

good morning

so in the last lecture we discussed the great rutherford scattering experiment in great detail and we also analyzed the results with the care

so what we found was that the experiment did not provide any evidence whatsoever for the picture that the positive charge in the atom is distributed over the volume of the atom that was the original picture people believed that atom was some kind of a solid semi solid in which the positive charge was uniformly distributed and the electrons which are much much smaller were embedded in that solid the rutherford experiment actually showed us that such a picture is not correct in fact it also showed us that the positive charge distribution is concentrated in a very very tiny volume inside the atom in fact the volume occupied by the positive charge distribution if you look at the size or the radius of the positive charge distribution it is about 10^5 times smaller than the radius of the atom it is that small this effectively overthrows the picture proposed by thomson now i have to make a corresponding model and the model as i told you was none other than the planetary model

so let us briefly recall what the most important features of the result are for the sake of completeness this is the schematic representation of the rutherford apparatus you have the radioactive source which is emitting alpha particles at an energy of 5.

5 million electron volts this is the lead shield to protect the person who is doing the experiment to protect the laboratory and then the alpha particle beam is coming here and it is getting further collimated by this lead plate and this thin narrow beam is hitting a gold foil which is very very thin about 10^{-7} meters in thickness and then you have mobile zinc sulphide detectors which move on the circle and every time an alpha particle hit this plate there would be a scintillation which could be observed through a microscope

so the number of scintillations will tell you the relative number of alpha particles that are hitting the shield at any given angle the figure is as i have told you repeatedly highly exaggerated and then this is the schematic representation of what is happening if the alpha particle is heading headlong towards the positive charge

so we have already accepted the planetary model or the fact that all the positive charge is concentrated at a small region then in this head-on collision it will be reflected backwards

so there will be a number of reflections close to 180° if it is far away from the nucleus because the potential the repulsive potential has become weak it will go almost undeflected otherwise it will get scattered

so this is the schematic representation

so what we have to do is to understand this and i showed you what the picture for a scattering from a point particle is and what the picture from a scattering from a distributed positive charge distribution is

so if the nuclear were really distributed over the atomic size it would have been showing humps and peaks and from these thumbs and peaks we would be able to establish the size of the positive charge that is the most important thing where the hump occurs where the minima occurs is dictated by the energy of the particle the scattering angle and most importantly the radius of the atom or the radius of the positive charge of the nucleus there was no evidence for this picture but you see there is a smooth smooth curve that is there of course if you are to send even higher energy alpha particles let us say suppose you send 20 mev or 30 mev it is possible that it may be able to probe even the final structure of the nucleus you will be able to see how the protons and neutrons are distributed in the nucleus what is important for us in this experiment is

that at this energy scale and therefore at this length scale there is no evidence for the size of the positive charge distribution you can put an upper limit and the upper limit that we arrived was about 10 to the power of minus 14 meters that is the upper limit that we got and we have to deal with it and remember the size of the atom is about 10 to the power of minus 10 meters so there is a discrepancy of 10 to the power of 4 .

this experiment which result which i showed you the earlier one was actually scattering of proton on different atoms this is the real geiger then result and you see that the experimental numbers and the theoretical calculation assuming that all the positive charge is a point distribution inside the atom there is a complete agreement and this is one of the great contributions of rutherford experiment

so rutherford immediately grasped the significance of this result and he gave this model which is again a schematic representation

so we are going to anticipate whatever that we discovered if few years later now we have the positive charge distribution concentrated in a very small region of course this shows both the protons and the neutrons the neutrons are blue the protons are gray and the electrons are going in orbits this orbit is also again schematic it need not be circular it could also be elliptic as we know from kepler's laws

so this was the situation when rather for did this experiment and as i told you this was a very pleasing thing because you have the same thing that is seen in the cosmological scale in the astrophysical scale repeated the replica is there in the atomic scale except that you have replaced the attractive gravitational force by the attractive coulomb force gravitational force is of course very very weak gravitational constant is very very small

so you need very massive objects in order to see the gravitational force and therefore you see you also need very large distances that is the reason why we look up whereas electromagnetic interactions are 100 times or probably even more 1 over 100 square times stronger than the gravity sorry electromagnetic interactions are something like 10 to the power of 30 times faster stronger than the gravitational force therefore you are able to see it in the atomic scale

so this was the situation and then we started taking a deeper look at the problems and the prospects associated with rutherford model

so this is a precursor for the bohr model

so let us start discussing that i will come back to this slide in a minute

so what we have is the consequence of rutherford model

so we made a picture that there is a positive charge and let us consider the simplest situation of the hydrogen atom because that is what we are going to discuss in the bohr model

so you have the positive charge proton sitting here and then there is an electron which is going ground in this orbit very nice very pleasing because it is exactly like the planetary orbit but then you know this coulomb force is equivalent to a centripetal force that is something that we extensively used when we were discussing gravitation

