

welcome to this class on solving problems in quantum physics of atoms this topic is a very interesting topic it laid the foundation of quantum mechanics a very elementary treatment of how an electron goes around the nucleus you can imagine it's like a planet going around the sun it has a classical trajectory and this is how people actually thought the electron was going around the nucleus now this approximation cannot take us very far the actual image or the actual perception of what is there is given by these wave function or rather the probability distribution plots the electron is really not going around in a classical trajectory but rather it has a certain probability distribution and it can be found here or there and as you can see in this image this is the distribution of the electron around the nucleus of a hydrogen atom in its different energy states

So the picture is very different as compared to our classical imagination there are no trajectories in quantum mechanics then once you have discrete energy levels what do we mean by discrete energy levels if you consider a ball that has been thrown or if you consider the earth going around the sun we treat the energy of this object as a continuous quantity it can take all possible possible continuum values of energy energy could be 2.1 joules 2.11 joules 2.111 joules 2.112 joules all continuum values but for the quantum system such as this atom the energy values that it can take are discrete So there are well defined energy levels and then there are transition between the energy levels when say the electron absorbs some electromagnetic radiation it can go to another level or it can emit the electromagnetic radiation and come down to energy another level and these levels are very well defined

So this absorption spectrum or emission spectrum one can get for example as the one shown here below which is the spectrum for the hydrogen atom and then there are very well defined precise uh lines associated with the spectrum such as the the lyman series the balmer series the bastion series and

So on we'll begin by solving some elementary problems before we take up a little more difficult ones

So here is the first problem the first line of the bomber series of the hydrogen atom has wavelength λ approximately 6550 angstroms the actual value is a little different but for ease of calculation i have taken this approximate value the question says find the wavelength of the second line of the larger frequency now what does it mean first line of bomber series of course there's a series but how do you know where to begin with is it this end or the other end high frequency end or low frequency end

So let's look at the obama series and how it originates you have these are the discrete energy levels and as you are close to the vacuum level the energy levels are spaced closer to each other and then when you go to lower energy levels or to the ground state you have larger separation between the energy levels

So the energy levels of the hydrogen atom this scale as minus w a constant i'll talk about later upon n square where n is an integer

So here we have n equal to 1 which is the ground state n equal to 2 n equal to 3 and

So on up till n equal to infinity we are talking about the bomber series and where exactly is this bomber series well let me draw first for you the lyman series the lyman series corresponds to transitions where the final state is the ground state

So you can have a transition like this you can have another transition like this a third transition like this and these are having different frequency because the energy associated with these separations are different and what about the bomber series

So let's have a look at that the bomber series corresponds to transitions to the first excited state

So you can have the following transition and the transition from the next higher level and to the next higher level and

So on

So this is the bomber series the question says the first line of the bomber series

So which is the first line should i begin from this one which has the shortest frequency or should i begin from the other end now if you look at the energy levels as i mentioned they are getting closer and closer to each other as you are going to the higher energy levels

So all of them actually have nearly the same frequency or same wavelength and that corresponds to λ limit

So the first line is actually defined to be the longest wavelength line and which is this one

So we have the final state for the bomber line as n_f equal to 2.

So let's apply this uh solution here for bomber series we noted n_f is equal to 2 and the n_i initial that is the initial energy level can be 3 4 5 etc we are given the first line has a certain wavelength and we are required to find the wavelength of the second line which has larger frequency

So we will make use of this expression uh which essentially originates from the fact that the energy levels scale as $1/n^2$ and $E = h\nu$ and $\nu = c/\lambda$

So you can see you can connect these for the n energy level and now we are looking at the difference of two energy levels

So from there it follows that $1/\lambda = R(1/n_f^2 - 1/n_i^2)$

So that's the general expression and of course R is your Rydberg constant

So let's solve this problem you have $1/\lambda = R(1/2^2 - 1/3^2)$ and that is equal to $R(1/4 - 1/9)$ or let me write it here $1/\lambda = R(5/36)$ similarly $1/\lambda = R(1/2^2 - 1/4^2)$ and what is that that is $R(1/4 - 1/16)$

