

good morning all of you

so in the last few lectures we have been looking at various aspects of the quantum behavior of matter the first aspect that we looked at was the wave nature of matter which is the counterpart of the particle nature of light

so essentially between planck and de broglie we have established a kind of symmetry between what we classically call as particles and what we classically call as waves

so this whole entire phenomenon this dichotomy is called wave particle duality and it played a very important and a crucial role in the development of early quantum mechanics the so-called old quantum mechanics which involves planck's radiation law einstein's photoelectric effect and then you have Compton scattering Stokes anti-Stokes line and then of course the Bohr model and de Broglie hypothesis that is what we have and after this there was a new understanding a better understanding of all these phenomena in the language of quantum mechanics which was accomplished by the work of Schrodinger and Heisenberg and also by Dirac but that is not a subject matter for you it is rather too advanced and you will do that in your higher classes when you go for graduation

so what we have to now do is to come back to the Bohr model which I had started discussing and what Bohr essentially did was to make an extraordinary and a highly courageous hypothesis that is a very important thing it was very bold on his part because he was bringing in various concepts of classical electrodynamics and the quantum hypothesis together it was there to a certain extent in Einstein and Planck but it was certainly not as pronounced as explicit as clearly expressed as in the case of Bohr and as I told you in the last lecture for this reason Bohr is considered to be one of the deepest and the most profound philosopher scientists of the 20th century he had an enormous influence

so let us recall what exactly Bohr did

so we had the problem of these spectral lines

so you see that here in this picture

so what we have is instead of

so what is it that we had classically the atom should not have existed at all but it should emit by continuous radiation but that does not happen atom is completely stable but however if you excite the atom it is not stable only in the ground state there is a minimum energy state as it we do not know where that minimum energy comes from even the Bohr model will not give you but once we excite it again the atom will radiate will come down to the ground state will get de-excited by emission of radiation but that again is not consistent with the Maxwellian prediction because classical electrodynamics says the radiate and energy should be continuous in frequency in wavelength or for that matter in energy but here you see there is a very good spectrum discrete lines that is what we are seeing here and we discussed at great length what the Lyman series were these are in the invisible region beyond the violet region then you have the Balmer series which is in the visible region then you have the Paschen series bracketed fund etc etc which are actually not in the visible region again because they go to the they go to the invisible in the infrared region that is what we got and I also should remind you that all these spectral lines the wavelengths were collapsed into a single formula by Rydberg and that is given here he found that $\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$ where n_1 and n_2 are integers obviously n_2 should be greater than or equal to greater than n_1 that means the number the quantum number n_1 should correspond to the ground state of the system n_2 n_3 n_4 etcetera should correspond to the excited states of the system Rydberg constant which is known to a remarkable accuracy as I told you almost up to 14 15 decimal places

was a phenomenological constant it was determined experimentally but the question was the big question was where does it come from

so these are the other lines as I told you the Lyman the Balmer and the Paschen
so you can see Paschen should be $n_1 = 3$ and n_2 should be 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100
this corresponds to $n_1 = 4$ Lyman corresponds to $n_1 = 5$ Balmer corresponds to 6 and this corresponds to 7 and this was discovered very very recently for which you need very very precise and very well resolved spectroscopy okay now Bohr came and clarified the situation although he did not completely solve the problem by making two hypothesis I will go through that very quickly

so that we have continuity the first statement that we made was that the orbits the classical orbits in the case of hydrogen atom are all not continuous but discrete only some discrete orbits are allowed when I say sum I do not mean finite

so let me remove this only discrete orbits are allowed and what is the condition for the discrete orbit and that is quantization of angular momentum

so what are we doing we are assuming at the time being that the orbits are all circular therefore we write $m v r = n \hbar$ is equal to $n \hbar$ identically equal to $n \hbar$ by 2π this is the condition this is not unique to quantum mechanics for example if you look at the vibration of the string you know that only some modes are allowed and then of course the most general vibration is a linear combination of those modes you have the fundamental then you have the first harmonic second harmonic third harmonic

so on and

so forth in fact this is not very different from that if you write down the equations of motion

