only do have a hot resorb wire i am extracting some heat from it and converting the entire amount of heat to work if that is possible then efficiency of an engine will be one ok remember we are assuming cyclic processes

So change in internal energy is zero

So to have a perpetual motion of second kind am i violating energy conservation no energy conservation is satisfied still i cannot have an engine with efficiency one still i cannot have a machine which is perpetually working by absorbing heat from a reservoir and converting it entirely to work okay this is not possible that is why second law takes us beyond the knowledge of mechanics mechanics we say dissipation less

So i have energy conservation i have all the processes which conserve energy here in the perpetual motion of the second kind energy is conserved total energy heat energy internal energy and the work done taken together is conserved ok but still i cannot have an engine whose efficiency is one okay

So now the formal definition of second law second law of thermodynamics can be put in two forms one form is in the context of engine second is in the context of a refrigerator this is due to two great scientists kelvin and planck planck you also know the father of quantum mechanics and the root of quantum mechanics was hidden in the study of thermodynamics namely the black body radiation well what is kelvin plong statement it says that no and this is a very important word cyclic no cyclic process is possible whose sole result is the absorption of heat from a reservoir and the complete conversion of the heat to work that is efficiency of an engine not equal to one rather it is always less than one ok

So this is kelvin planck statement of second law of thermodynamics in short you cannot construct an engine whose efficiency is one ok this is what i have written here efficiency will always be less than one whatever heat you supply you get less amount of work as the output now cross here statement clausius statement is in the context of refrigerator no cyclic process is possible whose sole result is the transfer of heat from a colder object to a hotter object that means i must do some work to make a refrigerator working ok no cyclic process is possible whose sole result is the transfer of heat that means q2 amount of it if you go back to previous light q 2 amount of heat it is taking from the cold reservoir and dumping q one one amount of it to the hot reservoir and clausius statement says it is not possible without some work being done on it ok and

So no refrigerator is possible whose w is 0 here w means work done on the refrigerator and hence i cannot have a refrigerator which is perfect with coefficient of performance tending to infinity okay you can prove very easily these two are equivalent statements if you take reversible engines it is very easy to understand reversible engine operated in reverse order gives you a refrigerator

So you can immediately argue for yourself that for reversible engines these two statements are actually completely equivalent ok let us now proceed to something which is phenomenal it is called carno engine carmo engine is a reversible engine reversible i remind you all the processes are quasi static quasi static and plus there is no dissipation there is a connection between the forward and reverse process which i have explained working substance i told you there should be a working substance i will choose it to be ideal gas not necessary you will soon see its not necessary but it makes calculation easier that's why we choose ideal gas and i choose one mole again you can do animals does not make any difference again any engine and refrigerator must work in a complete cycle and i will again choose two reservoirs hot reservoir t1 and coal reservoir t2 this is the definition of a reversible engine which is known as the carbon engine i will give you one realization of carnot engine which uses ideal gas efficiency here because it is non-dissipative should be maximum but not unity that is crucial this is the crucial point that even in this ideal situation efficiency is not unity but it has a beautiful universal relation which does not depend on the working substance and it does not depend upon the way you perform your thermodynamic operations ok

So in that sense it is universal remember universal means the efficiency which will calculate ok will be independent of the working substance and in the order i execute the thermodynamic operations ok now this is taken from wikipedia which i have clearly acknowledged here you can see nicolas leonard sadik arnold carno is the right

pronunciation he was a french military engineer and often described as father of thermodynamics this is carno was a military engineer and he wrote only one publication and in which he proposed this carno engine his work was more or less forgotten before clausius and kelvin these two names famous scientist you are already familiar with when i introduced to you the formal description of the second law of thermodynamics these two famous scientists actually resurrected carnos work and now carno is known as the father of thermodynamics because he gave us a procedure to find out an enzyme ok which could have maximum efficiency but not one ok

So let us define carbon engine needless to remind reversible it involves four processes isothermal expansion

So start from the point p one v one t one in the pv diagram ok first you have an isothermal expansion which takes you from p one v one t one to p two v two t one temperature is fixed second step is an adiabatic expansion which takes you from p two v two t one but now temperature is no longer constant as i told you repeatedly in the last lecture that adiabatic process is complicated in the sense all the thermodynamic variables namely pressure volume and temperature they change

So p two v two t one to p three v three t two ok now do an isothermal x compression

