so welcome all of you to our next set of lectures on the properties of the atomic nucleus
 so this is a good time for us to take stock of what all we have learnt during these lectures on the
 so called modern physics which was actually initiated by the seminal work of planck in nineteen hundred
 so we start with the problem of the black body radiation and then move down to the concept of the photon which was used very effectively by einstein to explain photoelectric effect
 so we pointed out the revolutionary character of the concept of the photon and how einstein was able to use that to understand the otherwise very very difficult concept of the very very difficult results of photoelectric effect the experiments performed by hertz its lennard etcetera etcetera from there we moved on to the properties of matter i mean microscopic matter and we started looking at the structure of the fundamental constituents of matter and we discussed both matter waves due to deep broly which found a brilliant experimental conformation in the experiments of davison and gomer and then of course the bohr model the bohr model actually was able to explain
 so many
 so many empirical laws involving the
 so called black headlines passion lines bartlett lines and it also was able to give us a fairly good picture of periodic table which was empirically arranged by the chemist in order to understand chemical properties the precursor for the bohr model was of course the rutherford model rutherford did his great experiments by scattering alpha particles against gold foil and he concluded from his experiments that most of the atom is empty in fact all the mass almost all the mass is concentrated in a very very tiny region in the atom about 10 000 times smaller than the size of the atom and then the electrons are orbiting at a large distance that is compared to the size of the region where the mass is distributed in some kind of a planetary orbit bohr was able to pick up on these things and develop your model from which he was able to explain this spectroscopic data if you combine the board model together with the public exclusion principle which i briefly described to you then we would be in a position to understand the periodic table qualitatively of course understanding the periodic table fully is a really difficult task because we have to switch on the interaction between the electrons we have to switch on the effect of what is called as the spin orbit coupling
 so on and
 so forth we need not worry about that but a qualitative understanding of the periodic table for example why there are these noble gases or the inert gases why there are these halogens why there are these alkalis what are their properties these are some things which we can actually understand from there what we did was to turn on our attention to the region in the atom where most of the mass is distributed
 so let us start with a description of these things we have discussed the part of it in our previous lecture but at this point it bears some some repetition
 so let us see how it goes
 so what we do is to start with a brief description of the rutherford model and then describe the famous experiments of chatwick that is what we would like to do
 so we are now going to study the properties of the atomic nucleus
 so remember what rutherford showed us was that there is a central region where most of the mass is distributed and then if you look at hydrogen atom for

example my electron is orbiting in this let us say circular orbit
 so to give you an idea of this scale this length scale is of the order of ten to the power of minus eight centimeter ten to the power of ten is ten meter that is of the order of it and what the rutherford experiment told us was that this region this region is of the order of 10 to the power of minus 15 meters
 so as i told you the region where most of the mass is concentrated is in a very very tiny region it is about 10 to the power of minus 15 meters and now the question that we are going to ask is how do we resolve this structure
 so in other words what i want to do is to zoom it let us say through a microscope and then i want to see what the constituents are this is the question that we have to answer we know that my electron is a negatively charged particle and if you look at a more complicated atom there are more and more electrons in various orbits therefore for surely this contains positive charge this contains positive charge now the question is how the positive charge is distributed in this small region in the space and whether it is only positive charges or whether other or whether there are other particles also which is in this region and this tiny region is what is called as a nucleus and we are interested in the properties of this nucleus to be sure we know the property of at least one nucleus and that is in the case of the hydrogen atom
 so in the case of hydrogen atom you have the nucleus you have the electron and this is carrying a positive charge
 so and we call it as a proton
 so for our purposes the completely ionized hydrogen atom the positive part is what is called as a proton and there are some interesting properties which people know that the mass of the proton is roughly 2000 times the mass of the electron and equally importantly the ma charge of the proton is identically equal to the charge of the electron charge of the electron that means atom is overall neutral now atom is overall neutral not only when we look at the hydrogen atom in fact it is neutral for all the atoms in the periodic table we have about hundred items hundred and odd atoms which are listed therefore we can conclude that if you look at an atom with z number of electrons then there are z number of protons in its nucleus
 so for example let us look at the helium atom what we are going to find if you look at the helium atom two
 so this is this at number of electrons let us say at this particular point
 so you have the nucleus and you have two electrons which are orbiting which are orbiting the nucleus
 so we know for sure that z is equal to 2 here as well this is the number of protons
 so if you go by our mass relation then mass of the helium atom should be roughly twice the mass of the hydrogen atom if my nucleus consisted only of the protons but this is incorrect what we have is that it is roughly equal to four times of the mass of the hydrogen atom
 so this suggests that there must be other particles which are sitting inside the nucleus what i am giving you is a very brief summary of the discussions that we had in great detail in the previous lectures that is what i am giving you and these extra particles were what were called as the neutrons to just refresh your memory what we have to do is to look at the slides which i showed you in the last time regarding the famous experiments of chadwick
 so let us start with them
 so here is a picture of chadwick here is a picture of chadwick who did these seminal experiments maybe i flashed it to them in the previous lecture but never mind
 so what chadwick did was to bombard light elements such as boron and beryllium

