

so here we start our second part of this topic on problem solving in quantum physics of atoms and this is question 5 with us scientists observing the absorption spectra from a distant galaxy hypothesized a hydrogen-like atom with the spectral lines matching those of terrestrial hydrogen but this hydrogen was different and in what way let us see in this unique atom the gravitational the electrostatic interaction was replaced by gravitational interaction in a hydrogen atom you have the electron and the proton interacting with each other with electrostatic forces or electrostatic potential that has been replaced in this unique hydrogen-like system by gravitational interaction

so both the species are electrically neutral but we for the sake of convenience we still call them e and p e and p don't stand for electron and proton which we just call them e as the particle which goes around the lighter particle and p the heavier particle which is at the center which interact with each other based on gravity gravitation now the question says the two electrically neutral interacting species called e and p had masses f times the masses of electron and proton respectively

so once again we have a heavier particle in the center a lighter particle around it based on this and the fact that the spectral line is identical even though the interaction is much weaker on that on that length scale find what should be f given that the mass of electron and mass of proton are as provided let's proceed with the solution of this problem to solve this problem we need to first note that that this what does it mean by hydrogen like why are we saying this is hydrogen like right the both the interactions the electrostatic interaction as well as gravitational interaction the forces go as $1 \text{ upon } r^2$ the nature of forces is identical and therefore the symmetry of the potential is identical whether you have the gravitational problem or you have the coulomb problem because this symmetry is identical your solutions of the hydrogen atom are also identical the hydrogen like atom which is based on gravitation

so it has identical solutions but which differ by certain constant values because the expressions for gravitation and the expression for coulomb interaction they both differ by some constant values

so let us apply this difference and see what we get you have the expression for radiation associated with the longest wavelength of the lyman spectral series of hydrogen

so i would write for that the general expression where n_f and n_i are subscripts identifying the final state and the initial state associated with the transition is equal to the readable constant with let me help you see this part i shall write this again at the bottom i hope that is fine

so the difference between the gravitational interaction and the electrostatic interaction would essentially lie over here in the redbird constant we will still get energy levels going as something upon some number upon n^2 for this system which has gravitational interaction because i mentioned that the force still goes as $1 \text{ upon } r^2$

so the form of the force is same

so the spacing between the energy levels is same and a similar expression would also hold if the interaction was gravitational in nature but the constants which are sitting in the ribber constant would be different and how different would they be let's have a look r for your hydrogen atom is given by $w \text{ upon } h c$ where e n is minus w upon n^2 or minus 13.

$6 e v \text{ upon } n^2$ w has the expression as $m e e \text{ to the power } 4 \text{ upon } 8 \text{ times } \epsilon_0 \text{ naught square the permittivity square times the square of the planck constant and this is how we get the 13.}$

6 ev

so when we have gravitational interaction how do we modify this w or effectively how do we modify this r whatever I have written here is for the electrostatic interaction which is your hydrogen atom this is not the hydrogen atom that the question states they discovered hydrogen like atom we'll come to that in the next slide

so your electrostatic interaction with all your constant terms would look like $\frac{e^2}{4\pi\epsilon_0 r^2}$ and that is replaced by $\frac{g m_1 m_2}{r^2}$ this part is common and if you look at $\frac{e^2}{4\pi\epsilon_0}$ this is the term that appears in r in the value of the Rydberg constant when comparing these two expressions which are written at the top I would say that it should be replaced by $4\pi^2 g^2 m_1^2 m_2^2$ if I make this replacement I would get the correct Rydberg constant of my system

so w becomes mass of the electron times g squared times mass of the electron squared mass of the proton squared and then you have this 4π the whole square divided by $8h^2$ and from there I can write the Rydberg constant as w upon hc now I can call this w with a tilde to distinguish it from the ionization potential of the hydrogen atom because this is a hydrogen like atom with gravitational interaction

so I've added a tilde over here to distinguish this from the pure hydrogen atom which is based on electrostatic interaction let's go to the next slide you may wonder is there a direct way of getting this expression I have just compared the two forces and said this is how their terms are different and that's how I wrote down the w with a tilde or r with a tilde but if there is a direct way what we would do is the following we would compare in this case the centripetal force with the gravitational force because that is what is keeping the atom stable there is an attractive force and then there is a force which is sending it out when you compare this you have another expression which is your Bohr's quantization condition and