so you already get an inkling that we are somehow importing the wave nature of particle as I told you historically the wave nature of particle came after Bohr proposed this model but since we have already discussed the wave nature of particle de Broglie and Compton experiment

so on and

so forth you can see that there is an inkling for this

so this you should compare with the modes of vibration right

so if you have a vibrating string for example with both the ends fixed the fundamental mode is like this then you have something like this

so on and

so forth and then you have the first octave second octave that is what they call it in music and then what you will do is to look at all possible superpositions this is something that you might like to revise although it will not be immediately used analogy with vibrating strings you can also have analogy with vibrating membranes

so on and

so forth let us not get into that

so that was the hypothesis and with this hypothesis Bohr was able to reproduce the mysterious Rydberg formula I need to derive that formula again because I want to illustrate a large number of results starting from that

so the two people become completely familiar these are what are called as scaling laws in a hydrogen atom

so let me repeat that

so derivation of the Bohr formula is what we have to write

so what is the Bohr formula now there are two ingredients $m v r = n \hbar$ please remember \hbar is h by 2π some people like to call \hbar as the Dirac constant but nobody uses it and the second one is to assume circular orbits and once the orbit is circular what do we have we have a centripetal force which is

equivalent to the applied attractive force

so what are we writing the centripetal force is given by $m v_n^2 / r_n$ in the n th orbit and this is equal to $k e^2 / r_n^2$ inverse squares law and what is my k my k is that $e^2 / 4\pi\epsilon_0$ ok by force of habit we put this z which is the atomic number of the nucleus but for our purposes z equal to 1 because we are interested in the hydrogen atom it is not easy to use this formula if you consider other atoms like helium or lithium or boron or beryllium or carbon because there you have a large number of electrons and you will have to worry about the repulsion between the electrons

so for that reason textbooks say these are hydrogenic atoms

so what you imagine is if you take helium you knock off one electron

so that it is almost like a hydrogen atom except that the positive charge is not two because there are two protons

so on and

so forth

so this z should not be confused for the description of a real atom where you know there are going to be set electrons we have to remember that and these two equations give us the bohr formula immediately

so what do we do i bring this r_n^2 here and multiply and divide by m

so i first do this and then i bring r_n^2 here

so this is brought here then what is going to happen $m^2 v_n^2 r_n^2$ squared is $n^2 h^2$ squared

so we are using the bohr quantization condition over $m r_n$ is equal to k

so we are asserting that the ratio n^2 / r_n is a constant that is the first consequence

so let me write that here

so n^2 / r_n is equal to constant

so this is the first scaling law that we have or if you feel like we can write r_n equal to $n^2 h^2 / m k$ that is what we have

so r_n varies like n^2 that is what we have and this is something which is completely new in the bohr model you will not find it anywhere else ok

so i have got this beautiful result r_n is equal to $n^2 h^2 / m k$ from this i can immediately find out what my potential energy is now i have to be of course careful because the potential energy is attractive my proton is positive my electron is negative and there is an attractive potential between them therefore my potential energy V which is attractive is given by $-k / r_n$

so we are asking what is the potential when the electron is at a distance r_n from the nucleus by the way when i am doing this calculation you people should remember that i am assuming that the proton is infinitely heavy compared to the electron it is 2000 times heavier there is no problem about that

so if i plug in this expression what am i going to get i am going to get $-m k^2 / n^2 h^2$ that is what you are going to get

so what happens is that as you keep on increasing the radius the potential keeps on increasing that is a very important point the magnitude keeps decreasing but then it is negative therefore the potential keeps on increasing and in the limit n going to infinity it becomes zero after all that is how we defined our potential to be k / r it goes to zero and therefore if a particle is at infinity and it has some kinetic energy the total energy is simply the kinetic energy that is what we have but our particle is not at infinity because it is at a finite radius and it is going around in a circular orbit

so what we should now ask is what is my kinetic energy kinetic energy is very easy to find that is nothing but $\frac{1}{2} m v_n^2$ that is what we have now i need an expression for v_n^2

so what will i do i will go back to bohr and i will write $m v_n r_n$ is equal to $n h$