with alpha particles

so the point that you have to notice at this stage is that when rutherford bombarded the atom with alpha particles they had a few kilo electron volt range let us say but here what we have is alpha particles coming with energies of about a few million electron volts is that okay

so they are much much more energy to compare to the alpha particles from there that bombarded the gold foil and where did you get these alpha particles from you got them from the radioactive decay of polonium and there the energies are typically of the order of 5 mev now when you bombard them essentially what happened was that the nucleus got completely shattered it broke and what you do is to apply electric and magnetic fields to isolate the particles which are charged from particles which are not charged that is the whole idea

so this experiment was performed by chadwick and what is it that chadwick found chadwick found that of course a large number of protons came out that should be equal to the number of yellow protons in the boron atom or in beryllium let us say and then he also found that there was a neutral radiation this neutral radiation had very great penetrating power

so what did they do they made the neutral radiation also bombard some other target and they found that the themselves could eject protons and electrons that is what they found out and therefore the big question was what was this neutral radiation consisting of it is interesting to know that by that time by this by the time these experiments were done there was already the concept of photon which was firmly established compton had given his explanation for the compton scattering effect by looking at what the momentum of the photon and the energy of the photon

so both peer curie and mary husband and wife couple they first actually wanted to identify these neutral particles with the photons very highly energetic photons however if you look at the atomic masses which i showed you just now it suggests that it should not be photon but it should be some other particle whose mass must be roughly the same as the mass of the proton that is what it is going to suggest and this is exactly what chadwick did chadwick imposed energy conservation condition momentum conservation condition and he knew the maximum energy which with a photon would be able to come out of a nucleus and he argued that there could not be photons but it has to be a new kind of a particle and by a careful analysis we do not have time to get into that what is it that chadwick found chadwick found that the new particles actually were very very close in mass to that of a proton

so as i am pointing out there mass of the neutron he called it as a neutron its about one point one five times the mass of the proton that was statwick's estimate and today very very careful experiments show that it is about 1. 001 times the mass of the proton that is the current value and given the experimental conditions of those days this is a remarkably good experiment that is what we have

so now what we will do is to accept the chadwick explanation and we shall go on to understand how we can start with the chadwick hypothesis of the existence of the neutrons in the nucleus the

so called atomic nucleus and use that to further understand the properties of the nucleus

so you should remember that physics started something with a human scale about a meter or two meters let us say order of magnitude then you go to a micron which is the size of a dust then we went to atom which is about 10 to the power of minus 8 centimeter

so another 4 orders of magnitude down now we are going another 5 orders of magnitude down to the very structure of the atomic nucleus that is the most