so mv^2 by r is some whatever that constant product of charges etcetera etcetera $\frac{1}{4\pi\epsilon_0}$ etcetera by r^2 that is what we are writing

so this is corresponding to a circular orbit this of course is equal to ma m is the mass of the electron

so we have the standard result all of you are familiar with it a is given by v^2 by r this is something that we have to remember because although in a circular orbit the speed may be constant v^2 may be constant

so the kinetic energy is a constant that does not mean that the velocity is a

constant at every point my direction of velocity is changing here it is moving upwards here it is moving downwards it is tangential

so the change in the direction of the motion gives rise to a change in the velocity v_1 minus v_2 that gives rise to acceleration and that is given by v^2 squared by r r is the radius of the atom

so this is common to both gravitational objects and electric objects electric charges

so why are we worried about we are worried about because maxwell equations predict that accelerating charges start radiating electromagnetic waves we know the wavelength of the electromagnetic waves can be very very small to very very large if it is very very large it is in the far infrared if it is very very small then you go to ultraviolet x-rays hard x-rays gamma rays

so on and

so forth for example radiation emitted from a nucleus is of the order of million electron volt whereas radiation emitted from an atom is of the order of 10^4 electron volts order of electron volts if you look at a molecule it would be a fraction of an electron volt

so on and

so forth it is there but whatever it may be they are going to emit the irrigation waves

so what is going to happen i take a charged particle and let us say i apply a constant electric field in this direction

so it starts accelerating

so you might think that the solution for this problem is very very easy because all that you have to do let us say uniform electric field all that i have to do is to simply solve the newton's equation of motion every one of you have solved it it will be a parabolic path like you solved it for the gravity earth's gravitational field freely falling body but maxwell tells you that it is incorrect because because of the acceleration whenever there is an acceleration there is a radiation loss that means you may be pumping in lot of energy but all that energy will not go into the kinetic energy a part of it will be lost as radiation that is what is going to happen

so that means that if you are not pumping energy and a particle is continuously accelerating that means it is continuously losing energy and if it continuously loses energy its velocity becomes smaller and smaller the speed becomes smaller and smaller and at some point it should come to rest that is what should happen

so if you look at this picture of atom what is going to happen this atom this electron which is going around the proton has an acceleration we have written that therefore if i locate the electron here it will start radiating away it will start losing its speed increases because its speed increases its acceleration decreases

so acceleration decreases means it will not come in whatever the initial speed was that is going to become smaller and smaller

so as this i should probably okay as the eventually what will happen because the velocity is going to be become smaller and smaller this electron instead of taking a circular path will start going closer and closer to the nucleus and eventually should collapse to the nucleus that is what is going to happen because this is a very very simple explanation that we have picture now you can actually compute the time scale for such a fall how long would it take for an electron which is about 10^{-10} meters away from the proton or the nucleus to fall into the nucleus you cannot work it out at this level but later when you study more of atomic physics and electromagnetic theory you will understand that could be the order of something like 10^{-9} seconds 10^{-9} seconds is enough for the electron to fall

into the atom but then we know the atoms have been there for the past billion years or

so a billion is something 10^9 and 1 year is 365 days each day is 24 hours and each hour is 3600

so you see the universe has been there for the past 10^{12} or 10^{13} seconds and an atom has been there over a reasonably large fraction of that but electromagnetic theory is telling me that within 10^{-9} seconds the electron should collapse and atom ceases to exist that is what should happen but that is not what we find at all one may argue that that is not entirely implausible because we know that beta rays are emitted in radioactivity okay you are going to study that at some length after we complete the bohr model and therefore maybe the electrons do fall inside a nucleus but that is incorrect because the energy of the beta rays beta minus is nothing but electron is much much larger than the energy of an electron in an atom

so you cannot confuse the electron that is coming from inside the nucleus with the electron that is going around the nucleus in an atom

so we have a large discrepancy but at this point i cannot just give you what maxwell's predict a maxwell's equation predict i should give you some observational experimental evidence for acceleration and here is the first slide which is coming from an accelerator this is a synchrotron where you know the charged particle something like a proton can get a very very large energy something like 30 gev

so it is a very very large energy as you can see and when it is going around is that okay it starts emitting radiation

so this is the energy of the radiation that is emitted and this is not small it is a almost 1gb radiation that is what we have okay that means the wavelength is a very very very small number because the frequency is very very large you can work that out and you see this is the effectively the number of the intensity of the radiation that is emitted and the intensity keeps on oscillating and keeps on as you keep on increasing the energy the intensity decreases again this is a logarithmic scale and on the logarithmic scale it is falling fast

so this is an evidence of what is called as a synchrotron radiation even a linearly accelerating particle can radiate that is called brimstra lung even that has been observed experimentally and what about other observational details this is a radiation that is caused by what is called as aurora boreolis