So go from p three v three t two to p four v four t two isothermal thats why temperature is kept fixed and its compression

So v four is less than v 3 and

So on finally complete the process with an adiabatic compression again p 4 v 4 t 2 to p 1 v 1 t 1 what is important you started with p one v one t one using four processes you come back to p one v one t one

So you are doing a closed loop and remember few statements which i have written below the processes can be executed in any order ok i am choosing ideal gas one mole ok ensuring that initial and final states are the same

So you come back a closed loop here these four processes ensure p one v one t one at the final step at the final step and also at the initial step we shall calculate the work done and the heat absorbed over a closed loop ok and the change in internal energy is 0 over a closed cycle

So i will not bother about internal energy i will just be considering work done and the heat absorbed in every process there is a possibility of change in internal energy for example here for example here and in an ideal gas internal energy depends on temperature

So in these two processes internal energy do not change but in these two processes two and four internal energy should change in such a way that overall the change in internal energy is equal to zero ok these are all words but we should actually go to pictures and i draw a carnot engine for you

So this is my pv diagram p v this is my initial point coordinates p one v one and t one i draw two processes ok you immediately know which one is adiabatic which one is isothermal ok from the slope at this point you know this is isothermal this must be adiabatic if i want to have an adiabatic from here this is my second point let us say which is ah p 2 v 2 but t is fixed t one

So this is my first process ok here again you know this process this process takes me to p three v three and from the slope again you know that is an adiabatic process which takes me to temperature t two and finally i do again an isothermal this two point curve should meet here and it should be more symmetrical i am sorry the drawing is not perfect but let me try to make it better well roughly this you can get better picture in your books this is a continuous line

So you see this is an adiabatic process which takes you to p 3 v 3 and t 2 temperature changes this is isothermal this is adiabatic this is isothermal contraction bringing you to p4 v4 but since it is isothermal its again t2 and then this adiabatic compression brings you back to the initial point p1 v1 and t1

So please remember this is a continuous curve i try to make it better well probably better now you see the first one this is your step one two three four let us recall what were our steps isothermal expansion this is this one adiabatic expansion this one step two isothermal compression this is my step three and then adiabatic compression which is this one ok now correspondingly i draw this basic picture of an engine which we have been dealing with t one t two q one heat absorbed q two heater released to cold reservoir at temperature t two w is the work done by the engine look at this

So i stirred the system in equilibrium with the hot reservoir at temperature t one then let it expand up to volume v two ok some work needs to be done which we will calculate

So it expands to the volume v 2 but thermally in equilibrium with the hot reservoir at t 1 ok i will calculate the work done etcetera but it is very simple to realize that it is in thermal equilibrium with t one and goes from volume v one to volume v two that is why i call it expansion and this is the heat absorbed in this process q one ok now this process is adiabatic you know by looking at the slope this curve should be very symmetric which i couldnt draw now you see this is adiabatic

So in this process there is no heat absorbed okay and this process takes it to p3 v3 and t2

So it comes to the temperature of the cold resolver now it starts dumping heat to the cold reservoir i allow compression up to volume v4 in this process q2 heat is released and then finally this process adiabatic process brings it back to p 1 v 1 t 1 the initial state and the cycle continues cycle continues ok

So you see started with p one v one t one in equilibrium with the hot reservoir then there is an expansion followed by an adiabatic process which takes temperature to t two which is the temperature of the cold reservoir then i allow a compression which brings it back to the volume v four then one more adiabatic process okay this two isothermal and then finally an adiabatic process brings me back to p one v one t one these two processes temperature is fixed here it is the temperature of the hot reservoir here temperature step 3 temperature is that of the cold reservoir and 2 and 4 naturally being adiabatic there is no heat exchange and this cycle continues i will calculate now the efficiency of this engine and

So it depends only on t1 and t2 to calculate the efficiency of this carnot engine i have to calculate the work done and the heat absorbed or released in every process in the case of work done also we have to be careful whether work done is on the system or by the system by the system is positive on the system is negative let us proceed

So first step step one step one what was that let us go this is the step one its isothermal expansion we have already calculated in gory details what is the work done work done is this and you can see from this picture ok v two exceeds v one v two is greater than v one

So this is of course a positive quantity

So is heat absorbed

So q one is also a positive quantity

So as i try to physically argue that here system does some work and absorbs the heat and amount of it which is q one from the hot resolver ok second process second process is adiabatic expansion ok