important thing now if you look at this slide let us summarize what all we have learnt from all the experiments that we have discussed very briefly
 so far today and from the earlier experiment from our earlier discussions where i discussed them in great detail
 so what is it that we find the atomic nucleus consists of two kinds of particles what are they the protons which have positive charge and neutrons which have negative charge and what is the most important a piece of information for us at this particular time the mass of the proton is roughly the same as the mass of the neutron in fact the mass difference between them is one part in a thousand
 so if i were to write i would write it in the following way mass of the neutron minus mass of the proton
 so that is what i want to write divided by let us say mass of the proton take the modulus value this is of the order of 10 to the power of minus 3 this is no accident in fact later when you study physics you will find that this particular idea together with other properties of the nuclear forces was responsible for introducing the concept called isospin we are not going to get into that now let us go back and look at look at our slides
 so if you look at your slide what is it that we are finding i said that the mass of the proton is roughly the same as the mass of the neutron that is what we found now there is no evidence for the existence of electrons in the nucleus this is very very important this you have to remember because later when we discuss the radioactive decay for example a neutron is not a stable particle it decays into a proton and an electron and an antineutrino you should not think that my neutron is containing an electron that is not true that is something that we have to remember
 so coming back to the slide there is no evidence for existence of electrons in the nucleus and of course there are always as many protons as there are electrons orbiting the nucleus this is a qualitative summary of whatever we have done now what we do is to look at it more quantitatively
 so as i told you the big question from the viewpoint of curie hypothesis and chadwick's experiment is what about the number of neutrons how many neutrons are there in a given nucleus and secondly chemists who studied you know all the elements in great detail and then starting with mendeley arranged all the elements in a periodic table they had these concepts of isotopes isobars and isotones
 so from this new understanding how do we understand isotopes for chemists isotopes were different elements having the same chemical property but in what sense are they different if they have the same chemical property in a similar manner what are isobars and you know what are isotones
 so these are matters of jargon empirical jargon when it comes to chemistry but starting with physics now we should be able to get a precise understanding of that and let us see how it looks like
 so this is the periodic table and in the periodic table if you were to look at it very carefully you find that each element is going to come with a large number of
 so called isotopes
 so our idea is that we want to understand qualitatively not quantitatively periodic table completely let me explain what i mean by that when you speak of the periodic table there are two aspects one is the chemistry aspect which can be understood qualitatively by appealing to the bohr model and of course quantum mechanics then there is the positive nuclear aspect you can call it nuclear chemistry or nuclear physics that is something that we have to understand if you understand the core that is the nucleus and if you understand the orbiting

particles that is the electrons then we have understood all aspects of periodic table that is what we would like to do and let us proceed first we need a notation

so the notation that i am going to employ i hope i believe that is the same notation which is employed in your 12th standard in crt textbook let us say the nucleus we are not going to look at the atom now will be represented by the symbol a x z and have defined all of them in this slide

so there are three symbols which are as characterizing every single nucleus if you hold both a and z fixed then x is called a nuclei

so please remember the word x is called the nuclei

so first x is the nucleus

so what are the examples of nucleus your hydrogen carbon phosphorous chlorine iron etc etc that is what it is now comes the symbol a which is at the top left top a is the total number of protons plus neutrons

so in chemistry it was called atomic weight is that ok but here it is the total number of protons and neutrons and there is a jargon again if you do not want to make a distinction between a proton and a neutron that is if you want to use a common word which describes either proton or neutron then what do you call it you simply call it a nucleon

so many times i will say a nucleon is roughly 2000 times heavier than a electron what do i mean by that that means whether you take a proton or a neutron it doesn't matter both of them weigh roughly 2000 times the electron that we have that is what we have in our mind

so a stands for the total number of protons plus neutrons in the nucleus and z stands for the total number of protons and that is what is called as an atomic number in chemistry now atom is completely neutral therefore z also stands for the total number of electrons in an ionized non ionized atom of course you can take an atomic and ionize you can take helium atom and you can knock off one electron you can take helium atom you can knock off two electrons that is not what i am referring to if you look at a neutral atom then this z stands for both the number of protons and the total number of electrons and then of course a minus z is the total number of neutrons