So it's the transition from $n = 4$ to $n = 2$ and that is equal to $3R/16$. In this case we are already given one of the lines we are given the first bomber line and the wavelength of that is given to us we need to find the other one and we don't even need the Rydberg constant to solve we have $1/\lambda = R(5/36)$ into $16/3R$ and that canceling out the terms you get $20/27$ I'll write it over here and from this $1/\lambda = 6.55 \times 10^7 / 27$ and that is equal to four eight five zero angstroms

So that completes the first problem and this line of course is called the H-alpha line and this is the edge beta line now let's go to the next problem this problem says if the first line of the bomber series has the wavelength 6550 find the wavelength of the first line of the Lyman series of hydrogen spectrum now this question is very similar to the previous one

So I could quickly go over it you have $1/\lambda = R(1/n_f^2 - 1/n_i^2)$ and here for the Balmer series what you have you have $n_f = 2$ and $n_i = 3$ because we are given the first line then for the Lyman series where we need to find we need to find this wavelength we are given $n_f = 1$ that defines the Lyman series this fact like just how $n_f = 2$ defines the Balmer series and it's again the first line of the Lyman series

So it will be $n_i = 2$.

So we can proceed with this and let's have a look here this becomes $1/\lambda = R(1/2^2 - 1/3^2)$ and if you plug in the values what you get you get the answer as $1/\lambda = R(5/36)$ and $1/\lambda = R(1/1^2 - 1/2^2)$ because now we are looking at the Lyman series and this gives us $3R/4$.

So combining these two equations I can write $1/\lambda = 5/27 \times 2/3$ and that is approximately 1210 angstroms

So this completes the second problem and now let's look at a different problem the third question says assume that the velocity of an electron in the first Bohr orbit can be defined by Bohr's angular momentum postulate if it is defined that way find its velocity and comment if the electron is relativistic now if it was relativistic there will be some kind of a self consistency involved here I will touch upon that but here as I mentioned the electron has a probability distribution if you go to the more exact theory the question says assume that it is going around like a classical object which it is not doing with that assumption you should try and find the velocity in the first Bohr orbit and we are required to make use of the Bohr's angular momentum postulate certain constants are given here to complete the solution

So let us begin with this solution of this problem as a short remark about what actually is happening in the atom is that you don't have the situation where you have nucleus and the electron going around in an orbit right what you have is the nucleus and there is a certain what can say probability density certain probability that the electron can be found here or there and when you learn advanced quantum mechanics you will try and find it using something that looks like no but this is for later you will find the expectation value of the velocity operator or the momentum operator divided by mass in the first

excited state because if it has a probability distribution it's really not going in a trajectory it's a good approximation to say it goes in a trajectory for certain purposes it does reproduce some results which are correct but does not reproduce all the results for today's class we will stick to the elementary theory and let us proceed with that So in the elementary theory you have a bohr's angular momentum postulate which says angular momentum mvr the magnitude of that is quantized as n times \hbar where \hbar is the planck constant and \hbar means you have to divide it by 2π the it is given that n is which is the integer is equal to 1 in this present case because we are looking at the first bohr orbit and we are required to find what is r what is the radius of this orbit So the centripetal force has to be equated to the coulomb force the electron is experiencing a coulomb force from the attraction to the nucleus they are oppositely charged and when it executes a circular motion there is a centripetal force when the two are balanced you have a classical picture but on the classical picture i am imposing this quantum mechanical requirement uh that the angular momentum should be quantized it can only take discrete values right

So this is the balance of forces and we will proceed here

So let's equate $m v^2 / r$ which is your centripetal force and this is equal to $e^2 / (4\pi\epsilon_0 r^2)$ and we can proceed and write $m v^2 r = e^2 / (4\pi\epsilon_0)$ or we can write $r = e^2 / (4\pi\epsilon_0 m v^2)$ right then the next step