So this is my process number two in this process q 2 is 0 obviously it is an adiabatic process and we calculated the work done if you remember this is the work done in this process there is a change in internal energy i recall delta q is zero but delta w is minus of d u ok these are all finite processes though

So this is the work done which i can calculate q two is equal to zero no heat absorbed let us go to the third process what was the third process third process is here i am doing an isothermal compression let us calculate what is the work done same expression as the previous one this one only thing here i am going from v3 to b4 this is v three this is v four well

So it should be v four by v three but remember as you can see from this picture v four is smaller than v three

So it comes with a negative sign where this thing is positive the sign is negative it tells me what is done only system and system releases heat

So this is releases heat it where to the cold reservoir which is at temperature t two this temperatures t one and two two they don't change because i have assumed my wires are very very big here this is heat absorbed here heat released and these four taken together gives me the net work done and net change in internal energy is zero in this process there is some change in internal energy which is compensated by the equal and opposite change in internal energy in step four ok here also its an adiabatic process

So delta w is equal to minus d u and the d u is equal and opposite in process two and process four now we have got everything can we calculate the efficiency of the cardinal engine now we can do it very easily actually we do not need to bother about the work done we can proceed simply this way efficiency of a carnot chain is work done by q1 but conservation conservation of energy tells me w should be equal to q one minus q two if i use this though i have calculated the work done this work done is not at all necessary for my purpose all i have eta is equal to q1 minus q2 by q1 which is equal to 1 minus t2

by t1 log of this and log of this how do i get this i simply get this using this expression and this expression once i use these two expressions i immediately get this result but now there is a problem problem is this expression is too complicated here it involves all the values that volume can take in a closed cycle that is v_1 v_2 v_3 v_4 the expression gets simplified only when i can get rid of this and express them in terms of the temperature that's quite easy these two processes apparently they play no role because i am calculating q1 which is involved in this process i am calculating q2 which is involved in this process

So apparently these two processes are not being useful though they are necessary to come back to p one v one t one at the end of the complete cycle but they do indeed play a very crucial role which i will show here you see adiabatic process connects two paths right step two and step four two processes now in an adiabatic path we always have $p_a v_a$ to the power gamma is equal to c_a that we have repeatedly discussed now you see that its an ideal gas

So $p_a v_a$ must be equal to $r t_a$ one mole that is why no n always on an adiabatic process

So i can substitute for p in terms of t here i can completely get p out of this equation and write the adiabatic path in t v plane if you like as $t_a v_a^{\gamma - 1}$ is c c is some other constant some other constant ok

So you can immediately see that i can write $p v^\gamma$ is equal to constant also as $t v^{\gamma - 1}$ is equal to c

So i was drawing always pv diagram and this is my adiabatic path say $p v^\gamma$ is equal to constant which is also implying that if i calculate temperature at every point $t v^{\gamma - 1}$ is also a constant ok t v to the power gamma minus one is also a constant not the same constant as c

So now go back these two paths this connects $v_2 t_1$ to $v_3 t_2$

So i must have this relation $t_1 v_2^{\gamma - 1}$ $t_2 v_3^{\gamma - 1}$ this should be always true on this path which i designated by step number two now go to step number four what do you have 4 t 2 is connected to v 1 t 1 also through an adiabatic path ok

So i must be having $t_1 v_1^{\gamma - 1}$ $t_2 v_4^{\gamma - 1}$

So this is corresponding to step two these corresponds to step four why it is useful then you can easily see it is very useful now i can write using this equation easily write v2 by v3 as this v1 by v4 has this once i have this what do i have i have here v3 by v4 and v2 by v one ok what i can do i can substitute everything

So you can see from these two equations i can immediately conclude v three by v four is equal to v two by v one

So once i have this v three by v four is equal to v two by v one when i consider these two equations together you can immediately see that v two by v v3 or i can write v3 by v4 is v2 by v1 i substitute it back here immediately i get efficiency is 1 minus t2 by t1 this is a fantastic result you see efficiency of a carnot engine is only given by t2 and t1 what is t2 and t1 i recall t2 is the temperature of the cold reservoir t1 is the temperature of the hot reservoir it depends on nothing else it does not depend on whichever way i carried the process through all i used i calculated eta through q1 and q2 which i have learnt by heart how to calculate then i ended up with a problem that involved v3 v4 v2 v1 okay that means all the values volume can take in a complete loop but that does not stop me because i know step two and step 4 they are both adiabatic processes in adiabatic process pv gamma is constant for an ideal gas i can always change it to t v gamma minus 1 is equal to constant ok immediately two adiabatic processes step two gives me this relation step four gives me this relation i immediately get v two by v three and v one by v four in terms of temperature only and that gives me immediately v three by v four must be equal to v two by v one if i substitute it back to here this expression substituted back to here i immediately get what is the efficiency it is 1 minus t2 by t1