so this compact notation a x z tells me everything about the nucleus nuclei if you feel like and also about the atom provided it is not ionized provided it is in the neutral states now once you did that whatever is empirical no becomes a matter of fact you are able to understand it completely and what is that if you take two nucleides with the same z they are called isotopes if you take two nuclei with the same a they are called isobar

so bar is like weight is that okay and two nuclei with the same a minus z that is with the same number of neutrons they are called isotones isotones are not very important for us either in chemistry or physics but isobars and isotopes are both very very important and let us give some examples

so a very good example of isotope in fact a set of examples for isotope are nothing but the well known hydrogen then you have deuterium and you have tritium

so how are we going to distinguish them now look at the notation one one h that means hydrogen atom has what a a equal to one z is equal to one

so what does it tell you it tells you that the number of protons is equal to one the number of protons plus neutrons is equal to one therefore the number of neutrons is equal to zero that is what it tells me now if you look at the next isotope 2h 1 what is it telling me it is telling me a equal to 2 z equal to 1 that means there is 1 proton that is my atomic number my atomic weight is equal to two that is what i have that means number of protons plus number of neutrons is equal to two therefore there is one neutron and of course the next one is tritium in a similar manner it has also one proton but its atomic weight is

three which means it has one proton and two neutrons now we understand why they are called as isotopes because all of them have the same number of protons therefore apart from very very small corrections if you look at the chemical property each of them has only one electron which is orbiting whether it is the hydrogen or deuterium or tritium that is what you are going to find therefore what you have is they have similar chemical properties in a similar manner you can look at helium and tritium you look at you can look at three helium which consists of two proton and one neutrons and tritium which consists of what two neutrons and one proton both of them have the same atomic number

so let me write that for you here

so i want to write for example 3 helium 2 and 3 tritium 1 that is what i want to write

so here my a is equal to 3 my set is equal to two and here also my a equal to three but that equal to one now you see this tritium is an isotope of my hydrogen atom that is why i use the same notation h because both of them have the same atomic number one but it is an iso bar

so it is an isobar of three helium because both of them have the same atomic weight that is they both share the sum of number of protons and number of neutrons inside the nucleus

so when it comes to mass it is the isobar when it comes to the number of protons it is nothing but the isotope that is something that we have to remember okay now we have dispersed off with the jargon now comes the next question as to what the size of this nuclei is now this is a very very important question for us on which i am going to spend quite some time

so what we are interested is in the size of the nucleus of the nucleus of an atom

so first comes the rough scale from rutherford experiment let us say

so if i look at the volume or the radius it really does not matter radius of the nucleus is of the order of 10 to the power of let us say minus 14 to 10 to the power of minus 15 meter

so that is what i have now what i want to do is to actually sharpen it

so what i what do i want to do i want to pick up a very very powerful microscope as i told you which will magnify the size of the nucleus

so what does it do it magnifies the size of the nucleus and i want to be able to estimate its radius assuming you know it is spherical for the time being in a more precise manner that means in the units of 10 to the power of minus 15 meters that is what we want to do and in order to do that what we have to do is to appeal to the experiments now never mind if you cannot see this figure completely but what i would like to convey to you this is exactly what i discussed with you when i was looking at the structure of the atom is that this is a recent experiment relatively recent experiment performed in 2008 and what you do is to look at the scattering of the electrons mind you electron not of alpha particles against various nuclei

so you have this nuclei for example the bottom most is lead with an atomic weight of 208 the one above that is oxygen 16 the one above is 99 zirconium

so these are heavy nuclei and what you do is to send electrons which are fairly energetic

so how energetic are these electrons the energy of the electrons is of the order of for example 500 db 374 muv and 300 mbv these are really energetic electrons but you take care to see that the scattering is elastic you know the difference between elastic and inelastic scattering what happens in an elastic scattering the body is intact both energy and momentum are conserved nothing goes into the internal energy no energy is lost in an inelastic scattering something can go into an internal energy or the energy can get lost and how does