so if i remember that i have to need i need this expression in some point

so my v_n^2 will be $m h^2$ over $m^2 r_n^2$

so this quantity is half $m v_n^2$ h^2 $m^2 r_n^2$ please don't be under the impression that my kinetic energy keeps on increasing as i go further and further away that is not what is happening what happens is that i have to substitute for r_n^2

so let me write that again here my kinetic energy t_n is half $m v_n^2$ h^2 $m^2 r_n^2$ square over m^2 i will simplify it and i have one over r_n^2

so i need the expression for r_n^2

so this quantity will be $m^2 k^2$ we have to pay good attention to everything n to the power of 4 h^2 to the power of 4

so now we have the full expression

so if i were to substitute and simplify what am i going to get this m^2 and this m^2 cancels

so n^2 this becomes 1 over n^2 to the power of 4 this becomes 1 over h^2

so i am going to get half $m k^2$ i hope i am doing the right calculation n^2 h^2 this is my t_n

so i am doing the calculation as i am teaching you

so you people should also work it out

so now what we have to ask is excellent but what was my potential energy if you look at it my potential energy was actually minus $m k^2$ over n^2 h^2 squared

so what are we saying we are saying the magnitude of the potential energy is twice the magnitude of the kinetic energy and therefore my total energy is a negative quantity as it ought to be if my total energy is positive you know from classical mechanics or classical even electricity and magnetism the particle would be free to go to infinity it will not be in a bound state therefore the total energy should be negative and from these two i will write e_n is equal to minus half $m k^2$ over n^2 h^2

so we got the red bar formula from this by making use of the fact that when there is an emission then the energy carried by the radiation is nothing but the difference between the two energies multiplied by $h \nu$ whatever ν that is the energy therefore you could get the frequency and therefore you could get the wavelength i am not going to get into that

so what are we going to find that is a very very important thing

so let me collect all the results let me collect all the results the first one is that r_n behaves like n^2 there is a very very important thing whereas v_n behaves like 1 over n my kinetic energy and potential energy both behave like 1 over n^2 therefore my energy also goes like 1 over n^2 this is the origin of the red burglar and very very importantly my t_n is minus half v_n my t_n is minus half v_n it is only half of v_n with the opposite sign therefore my e_n less than zero for all n

so when i was showing you that list hund humphreys etcetera etcetera basically they were accessing very very large values of n that means the emitted wavelengths were very very large and therefore it went into the deep infrared regime and these are called scaling laws these are called scaling laws and from these you can derive a number of conditions without even completely solving the problem you do not have to go please remember this because these are also typically the questions that are asked of you in your examination

so this is something very interesting about the bohr model now every model needs a confirmation because a model is proposed to understand one particular

system

so let us ask how realistic is the bohr model you should not think that because bohr was able to reproduce the ritual constant was able to give the spectral lines etc it is sacrosanct and everything is great what we should be able to do is to test it in a different system

so what is the method method of testing method of answering that question is to test the model in a different settings in a different setting and if the model fares well then we can believe if it does not fare well it is not going to believe

so what we are going to do is to look at two examples two examples first one is the celebrated frank hertz experiment and the second one is vibrational states of molecules these two examples have a lot of lessons for us apart from giving us some numbers

so what we will do is to proceed slowly and try to understand frank and hertz performed this experiment immediately after bohr proposed this model bohr model came in nineteen thirteen and the first experiment of rank and hertz were reported in nineteen fourteen the frank experiment and what was the experiment let me define discuss that

so what frank and hertz did was to look at a completely different element namely mercury the chemical symbol is hg the chemical symbol is hg that is what they did because people knew what the spectral lines of mercury were now we should remember that mercury is a metal

so it is sitting somewhere far away in the periodic table far far away from the hydrogen atom and mercury has a large number of electrons whereas the bohr formula is valid only for the single electron in the field of a nucleus

so what i want to tell you is in spite of the fact that the frank hertz experiment was hailed as one of the most beautiful experiments that verified the bohr hypothesis because soon after frank and herz performed their experiment bohr himself wrote a paper saying that it is consistent with this model and at some point einstein is said to have remarked that this experiment is