so what happens is that whenever there is a large solar activity lot of charged particles are emitted and as soon as they enter the atmosphere then they start decelerating and they also start accelerating because of their gravitational field and because of this acceleration they produce this beautiful electromagnetic radiation now this is some kind of a schematic representation but the next picture actually shows the radiation that is emitted you can see how these charged particles these are plasma particles actually they start radiating this is what this is the famous van allen belt this is the famous van allen belt that you will read quite a lot about it

so what observational this experimental is do is to find the intensity of the radiation you can send balloons you can do a lot of experiments and then show it schematically how the radiation is distributed

so the fact that accelerating charged particles radiate is something that is established both in the accelerator and also in the upper atmosphere now this is a radiation picture curve corresponding to axial related particles in what are called active galactic nuclei there the electrons or the protons get accelerated very to a very very large velocity and in that process they emit radiation over the full wavelength

so you see in the radii in the radio regime infrared visible ultraviolet index right this is the synchrotron characteristic emission

so it is not coming from the synchrotron radiation you can see they are accelerating and people actually try to understand the dynamics of these new galaxies by looking at these curves these are some of the examples i think i had one more picture

so this is the radiation jet which is color mapped because of the galactic nuclei and this is the qualitative picture that we saw

so again as in the case of wave nature of light maxwell's equations and their predictions are very well established verified by experiments and observations

so we are infactible now does it mean that atom does not radiate at all we do not say that there is a catch what happens is that when you for example heat a material then the atom will start getting excited because the electrons will start getting energy and they do start radiating it is a very very important piece of information for us and how do they radiate that is the question that we are asking

so if you look at the previous curves for example the radiation is continuous as you keep on changing your frequency or wavelength the intensity continuously changes there are no gaps look at this curve there are no gaps there are minima and maxima all right but there is a emission is continuous in wavelength but when spectroscopists started observing radiation emitted by an atom these are all observations made on the hydrogen atom you find something very very interesting the most important thing that you find is that the lines are all discrete maxwell would predict classical electrodynamics or electromagnetism would predict that when the charged particle is accelerating the spectrum of the radiation emitted that is the distribution with respect to the frequency must be continuous like for example you see in the black body radiation or when you take a metal and heat it red hot or white hot the all frequencies will be emitted in a continuous way you do not choose discrete frequencies but what do you find here you see that this is the increasing wavelength as we go from this direction to this direction it is discrete you will emit radiation at something like 12 16 angstroms one angstrom is 10^{-8} centimeter that is 10^{-10} meters that is what we have and then you have one at 10 26 one net in the 972 and etcetera etcetera and it stops somewhere here in at 912 angstroms the important thing here is that this spacing between different spectral lines is not uniform in fact there is a pattern which we are going to discuss in a while there is a large gap here the gap becomes smaller the gap becomes even smaller therefore as you go to shorter and shorter wavelengths the gap is becoming what smaller and smaller that is something that we are finding and this series is what is called as the lyman series now this is bomber series which is exactly the same way

so you are starting with the red

so it is in the other direction that we are moving ok

so there is a large difference and then you come to the blue and you come to the violet the difference between lyman and bomber is lyman is not in the visible range that is all in the ultraviolet and x-ray range but here you start with the visible range while it and you go all the way up to the red and you can again see all the wavelengths are returned 6562 etc etc and low is of course somewhere around 5800 tank storms or whatever

so this is the bomber series you see exactly the same pattern but if you look at the spacing this spacing will be different the characteristic spacing will be different but the pattern is the same and here is a consolidated picture of many many lines that people have seen

so different people observed these patterns in different wavelength regions

so it has all been named after them

so we already looked at lyman we already looked at bomber now you also have passion ritz passion bracket and fund that is what we have and these people actually started looking at in the infrared region where probably you will not where it will not be visible to the naked eye

so they have to use special spectroscopy but in all these cases you see a regular pattern and therefore what you have to do is to perform an empirical study is there a regularity of course the regularity seems to be there can they all be collapsed into some nice equation then if i can write it as a nice equation then i will be able to at least make an attempt to understand the pattern

so what are we doing saying give a quantitative form to this series that is the question that we have to do and this was done by this gentleman rid burke