it happen in the case of electron atom scattering at this energy let us say 500 mmv it cannot cough a proton it can knock off a neutron
 so on and
 so forth nothing like that happens my initial state is the nucleus and the final state is the same nucleus that is what we are interested in and what you do is to look at what is called as the scattering cross section that is you look at how many electrons are scattered in different directions
 so here is the experiment here is my nucleus let us say 208 pb that is what i have
 so let me show you to you clearly this is my 208 pb here comes the beam of electrons of let us say some energy even 50 muv should be do go ok with us and they get scattered and we should assume that they are getting scattered inside the nucleus we are not interested in these fellows we are interested in those that are getting scattered from within the nucleus because i want to see this structure
 so i am interested in this i am interested in this now what i do is i put a detector here i put a detector here and count the number of particles which are coming as a function of what scattering angle
 so let me show that again here in a much neater figure you have the target
 so here is an electron which is coming like this
 so this is my zero scattering angle this is my theta and i am asking how many electrons are coming out in an angle theta
 so in fact i can call it as n scattered at an angle theta this is essentially going to give me information on this structure that is what this experiment is going to be all about now what do i do i look at this carefully here
 so you have to look at this slide now let me show you this slide if you look at this slide you see that the as i look at larger and larger scattering angles the cross section that is the number of particles that are getting scattered is falling very very rapidly this is a logarithmic scale this is not a linear scale 10 to the power of minus 1 10 to the power of minus 2 minus 3 minus 4
 so on and
 so forth it falls off very rapidly but the important thing is that when it falls off you see there are this oscillations there are this minima maxima minima maxima and if you people remember the wave optics that you studied in your optics course whenever there is a diffraction for example you are going to find minimum maxima minima maxima and what did davis and germa and deep brawley teach us davis and germa and d broglie taught us the most important thing about electrons that is not only do they behave like particles they also behave like waves
 so now if you imagine that actually this is like a diffraction pattern which is coming because of the scattering of this waves that gives me an idea of the radius just as there you get an information on the for example the properties of the slit when you look at the diffraction here you are going to get information about the radius or the size of the nucleus that is the great experiment of these people of course there is a long long experiment starting with the work of half statter we do not have to get into that
 so what i will do is since we cant get into a very very detailed analysis what we will do is to convert it into a statement of charge density how does the charge density behave
 so i am going to convert the information coming from the scattering cross section into charge density and here is the important thing
 so you make use of these diffraction patterns and you will find that the charge density is roughly constant and falls off very rapidly that is what you are going to find i am using the word very rapidly i am not using the word abruptly

that is something that you have to remember and this is not the result of a single experiment actually it is a result of a large number of experiments these are the names of the groups g two n l three f s u d d s l y etcetera etcetera this is an oxygen 16 and all these experiments agree that my charge density roughly remains the same and it falls off very rapidly that is what it is going to show me and this is in the unit of 10 to the power of minus 15 meters that is what i have shown here in 10 to the power of minus 15 meters

so i am able to break this into that many units 1 into 10 to the power of minus 15

so on and

so forth if you got a heavier nucleus let us say 90 zirconium everybody remembers what 90 stands for that is the atomic weight the number of protons plus the number of neutrons you see again my charge density roughly remains the same for a large distance and it again falls off and if you go to lead nucleus the charge density remains roughly constant and again falls off and what is the most important thing the range over with my charge density remains increases i move from oxygen 16 to zirconium 90 to lead and now we are interested in the relation between the atomic number the atomic weight and the distance over which my charge density remains the same now if you assume that the neutral particles are also distributed uniformly inside the nucleus namely my neutron that means my mass density is distributed uniformly over the nucleus however this up fall this rapid fall which is not an abrupt fall tells me that my nucleus does not have a fixed boundary it slowly tapers off what is an example for that a very good example is our atmosphere if you imagine that earth means only solid earth and if you for example ignore the mount everest or the valleys let us say grand canyon or whatever grand canyon or whatever then earth is a solid with a fixed radius that means it has an abrupt ending but rightly speaking we should also include the atmosphere in the earth now we know the earth does not have a fixed radius as you go higher and higher what happens the pressure keeps on falling and therefore the density keeps on falling and if you go about 200 kilometers above probably there is practically nothing but we can never ascribe a definite boundary to the earth when we include the earth in a similar manner if you look at these experiments you see that you cannot describe a different boundary but if you look at a benchmark let us say half the distance at which you know by the charge density falls off to half that can be called as the nuclear size or the nuclear radius