So it does not depend on the process the way the order in which i have executed my thermodynamic processes now question comes it is always less than one why if you want it to be equal to one you must have t two is equal to zero but you know absolute zero cannot be reached we know we cannot reach the absolute zero if i cannot reach absolute zero i cannot have a cold reservoir which is at temperature absolute zero if i cannot reach absolute zero and i cannot get a cold reservoir which is at temperature absolute zero in kelvin scale as we know it is cannot be reached then i cannot have a carnot engine with efficiency one

So a reversible engine i will always have efficiency less than one this tells me whatever was the statement of second law of thermodynamics in kelvin pump plant form you can similarly run the kernel engine in refrigerator form and reach the similar conclusion it stems from the fact that you cannot reach absolute zero and then you cannot have an engine whose efficiency is one you cannot have a refrigerator whose coefficient of performance is infinity well this tells us lot about cardinal engine later i will also calculate this efficiency using some idea called entropy which is an extensive variable and something called ts diagram

So far i have been talking about p v diagram and if you know the equation of state of the gas you can immediately construct the v t diagram or p t diagram given your p v diagram ok but i will introduced a new extensive variable which is called entropy and state to you the second law of thermodynamics using the concept of entropy and redo kerno engine for you and arrive at the same result for the efficiency okay now let us propose something which is known as carnot theorem given two heat reservoirs a carnivore engine reversible engine has the maximum efficiency ok soon i will try to elaborate this first part of kernel theorem ok using some special setup and arguments second part of the theorem says efficiency of all reservoir reversible engines working between two given reservoirs given resolve as that means t one t two given two heat reservoirs that means fixing t one and t two similarly efficiency of reversible engines working between two given reservoirs fixing t one and t two is the same is the same ok these are two parts of the kernel theorem ok but what is more important in the second part regardless of the working substance it does not matter whatever you choose as the working substance i have chosen ideal gas but one could have chosen van der waals but efficiency does not change that simplest universal form of efficiency t two by t one it really does not change and regardless of the working substance employed or the operational details which means as i have been repeatedly telling you this means the order in which you perform your carnot cycle is not important for example one could have started from here p 3 v 2 t 2 or from here or from here it does not matter which order you perform the operations efficiency will be again 1 minus t two by t one and always less than one because t two cannot be absolute zero well then you can ask me why are you choosing ideal gas because it is very easy to calculate all these work done and heat absorb we have done it at length in our previous set of lectures we know them by heart its very easy what makes it easy that specific heat is independent of temperature if it is a mono atomic gas which i am always considering it is three by two n k b which is k v is the boltzmann constant

So you see it is independent of temperature well can i do similar calculation for van der waals yes you can do that but van der waals gash will be complicated here cv is a function of temperature if you assume that still life is not very complicated but it could be a function of volume also

So equations and calculations become complicated that is why we stick to ideal gas and believe me the efficiency does not matter its always determined by temperature of the hot reservoir and the cold resolver ok with this i will try to prove for you ok one part of the kernel theorem ok what is that part that part is that if i take a reversible engine and an irreversible engine which has dissipation ok then the reversible kernel engine will always have efficiency higher than the irreversible engine this part i will try to prove for you rather argue in thermodynamics the beautiful thing is its mostly based on arguments its not very mathematical all we have used mathematics

So far a differentiation and sometimes not mentioning i have used partial differentiation ok

So what is the idea consider a carnot engine c which is operated as a refrigerator which is very important the kernel engine is being operated as a refrigerator and irreversible engine i carno engine is denoted by c here universal engine is denoted by i which is here okay both are being operated between a hot reservoir temperature t one and a cold reservoir temperature t two same two disorders this is important because the condo theorem always mentions given two resolvers that means fixing t1 and t2 if you change these always change the temperature of the reservoirs carnot theorem doesn't hold true but to be precise you cannot talk about a carbon engine always you have to operate your engine and refrigerators between the same two resolvers ok