so beautiful it makes one feel weep with joy okay that means this is an experiment which has had a tremendous impact on the psyche of the physicist those days of course it is a beautiful experiment but still we should remember the applicability of the frank hertz result is not straight forward to the bohr model therefore it is bound to be qualitative and not quantitative that is something that you have to remember that was the state of affairs in 1914 1915 there were the two years in which frank and hertz did their experiment and in either in another 10 or 12 years they had been decorated with a nobel prize and no wonder about that because this is one of the most beautiful experiments

so what is this experiment

so what you do is to look at the mercury vapor

so in a discharge tube people know that when the mercury is vapor even the mercury heated then obviously the mercury atoms are going to get excited the electron is going to excite it

so in discharge tubes i could have brought a picture for that anyway never mind there is one vacuum tube mercury emitted radiation of about if i remember correctly 250 nanometers and this was a sharp line that is something that you have to remember this was known experimentally prior to frank and hertz adventure into this area

so now we have to make use of it and we have to see what can be done

so what is it that we did what frank and herz did was to look at the evacuated took a look at a vacuum tube

so a picture of the vacuum to be see here and if you people are going to your lab you people can realize the anode and then there is a grid here and then

there is a cathode basically what you do is to send cathode rays which are nothing but electrons and accelerate them or look at their motion that is what you want to do

so what frank and hertz did was take a small quantity of mercury it was in the vacuum tube and evaporate this this is very very important

so what you do is to essentially produce a low density mercury vapor in the vacuum tube and where is the mercury vapor it is distributed all over the vacuum tube distributed all over this is very important for us because this is the counterpart of the rutherford experiment where the target was a very very thin foil

so when the target was a very very thin foil i argued that each alpha particle could have undergone at most one collision that is the statement that i made but then if the target is distributed over a certain volume in space then you cannot make such an assumption and indeed frank hertz experiment actually gives a teach to it that we should not make such an assumption okay

so before i come to that what did they do they injected accelerated electrons this is a simplified description in the tube now we have to know what the range of these energies were

so this energy range this is very very important for us was a fraction of an ev to about 80 electron volts

so let us say one electron volt to 80 electron volts that is what they did again we are going to assume that mercury being a very heavy object mercury nucleus being very very heavy we are now going to look upon mercury as what we are going to now look upon mercury as infinitely heavy object the rutherford experiment had the alpha particle energy in the range of mgb million electron volt here we are speaking of 1 to 80 electron volt therefore the energies are 10 to the power of 5 to 10 to the power of 6 smaller than the alpha particle energy and at this energy we already know from the bohr theory which i wrote the ionization energy is 13.

6 electron volt

so on and

so forth therefore electron volt is the characteristic energy for an atom therefore my electron will not be able to reach the nucleus at all it will get rippled by the other electrons in the mercury atom and it should get scattered that is the idea

so this is the contrast between the rutherford experiment and the frank hertz experiment

so the cathode rays cathode ray particles and their electrons interact that is the most important thing with the electrons in mercury atom but the only problem is if the bohr hypothesis correct my electron cannot take the energy from the coming incoming electron my electron in the atom cannot take the energy from the incoming electron unless it matches with the energy level description

so what are we going to write

so let us say this is my ground state energy of the mercury atom then you have the first excited state second excited

so on and

so forth we do not know what the spacing is

so let us say this is e_1

so with the minimum energy that should be carried by the incoming electron is $e_1 - e_0$ if it has to go up if the energy of the incoming electron is less than that then the whole atom scatters elastically because you are not able to exit what is happening when you excite the kinetic energy of the incoming electron will be converted into the internal energy of the atom because it is getting excited but then what we are going to assume or what we have assumed is

that the atom is infinitely heavy

so in that elastic scattering if you look at the scattered electron its momentum may change but its energy will not change it is like a ball going and hitting a brick wall okay the momentum will change

so momentum will be transferred to the wall but no energy will be transferred to the wall therefore the scattered electron from the cathode ray tube will carry the same energy that is the principle

so here is a very very beautiful experiment of the scattered electrons by mercury i am going to look at it in great detail

so these are the incoming energies and i know as soon as the incoming energy hit 4.