so if you did that what we are going to get is a rough idea of what the charge of the nucleus is and that information can be very succinctly very very succinctly captured in this please remember the results that i am showing you are results which are based on a large number of experiments a large number of empirical observations and all of it can be summarized in this slide and you see that of the two parameters a and z only one parameter plays a role namely the atomic weight and the not the atomic number this is very very important for us in studying the properties of the nuclear forces and we will come to that in a while that is something that we have to do

so what we have to do is to look at this structure what we are going to assume is that my nucleus is roughly spherical in fact more careful experiments also reveal the ellipsoidal nature of some of the nuclei we are not going to get into that the important point is that the radius goes like one third of the power of the atomic weight

so if you assume that the mass is uniformly distributed then mass goes like linear function of a and volume also goes like a linear function of a now what is it that you are finding there are there is an empirical parameter r naught and this r naught is given by 1.

25 into 10 to the power of minus 15 meters

 so we are looking at everything in units of 10 to the power of minus 15 meter and there is a special name just as one angstrom is 10 to the power of minus 8 centimeters or 10 to the power of minus 10 meters this has a special name after fermi the great enrico for me

 so 10 to the power of minus 15 meter is 1 for me and just to tell you that this formula is empirical my radius of the proton is 0.

85 for me it is not exactly 1.

25 that means this formula does not hold very accurately for a proton it is some kind of a rough estimate of what the size of the nucleus is

 so what we have is the following relation which is very very important for us and from this you can make a whole lot of estimates

 so what is it that i wrote r is equal to r naught a to the power of one third

 so r of a and mass is given by m naught into v why is it

 so because my mass is the density multiplied by volume we already said that this density is a constant therefore this is rho constant into r cubed of a which is rho constant into r naught cubed which is another constant into a

 so this is my m naught that is what i have this is very important for us and from this you can estimate a whole lot of things

 so for example if you are given radius of let us say 16 o we should be able to get let us say radius of 12 carbon we should be able to get that how do i do that radius of 16 o is r naught into 16 to the power of one third and radius of twelve carbon is r naught

 so let us say i know this into twelve to the power of one third what do i conclude from this from this i conclude that radius of twelve carbon divided by radius of oxygen is nothing but 12 by 16 to the power of one third

 so experimentally if somebody measured this that means if somebody determined or not you will conclude radius of twelve carbon is twelve by sixteen that is three by four to the power of one third into radius of sixteen and this is very very close to one whatever

 so there are many many minute changes but this is something which you can actually experimentally verify in fact this formula was derived by looking at the experimental data

 so this is a very very important thing for us

 so what do we know about the nucleus

 so far protons neutrons both of them have size of about 10 to the power of minus 15 meters that is what we have and the nuclear size is r naught a to the power of one third notice when i am writing a to the power of one third that means i am sensitive to changes in the scale of femtometer for me

 so you can read fm as either femtometer or fermi because for me stands for a femtometer now that means the initial statement that i made my proton and neutron have roughly the same mass that wont work out anymore because now i should be equally careful about the masses of the nuclei