So remember first i have a carnot engine this is c this carbon engine is being operated as a refrigerator

So let me first focus on this hot reservoir t1 and this cargo engine which is being operated as a refrigerator what will it do it will dump heat to the hot resolver let us

say this is q1 and since it is refrigerator second law already tells me that i have to do some work on it its w then how much heat it must take from here conservation tells me that will take a hit q one minus w

So this is a carno refrigerator carno refrigerator well what does it do it absorbs this amount of heat from the cold reservoir which is q one minus w w amount of work is being done on it and it dumps q one amount of heat to the hot reservoir which is at temperature t one now comes the irreversible engine ok both of them are working in a complete cycle please remember it is absorbing a heat q1 from hot reservoir it is giving out a walk which is w prime and conservation tells me it must give q one minus w amount of heat to the cold resolver

So this is my irreversible engine i

So this is irreversible and this is engine this is being operated as an engine you can see it takes q one amount of heat from the hot reservoir giving out w prime amount of work and rest of the heat q1 minus w prime is being dumped to the cold reservoir at temperature t2 now the karma one is being operated as an refrigerator takes heat q one minus w from the cold resolver w amount of work is being done on it and q one amount of heat it releases to the hot resolver

So now we look at this composite system i'll let me tell you the most of the arguments of thermodynamics are based on this composite system structure and hence the having two reservoirs same is very important otherwise these arguments do not go through

So look at the composite system and let us assume the blue prime is w and ask the question is it possible what is w prime w prime is the work done by the irreversible engine what is w work done on the carnot refrigerator okay to ensure that carnot refrigerator releases q1 amount of heat to the reservoir t1 whereas irreversible engine extracts q1 amount of heat from the hot reservoir question is what is the composite system ok in a closed loop what is the change in hot reservoir ok q 1 heat is released by the carnot and q 1 heat is extracted by the reversible engine

So net change in hot reservoir is zero no change no heat absorbed no heat released now heat absorbed from the cold reservoir lets see okay this is absorbed q one minus w by the carbon refrigerator this is released to the cold reservoir by the irreversible engine

So to the cold is or where this amount of heat is released this amount of heat is being extracted from him

So what is the net net is this heat absorbed from the cold resolver it is absorbed because i have assumed w prime is greater than w

So this fellow is greater than 0 well what is the net work done that is very simple w prime is the work done by the engine should be positive w is work done on the kernel refrigerator that should be negative

So this is this is the network

So what is the heat absorbed ok heat absorbed is this from the reservoir at temperature t2 and network is this remarkably they are same and you know it is not possible

So the composite system is actually like an engine which absorbs w prime minus w amount of eight and converts the entire head to work is this possible no second law of thermodynamics says this is not possible ok second law of thermodynamics tells us this is not possible i cannot have an enzyme which extracts some heat from some reservoir here the reservoir at t1 plays no role because no heat absorbed from it or no head released from it in total

So what we have network is equal to the net heat absorbed from the reservoir at temperature t 2.

So this is an engine that values second law

So it violates second law which means w is always greater than w prime otherwise i will violate second law this implies w by q one is greater than w prime by q what is this quantity this quantity is nothing but the efficiency of the carno engine when it is operated as an engine and what is the this quantity this quantity is the efficiency of the irreversible engine which i am already using as an engine remember there is a crucial point w by q one is the efficiency of the cargo engine which in this argument i used as a refrigerator but if i operate a carbon engine as an engine then i know this is the efficiency my mathematical arguments tells me that this quantity is greater than this quantity

So efficiency of a carnot engine must be greater than the efficiency of the irreversible engine

So this is the point i will stop the lecture today what i have discussed i have told you

about the possibility of perpetual machines of two kind first kind is forbidden because it violates energy conservation that means the first law of thermodynamics second one is violated because of the second law and then i showed to you that efficiency of an engine is maximum for a carnot engine and that has a universal form which is given simply in terms of the temperature temperature of the cold reservoir t2 and hot reservoir t1
So efficiency of a carbon engine is simply given by 1 minus t 2 by t 1 and this is the maximum take any its efficiency will be less than that of the kernel engine furthermore efficiency of a carnot engine will never be equal to one that demands t two should be equal to zero which means that i must reach absolute zero temperature which i cannot and hence efficiency of a carnot engine or for that matter any reversible angel will always be less than unity this is a fundamental law of nature
So this is where i will stop today's lecture you