9 electron volts it immediately dropped

so what you do you keep on sending energies of different energies but as soon as it is 4.

9 electron volt that is the magic number for me it immediately comes down here and then what happens if i continue to accelerate it will keep on going as soon as it hits 9.

8 electron volt which is exactly twice four point nine electron volt again there is a certain drop and a sudden drop to actually close to four point nine it should be but never mind about that and again it will keep on going when it hits the energy three point nine this is the most beautiful curve that is taken directly from the frank hertz experiment and we should understand that

so what we are saying is that there is this atom mercury atom there is this electron here and there is this excited state here

so there is an electron here and there is an incoming electron here that is what is happening now this energy difference is 4.

9 electron volt if the incoming energy is less than 4.

9 electron volt the electron will simply get scattered its momentum will change its energy will not change

so it will keep on going and the incoming energy will be the same as the outgoing energy the direction may be different but you are not sensitive to the direction like in photoelectric effect you are not sensitive to the direction of the emitted photon you only gathered its energy

so in that way it actually bears a good resemblance to photoelectric effect but if the electron has an energy of 4.

9 electron volt now this is the cathode ray this is the cathode ray the electron in the atom can absorb all the kinetic energy and can come to this state because the difference is 4.

9 electron volt

so energy of scattered electron is zero now the question is what is the cathode energy range cathode electron energy is seven electron volts no problem you make use of the conservation of energy

so the electron will take four point nine electron volts and the scattered electron will have an energy of what seven minus four point nine which is 2.

1 electron volt and if you look at this picture that is what is happening it is not becoming 0 okay except the one which is at 4.

9 the energy keeps decreasing now this 2.

1 electron volt electron as it is propagating in the vapor it does not have energy to ionize the mercury atom further and it will be detected as a 2.

1 electron volt electron now what i will do i will say that the incoming energy is let us say 9.

8 electron volt now what happens in the first collision it loses 4.

9 electron volt continues with a kinetic energy 4.

9 electron volts but then it is moving through the bulk it is moving through a

gas another atom takes away the energy and the final electron has zero kinetic energy there is what we are writing

so now that means that instead of nine point electron volt if it had an energy of ten point eight electron volt incidental energy then final energy is $10.8 - 9.8$.

which is 1.

0 electron volt again it is not enough to excite the atom further therefore it will simply keep on getting elastically scattered its momentum may change but the energy will not change and here you see there are three such peaks corresponding to four point nine nine point eight and four point nine into nine point four point nine into three that will be three lines are twenty seven three four three twelve fourteen point seven electron volt in fact this last number is fifteen electron volt this is fourteen point seven electron volt i don't know if you people can see this or not but take my word for it this is one of the most beautiful demonstrations of course frank and herz did not stop there as i told you they went all the way from 0 to 80 electron volt you see a number of peaks and these correspond to various excited various scatterings and various excitations and this gave a great qualitative evidence for the bohr model because the frank heads experiment cannot be understood if the energy levels are continuous

so although at this particular instant of time i do not know how to work out the energy levels of a mercury atom for that matter i do not even know how to work out the energy levels of a helium atom this gives a qualitative evidence frank and hertz did something else they waited you don't have to wait for long and they found that the excited atom is going to come down excited vapor emits light of energy 4.

9 electron volt that is the most beautiful thing it trades emits light prominently in the 4.

9 electron volt

so that gives another good evidence and now if you today look at the spectroscopic data this is very very well understood you will see that these are the spectral lines this is what corresponds to about 252 nanometers exactly corresponding to 4.

9 electron volt therefore by hind side with hindsight we can say that frank hutz experiment was one of the most spectacular and beautiful confirmations of the bohr model in 1914 or 1915 it was a qualitative confirmation today it is a quantitative confirmation because these things are very well measured if you go to higher studies then you may end up doing an experiment on frank hertz there are many many universities and institutions where it is done all over the world including our country

so what are modern day experiments modern day experiments people do not use mercury but people use neon neon does not have to be evaporated neon is already a gas