 so what we shall do is to go down today go on to the next topic of great interest and that is masses of nuclei in fact the fundamental problem in nuclear physics is to determine the mass of a nucleus the magnetic moment of a nucleus the total spin of a nucleus and how the orbits of electrons and protons inside nucleus are there what is the complete structure and that is a very very difficult problem but we do not have to worry about it at this particular stage because our interest is actually in obtaining a qualitative understanding with basic ideas of scattering wave particle duality uncertainty principle

 so on and

 so forth that is what we are interested in and in order to understand that we need a break that is a very very important thing and what is that break that

break is of relativity this break is that of relativity but then you know and i am equally conscious that you are not going to study relativity in any of your courses but never mind if you look at your 12th standard ncrt book the famous mass energy relation is given there

so what we can do is to try to get an understanding get a feeling of how you get the mass energy equivalence and what are the changes responsible for that and how to formulate it we are not pretending to teach relativity to you but we are just giving some basic facts

so that you get a better feeling for what is happening

so let us start with some simple relations and let me give you a few matters of fact and let me collect them all of you would have read popular books on relativity heard popular lectures and relativity and what is the first thing that you know the first thing that you know is that no material particle can move with the speed of light let alone exceed the speed of light now notice i carefully use the word material particle what do i mean by material particle electron proton neutron atom nucleus molecule earth ball whatever whatever what is an immaterial particle then whatever we thought was waves for example electromagnetic waves when quantized become photons of course electromagnetic wave moves with what the speed of light and therefore photons also move with the speed of light

so we want to make a distinction between that kind of a particle from the material particles whatever we understood from classical physics therefore we say no material particle can move with the speed v greater than or equal to c however there is no restriction on the energy that your body can possess why if i keep on pumping energy the energy of the body keeps on increasing there is no restriction

so we have been a problem and what is the problem if speaking classically let me come back to this if i look at for example newtonian case p equal to m v and d equal to half mv squared that is what i have where v is my speed velocity if i put a vector sign now if i take a particle and put it a uniform field what do we know my v equal to a t therefore my v will continuously increase and eventually exceed the speed of light that is the newtonian case but mr einstein tells us or even better experiments tell us that no material particle can move with a speed greater than that of light that means this expression is wrong as it is written now why is it that v is equal to a t remember a is the acceleration

so what you do is to write dp by dt is equal to m dv by dt is equal to f that is what you are writing and then you write a is equal to f by m that is what you write and the most important thing is you take m to be independent of speed independent of speed that is what you do and therefore you will write v of t is equal to f by m into t plus some v naught that is what you write

so this integration that is the most important thing assumes that my m is independent of speed but if my m is independent of speed this can become greater than c for sufficiently large values of time

so what do we conclude from that if you come back to this slide what you will notice is that my momentum cannot be given by m v where m is independent of speed or velocity by for the same token my energy also cannot be given by e equal to half mb squared

so the solution to this problem that i can keep on increasing the momentum of a particle that i can keep on increasing the energy of a particle but i cannot keep on increasing the speed of a particle such that it hits c or exceeds the value of c is that my inertia or mass should depend on the speed this is the basic idea

so once we admit that let us assume that mr einstein did all the hard work for us you make your inertia as a function of velocity

so when you make your inertia as a function of velocity the new definition of m at any velocity will be given by m_0 into 1 over root of $1 - \frac{v^2}{c^2}$

so there is an additional factor 1 over root of $1 - \frac{v^2}{c^2}$ that means that the usual not on a notion of inertia which newton gave us holds only when v equal to c or when v is very very close to c

so let us come to that what is it that i wrote i wrote $m(v)$ is equal to $m_0 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ now if v is very small what does v equal to c small small mean $\frac{v}{c}$ is much much smaller even the earth which is going around the sun with a very very great speed even for that $\frac{v}{c}$ is very very small you can verify that speed of that is about 30 kilometers per second or whatever you can make a binomial expansion and what do you get you get $m(v)$ is approximately m_0 into $1 + \frac{1}{2}\frac{v^2}{c^2}$ which is roughly m_0 for small $\frac{v}{c}$