So here we have we can write $m v r = n \hbar$ from bohr's quantization of angular momentum quantization or bohr's postulate now we have from here all that we need to solve for the velocity and what is the expression for velocity it becomes $e^2 / (4\pi\epsilon_0 m v^2) = n \hbar / m v$ and that is equal to your $n \hbar / (2\pi m r)$ from bohr's quantization of angular momentum

quantization or bohr's postulate now we have from here all that we need to solve for the velocity and what is the expression for velocity it becomes $e^2 / (4\pi\epsilon_0 m v^2) = n \hbar / m v$ into h So we can plug in the values we are already given charge of the electron we are given the permittivity of free space and we are given the planck's constant and when we do that what do we get we get a value point 2.19×10^8 meters per second So that's the velocity that the electron has in this orbit the first orbit the first bohr orbit this value can be seen to be approximately three times the velocity of light upon 137

So it's actually much much less than the velocity of light therefore it's non-relativistic and it is therefore self-consistent because we never assumed it to be relativistic it's clearly a case of non-relativistic where your velocity is much much less than the velocity of light the velocity of light as you know 3×10^8 meters per second this question can also be done in a different way and the different way is if we know uh what is bohr radius

So to be exact the bohr radius will be introduced to you in uh advanced courses where you will learn that this is the place where the wave function goes to 0 as a function of r and this is your bohr radius r_n if you look at the solution of the hydrogen atom problem but then it gives an idea of the extent of the size of the what you call the the wave the wave function or the radius of the orbit and if we take the first energy level and assign the radius to be equal to the bohr radius although it's actually a probability distribution i can solve this question a little more easier it's $n^2 \hbar^2 / (2\pi^2 m r^2)$ and if i know r i can solve this problem and i can obtain the same answer the bohr radius is 0.52 angstroms

So let's go to the next problem which is question four here collisions knock off the lowest energy electron in a helium atom and raise the remaining electron to various excited states if the resultant helium ion is irradiated with a broadband wavelength source that is it contains a lot of wavelengths then find the largest wavelength that would be absorbed and to help us we are given the value of the planck's constant in different units in joule seconds or in electron volt seconds and we are given the speed of light

So let us see how to solve this problem we first note that helium is the $1s^2$ system it has two electrons but then we have knocked off one electron

So it becomes He^+ and that has only one electron

So it is isoelectronic it has the same number of electrons as the hydrogen atom

So the physics of the hydrogen atom actually the symmetry of the problem because there is only one electron the symmetry of the problem is preserved and you get identical solutions but there is a difference that the nucleus still has double the charge and then you will write the energy levels of this problem to be equal to $-Z^2$ Z being the

nuclear charge uh times on the nuclear uh charge associated with the fact that there are two protons there times the Rydberg constant $1/n_f^2 - 1/n_i^2$ and z is the number of protons with this we can know the transitions involved in the helium ion which is isoelectronic now we have we are given the fact that what is we have to find the largest wavelength which will be absorbed

So among all the wavelengths what would be the largest wavelength we have written out the general expression and let me write down this expression uh with the numerical values $n_f n_i z$ is known to us becomes 2^2 and the Rydberg constant is not given but if we don't know we can use the value 13.6 eV which is the ionization potential of the hydrogen atom divided by h into c if you look at the solution of the first problem I have discussed that the Rydberg is actually in terms of this ionization potential divided by h times c which comes from the dispersion relation of electromagnetic radiation into this expression $1/n_f^2 - 1/n_i^2$ because we are looking at the longest wavelength we will have to set the final n_f to 1 and the initial n_i to 2 and that would give us the longest wavelength with the quantities that are given to us we can plug it plug it in and we will obtain $1/\lambda R$ and it is equal to $4.14 \times 10^{-15} \text{ eV} / (4.14 \times 10^{-15} \text{ eV} / (3 \times 10^8 \text{ m/s})^2) \times (1 - 1/4)$ and this will be meter inverse and let's obtain the numerical value which will be λ in minimum will be sorry the λ maximum will be approximately 300 \AA now there is another way of solving this problem a second method in case we know the Rydberg constant then R is $1.097 \times 10^7 \text{ cm}^{-1}$ or if I can write it in SI units $1.09678 \times 10^8 \text{ m}^{-1}$ approximately meter inverse

So if you know the value of the Rydberg constant then the solution becomes much shorter I will write $n_f n_i$ is equal to here it is $4R$ times $3/4$ or it is 3 times R and from this you will get λ will be $1/3R$ and that is again about 300 \AA

So that completes the solution of this problem and I will continue the solutions of the more problems related to this topic in part two you