so they fill the vacuum tube with that there will be some pressure that is quite a small pressure if you go and look up the paper of frank and hertz you will get the complete data and now again you scatter neon atoms of the electron the beauty of this is that the energy gap is much much smaller and therefore emitted radiation in the is in the visible range whereas 252 nanometers is not in the visible range that is the statement that we want to make and i want to show you a picture of such a thing this is a beautiful picture of what is happening in the vacuum tube it is actually in the orange region and you see this orange hue

so not only do you make a measurement but you can actually see with your own eyes that the radiation is being emitted in this particular reason therefore if

you get an opportunity to perform such an experiment you should not miss a chance because in that case you would be essentially recreating one of the greatest experiments of the 20th century the original paper is written in german but i would still advise you to go through that because again at least look at the equations and you can look at the graphs and those of you are studying german language would also enjoy reading that in the original and i am sure it is very very beautifully written

so this gives you a confirmation of the bohr model but we want a confirmation in an even more quantitative way and for that what we shall do is to look at vibrating molecules this requires a little bit of preparation but we are not in a hurry

so let us start with that

so the basic idea behind this analysis and which is universal in fact it is ubiquitous and you see the same thing occurring in molecular physics the same thing occurring in atomic physics the same thing occurring in nuclear physics in fact even in quark physics if you imagine that proton is made up of quarks people will look at these vibrational molecules

so what is the idea now suppose you are given a potential which is very complicated let us say something like this this is quite different from the hydrogen atom position potential that i wrote because i am interested in atom atom or molecule or molecule

so this is my potential and this is my separation now what happens if two atoms will come very very close to each other the electrons in one atom will start rippling the electrons in the other atom if you want you can even bring in poly exclusion principle the electrons cannot occupy the same space

so very close to the origin the potential raises very rapidly but the atoms are neutral therefore if you go far far away you should fall off very rapidly

so it comes here and it will fall off very rapidly in fact we can arrange it such that it is going to ∞ here is that ok this is not a very good picture i am sorry because it is giving a misleading interpretation picture

so the correct picture should be something like this comes here and falls off very rapidly and it will go to ∞ .

so if you take the derivative or you can see it from this this is repulsive here it is attractive here it continues to remain attractive but becomes very weak as you go farther and farther away in r and there is a minima which is located at capital r

so one very good example is the van der waals force i think it will look like a $\frac{1}{r^6} - \frac{b}{r^n}$ where a and b are positive constants if you plot that this is how it is going to look this is nice now what we will do is to not try to apply the bohr model to this directly because i do not even know the form of the potential but i assume that whatever is bound here the two atoms are here they are very close to equilibrium position close to equilibrium position at equilibrium position it has the minimum energy

so we are looking at small perturbations or low level excitations that is what i want to do whether it is classical or quantum mechanical

so what are we going to do we are going to say that v of r has a vanishing derivative at r equal to r this is the equilibrium separation let us say between the two atoms and which is a minima position for minima what do you mean by that all these are important minima means that $\frac{d^2v}{dr^2}$ at r equal to r is greater than zero

so these are the conditions that i have

so i am sitting in the minima and the way i have written this picture this minima is a global minima there is no other minima actually ok now if that be

the case i can make a taylor expansion of v of r around r equal to r

so if i write $\frac{d^2 v}{dr^2}$ at r equal to r equal to k greater than 0 the first derivative is vanishing therefore my v of r in the immediate neighborhood of r will be capital r equilibrium v plus half k into r minus r whole square it is not the end of the thing plus higher order you do not have to worry about that

so if i am looking at the separation let me call it as some ρ which is r minus r mod r minus r my potential is essentially some constant plus half k ρ square and all of you without an exception will recognize this to be the harmonic oscillator potential

so this should be a capital k the way i have written and your k the spring constant is nothing but the second derivative of the potential at the minima

so what are we saying mathematically speaking what we are saying is that wherever the derivative is 0 and the second derivative is positive you can approximate the function in the neighborhood excellently by a parabola that is all what we are saying of course if it is a maxima it will be approximated excellently by an inverted parabola it will become a minus half k r square that would be a position of unstable equilibrium whereas this is a position of stable equilibrium whatever we have written here this is a position of stable equilibrium and that is all what we have to worry about