so that means in writing the correct equation we are not completely dismissing the newtonian laws after all we know that they work extraordinarily well in the laboratory scale in the terrestrial scale that is because this $\frac{v}{c}$ is a very very small quantity that is what we have in a similar manner what we can do is to know write the expression for the momentum

so expression for momentum is now very very simple i will write $m(v)$ into v if i write this expression no you see there is no contradiction at all i can still write $\frac{dp}{dt}$ is equal to f constant but when i try to integrate this $m(v)$ will also come

so $p(t)$ is equal to $f(t)$ into t is equal to now i will be careful $m(v(t))$ into $v(t)$ that means but $f(t)$ is equal to constant

so i should not put this at all i am sorry about that otherwise i will have to integrate that $f(t)$ is equal to constant therefore $p(t)$ is equal to f which is a constant into t this is the correct expression you see my momentum will be increasing linearly as a function of time but my velocity will not be increasing linearly as a function of time because p increases linearly but when p is increasing my mass increases my speed increases or velocity increases let me put a vector sign over this such that the linearity in p because does not become a linearity in v and as time passes my velocity will asymptotically approach the speed of light the velocity of light those of you who can do this integration are welcome to do it it is not a very very difficult thing and this is something that we are going to get this is a very important relation for us you may be wondering why i am discussing all this in this particular set of lectures the reasons will become very clear to you in a minute and by the same token my expression for energy as a v will now be given by $m_0 c^2$ divided by square root of $1 - \frac{v^2}{c^2}$ that is what i have this is the famous einstein mass energy relation e equal to mc^2

so if you want i can write it as $m(v)$ into c^2 again if you were to make a binomial expansion of this quantity what will this quantity be this quantity will be nothing but $m_0 c^2$ approximately into now i will be careful $1 + \frac{1}{2}\frac{v^2}{c^2}$ plus higher order terms which is $m_0 c^2$ plus half mv square $m_0 v^2$ and this you recognize to be the newtonian expression what was the genius of einstein the genius of einstein was that he did not treat this as an unimportant constant which cannot be measured because we know only that energy differences are measurable but energy is not measurable but he gave a precise meaning to this and he said even when the particle is at rest it carries a lot of energy given by $m_0 c^2$ for which actually there is an extraordinarily good experimental evidence is that okay pair production pair annihilation and

so on and

so forth but again if you look at processes where the identity of the particle is maintained that is it is maintained what matters is only half mv square for very very small speeds therefore mr newton is again in safe place all that is summarized in this particular slide we shall look at it

so the first expression is the expression for the relativistic mass m naught into 1 over root of 1 minus v squared by c squared by the way this quantity is sometimes called gamma and v by c is called beta do not worry about that then my energy is m of into c squared i describe to you my momentum is m of e into v if you combine these two relations together you get a very beautiful relation e squared equal to p square c squared plus m naught square c to the power of four now this is something which is very very beautiful i am not going to get into it because although we got this relation assuming that m naught is not equal to zero this has non trivial solution v even when m naught is equal to zero that is e squared is equal to p squared c squared plus m naught square c to the power of 4 has non trivial solutions even when m naught equal to zero because i will get e equal to p c

so if you write a equal to p c and if you write your velocity to be d e by d p what do you get this is nothing but c that is what you are going to get therefore this takes care of light also that was the great great accomplishment of einstein when he gave this relation this is also called mass energy relationship

so now whatever i showed you through the binomial expansion i want to show you through a slide

so that you people understand what is happening

so in this figure you see my beta v by c is changing the maximum value that it can take asymptotically is 1 when v equal to c over a long range by m by m naught my mass does not change at all and only when it hits something like 0. 8 that is when the speed of the particle is point i times the speed of the light it starts increasing therefore neutron in mechanics is safe what we want to now do is to start with this combine it with the mass energy relation introduce the concept of mass defect and show you how the mass property of the nuclei can be understood and from that how fission fusion radioactivity can be understood which we will take up in the next lecture good you