so these are the two expressions which are available with me I can uh solve the first one and I will arrive at from 1 I will get v is equal to $2\pi g$ mass of electron mass of proton divided by nh and further I can get the r to be $n\hbar$ upon $m v$ but here it's mass of electron there are two masses times v and this will give me $n\hbar$ into $n\hbar$ upon $m_e g n m_p$ you will note that this part of the expression has been used over here and the result for the velocity has been used over here

so what does it tell for r r is $n^2 \hbar^2$ upon $g m_e^2$ times mass of proton that brings us nearly to the end of this calculation let us write down the expression for u

so u which is the potential energy under gravitation will be $g m$ of electron mass of proton upon r the gravitational potential the kinetic part will be minus half of u and your total energy will be which is E is K plus u and that will be minus g mass of electron mass of proton upon 2 times r we have found the value of r in the previous slide and let us plug that in here to obtain that your total energy is minus g^2 mass of electron cube mass of proton square upon $2\hbar^2$ and the important thing here is n^2 that tells us that the dependence of the energy levels uh with the quantum number is just the same and here I will write down the expression for the wavelength radiated by this atom and that will be given by λ upon hc into 1 upon and final square minus 1 upon an initial square and this system is quite similar to the hydrogen atom except the difference sits over here and another important thing to note it is given in the question that the masses of these particles which have no charge are in the same ratio but much higher and higher by a factor f now if each of this mass of the e particle and the p particle are f times higher then the total

mass would be higher by f raised to power 5 and this is the factor that would sit inside ω tilde w tilde going to the next slide we can write λ of the hydrogen atom divided by λ of this new atom is in the ratio of w tilde upon w and this will give us $g^2 m_e^3 m_p^2 f^5$ upon 13 .

6 eV and all the constant values are given to us

so we can plug them in and obtain the answer as 0 .

2×10^{-78} and f^5 .

but then it's given that the wavelength of the transitions are actually the same that even the system where the fundamental particles had no charge and they only interacted with gravitation they emitted the same radiation

so this is equal to 1 and from here it follows that f is 5 times 10^{-77} raised to power 1 upon 5 or the masses are in the ratio of 3.

4×10^{-15} which completes the problem and this shows that the gravitational interaction is extremely weak though it has the same symmetry and it would lead to the same kind of structure in terms of the spacing of the levels if such a system could have existed it's only a hypothetical system but then to get a comparable wavelength of transitions or to get a comparable spacing between the levels the masses should be extremely large and at the same time they should be confined to a very small space

so this the mass of the electron and the mass of the proton should be heavier by

so much amount that's that's a huge amount let's go to the next problem

question 6 asks a different question what is the percentage error in the estimate of the frequency of the first bohr line if the proton is taken to be fixed at the origin instead of having a motion about the electron proton center of mass it's a two body system

so you would expect that not only the electron goes around but there is certain motion associated for the proton as well

so this is what you expect for a two body system but the mass of proton is much much larger than mass of electron

so for most purposes you can just neglect this small motion on the other hand there will be some error when we do this neglect when we neglect this motion and this question asks us to find what kind of error would be there in the frequency value of the first bohr line

so the frequency of the bohr line λ_{bohr} is 2 and the first bohr line is given by the final state 3 is equal to the redbird constant and 1 upon 2 square minus 1 upon 3 squared and my error will sit in r because r is given by a mass of the electron times electron to the power charge to the power 4 8 times epsilon naught square $c^3 h$ and my error is sitting over here

so let me calculate that in the next slide we have to begin with the effective mass $1/\mu$ which is $1/m_e + 1/m_p$ and that is equal to $m_e + m_p$ we can plug in the values and what we obtain is μ is 9.

0.99×10^{-31} kg and noting that the mass of the electron is actually 9.

1×10^{-31} kg this is the difference which accounts for a slightly shifted value or estimate for the radiated line and by how much

so let's find it out numerically your $h\nu$ is r times hc 1 upon n_{final}^2 minus 1 upon $n_{initial}^2$ and your $\Delta\nu$ by ν will be Δr by r because the other quantities will be constants and within Δr you will have the error associated with Δm alone because again in r the other quantities such as the charge of the electron or the permittivity speed of light planck's constant are all taken to be constant values

so this is clearly point zero zero one upon the mass of electron being 9.1 this is the deviation from the mass of electron if you consider the reduced mass instead of the mass of electron and that gives us a value about 10^{-2} raised to minus 2 percent