so now what we are claiming is that when two atoms come close to each other if i model it by that kind of a potential if i model it by this kind of a potential r then if it is described by a quantum phenomenon then the energy levels the quantized energy levels must be given by what the bohr orbit rule to harmonic oscillator

so what are we saying apply the bohr model hypothesis to simple harmonic oscillator let us call it s h o and what i am going to do in the next 10 minutes is to work it out it is in fact even simpler than the hydrogen atom one actually maybe it is as easy as that and let us see how we are going to get that again we shall assume circular orbits and if you people feel like you can even work it out in one dimension

so my total energy e is given by half m v square plus half k x square that is what i have

so it is convenient to work not in units of k but in units of the angular frequency therefore we will write it as half m v square plus half m ω squared x square

so everybody knows what ω squared is ω squared is k by m because this is a natural unit that is what we have what is my force my force is oh i am sorry i should write it as a position vector let me take a different sheet my e is half m v square in three dimensions is what i am going to write plus half k r square which is identically equal to half m v squared i can put a vector sign here also plus half m ω squared r square

so it is an isotropic oscillator that is what we are saying

so you displace it in any direction from the equilibrium position it will be pulled back in the same way and how what is the same way my f is therefore given by minus k r

so this is hookes law in three dimensions

so you people have studied the oscillation in two dimensions oscillation in three dimensions lisa jose figures how you get ellipses

so on and

so forth

so what i will simply do is to plug this equation make use of the bohr hypothesis and see what is it

so we are going to assume again circular orbits plus bohr quantization is what

we are going to assume

so circular orbit means mv^2/r is equal to $-k/r$ all the directions are taken care of the force is radially inverse it is a centripetal force and that is what we have written here

so mv^2/r is also equal to $-m\omega^2 r$ let us write both of them and the bohr quantization condition is as always irrespective of what the potential is in \hbar

so that is what we are going to get and again we have to combine these two and we have to get a meaningful equation what is the solution there are many many ways of doing this one thing that we can do

so i am going to put a n and a n and a n here i should not put a plus sign here because the signs are taken care of there is no question of because this is for the magnitude

so what we have is that $m v^2 n^2 / r n^2$ is equal to constant and from the second equation $m v^2 n^2$ is equal to $n \hbar^2 / m r n^2$

so that is what i have to use therefore $m v^2 n^2$ is $n^2 \hbar^2 / m r n^2$ there may be many many ways of doing it even more elegant ones but never mind that therefore $m v^2 n^2 / r n^2$ is what i am going to look at is nothing but $m n^2 \hbar^2 / m^2 r n^2$ that is what i have ok i have to divide it by one over $r n^2$ this is constant i hope i have done the calculation correctly otherwise we will have to repeat the calculation you see equal to k i think i am doing well therefore we get the condition $r n^2$ to the power of 4 is $n^2 \hbar^2 / m k$ and this k is $m \omega^2$ therefore this is $n^2 \hbar^2 / m^2 \omega^2$

so i got a relation $r n^2$ to the power of 4 is $n^2 \hbar^2 / m^2 \omega^2$ i am interested in $r n^2$ because it is the potential energy $r n^2$ is $n \hbar^2 / m \omega$

so this is a very important result for me because we are seeing quantum mechanics rather bohr model is giving a new length scale in fact the same thing happens even in the case of the hydrogen atom which was your bohr model for your bow radius θ .

5 angstroms actually

so you notice that $r n^2$ is proportional to n this implies my potential is proportional to n there my potential was proportional to $1/n^2$ here it is proportional to n in the next lecture i am going to show you that kinetic energy is also proportional to n the total energy is proportional to n and therefore unlike the hydrogen atom you know where the lines were all getting bunched up as you kept on moving and farther and away because one over n^2 here the energy levels will all be equally spaced and what we will do is to look for the experimental evidence in your atomic spectroscopy the experimental evidence has to be carefully interpreted because the spectral lines are quite complicated but i will tell you and after concluding this in the next lecture we should take us about 15-20 minutes we will proceed to discuss what is sitting deep inside the atom the nucleus its structure its properties radioactivity fission fusion and that should complete the course

so thank you very much have a good day you