so we can get the value of the shift in the bomber line based on this correction that essentially completes this problem and let's move on to the next one the emission spectrum associated with the de-excitation of hydrogen atoms is recorded using a grating for transitions associated with the first line of the bomber series a first order maxima is observed at an angle 20° degrees what is the spacing between the slits of the grating the question has two parts in the first part we just have to find what is the wavelength involved because this is the first line of the bomber series then this light is incident and we observe a first order maxima at a certain angle and then we are required to find what is the spacing between the slits of the grating which gives us this maximum

so this question is actually trying to combine two concepts one associated with the quantum physics of the atom and other associated with the optics of the system where you have electromagnetic radiation incident on a grating and you look at the diffraction produced by the grating

so the first part should be simple we have bomber line

so the n is 1.

and your wavelength associated with this line one indexed with the final comma initial state will be r times $1/2^2 - 1/3^2$ and this will give us $5/36$ okay let me go to the next slide to continue this problem

so we have $1/\lambda$ is $5/36$ times r and we can plug in the value of r here the Rydberg constant 1.097×10^7 centimeter inverse and what we get is 1.52×10^7 centimeter inverse or the wavelength is approximately 6563 angstroms let's go to the next slide having found this wavelength you have now this array of holes which form the grating these openings which form the grating and we don't know what is the separation d between the gratings between the openings of the gratings and we have a screen over here

so let me draw the incident radiation and this radiation gets deflected at an angle θ the diffracted ray because each point is a source of secondary wavelets

so at angle θ we have to see whether we get constructive interference or not and one can do that the expression for the uh constructive interference is this where m is m is an integer and here it is given to be first order

so m is equal to 1.

with this and knowing the λ we can find uh what is the spacing between the slits for which i shall go to the next slide

so you have d is equal to $6563 / \sin 20^\circ$ because m is equal to 1 divided by $\sin 20^\circ$ which is 0.34 and it gives a value 19320 approximately angstroms or the spacing should be 1.932 micrometers

93 micrometers give us the first order maxima at 20° degrees let's go to the next problem assume a classical collision between an alpha particle and the gold nucleus alpha particles emitted from radium with average velocity of 2×10^7 meter per second are brought to a momentary halt at a distance d from the gold nucleus if Z is 79 for gold find the value of d and we are given the charge of the electron and we are given the permittivity value of free space before we imagine the solution of the problem i would like to make a comment that this is a very much a classical solution it's not a quantum mechanical scattering problem which would be a more exact way of doing this nevertheless we will proceed with this simple version you have this alpha particle and this is

the nucleus if it comes very close to the nucleus it can of course get back back deflected and there is a very small probability a very small number of alpha particles are back reflected and this is a very surprising experiment when it was done for the first time by rutherford he said that it's you can imagine it that you are firing cannon balls onto a wall and some of them just rebound back and come to you and which is something that usually does not happen in the world around us but in the quantum world it does happen and a very simple classical explanation of this is that here this alpha particles they have certain kinetic energy when they start off and the gold nucleus you can you know there are arrays of gold nucleus if it's a crystal the gold nucleus are stationary the gold nuclei are stationary and their kinetic energy is zero whereas the potential energy of their interaction between the alpha particle and gold is zero because they are far away when the alpha particle comes very close there is a lot of interaction in terms of the electrostatic interaction between the alpha particle and the gold nucleus and the kinetic energy is converted to that interaction potential energy

so we will be equating the kinetic energy to the potential energy of electrostatic interaction the initial kinetic energy is simply mass of the alpha particle times the velocity of the alpha particle square upon 2 and this will be equated to the potential energy which is the charge of the alpha particle charge of the gold nucleus divided by $4\pi\epsilon_0 d$ and our interest actually lies in this quantity called d we have to find at what t they become equal because when they become equal entire kinetic energy has become the potential energy

so let's go to the next slide and where we will try to find what is d from the last expression d will be equal to $\frac{2e^2 z^2}{4\pi\epsilon_0 m v^2}$ and noting that the alpha particle is helium 2 plus or you can write it as $4He^{2+}$ we have put this $2e$ for the charge and $z e$ for the gold and plugging in all the values which are given to us and noting that the m here will be four times the mass of the alpha particle will be four times the mass of proton plugging all these values we will get d equal to 2.8×10^{-14} meters i leave that as an exercise that you can plug this in and arrive at this answer you