so ridburg was a smart man he must have tried many many formulas and many many fittings just as kepler tried many many formula you know first he had one picture then he had another picture then he came then he moved to the frame in which the sun is at rest and then he got these beautiful ellipses in a similar manner redwork must have tried many many things and he found that what one should do is to not plot the wavelength but to look at the inverse of the wavelength inverse of the wavelength has a name in atomic spectroscopy it is called the wave number sometimes people say one over lambda is the wavelength sometimes p by wave number sometimes people say two pi by lambda is the wave number atomic physics is always called one over lambda is the wave number and for all these series he showed that the spectrum is characterized by one universal number that is called as a reader constant and what about the spacing lyman series bomber series bracket series fund series humphreys series all of them could be collapsing the formula $1 \text{ over } n \text{ squared minus } 1 \text{ over } n^2 \text{ square}$ left hand side is of course a positive number therefore what is what should it be n^2 should always be greater than equal to $n - 1$ otherwise we will get into trouble the most important thing that you have to notice is that it is no ordinary fit and this is a very very important experimental observation for us because i do not know the number of digits that rydberg had when he gave the formula but today it is known to a remarkable accuracy okay obviously one over lambda has the dimension of one over length $n - 1$ and in two are dimensionless numbers

so this red bar constant must have the dimension of inverse length and that is given by one point see let us count how many digits are there one two three four five six seven eight nine ten eleven twelve thirteen fourteen

so this is a number which is known to fourteen decimal places this is very very important for us and the uncertainty is in the fifteenth and the sixteenth decimal place

so what we are saying is that r_y by Δr_y that is a dimensionless number that is of the order of 10 to the power of minus 15 this is a remarkable number this number played an important role in the development of quantum mechanics to understand explain fine structure constant to explain what is called as a lamp shift

so on and

so forth lyman alpha line is very very important in that context

so experimentalists have measured this to a great extent and when we propose that there is something called a bohr model we should actually be able to reproduce this number but no model probably today has the capability no theory has the capability to reproduce this number with the same accuracy it is very very tough

so it is one of those great experimental numbers which acts as a standard or a

benchmark for any theoretical approach

so this is very important

so i told n_1 and n_2 are integers let us try to understand what they are redwork found out that the lyman series corresponds to $n_1 = 1$ and $n_2 = 2, 3, 4, 5$ etcetera the bomber series corresponds to $n_1 = 2$ and $n_2 = 3, 4, 5, 6$ etcetera passion series corresponds to $n_1 = 3$ and $n_2 = 4, 5, 6$ etcetera obviously bracket will correspond to four fund will correspond to five humphreys will correspond to six and in 1970s or 80s i think i don't remember exactly in mit people were able to do very careful experiments these are called strong and samsung series what does it correspond to this must be $n_1 = 3, 4, 5, 6, 7$ and $n_2 = 7$ and n_2 will start with 8 the spectral speaking is

so small and it is

so far far away from the visible region very very large wavelength it is not easy to measure them you need spectroscopes with remarkable resolution and they were able to achieve that and all of them fall within the redwood formula

so redwork formula for us is the magic formula something we should break this mystery we should understand if we can understand this together with another model maybe we will be able to get into the mystery of nature

so this is a very important experiment that is what i have

so physics was in a strange dilemma it was on the cry it was in a crisis horns of a dilemma as they say because it was it appeared as if it is impossible to reconcile all these conflicting data atoms should not be stable but there is a stability and what is that stability the stability is that the radiation will occur only if there is something called an n_2 eventually all of them will correspond to $n_1 = 1$ and after that the radiation stops that means the atom has a ground state a minimum energy state only when you excite it above the minimum energy state the atom will come down but once it comes down to the minimum energy state it will not fall down further

so there is a partial agreement with maxwell there is a partial disagreement with maxwell the partial disagreement is that maxwell says you should continue to lose energy until you fall inside the positive charge but these numbers tell you no no there is a minimum energy after which it will not fall again there is a partial disagreement with maxwell maxwell says that when you are excited the radiation emitter should be continuous it is showing what ah it is showing the sorry maxwell said that there should be a radiation radiation emission it is showing radiation emission but there is a partial disagreement again maxwell threats should be continuous it is not continuous

so it is like sometimes you pay maxwell sometimes you do not obey maxwell it is exactly like the wave particle duality

so we need something as radical as what blank einstein did in the case of light we need something equally radical and the interesting thing is just as einstein was able to make use of one great constant pranks constant to understand photoelectric effect bohr also made use of that bohr also made use of that why prank said that the radiation is discrete $E = h\nu$ and if we put it in a box the number of modes that are allowed will become discrete in a similar manner here for this atom also the radiation is discrete bohr proposed a model and that is known as the bohr model and here is the gentleman who actually shaped the 20th century physics to a very very large extent probably his contribution is profound and even more profound than that of einstein in some sense for the development of quantum mechanics because not only he gave the model he collected a whole lot of disciples around him almost everyone heisenberg paulie all of them were his disciples they went to him they discussed with him and he was active till 1950s when he and his student rosenfeld wrote a

very very important paper on measurement in quantum mechanics and he was also instrumental in understanding the nuclear process nuclear fission in fact he wrote a very very important paper on nuclear fission which later became the basis of developments of nuclear reactor or whatever is happening given in the case of destructive weapons

so here was a great man who was a philosopher and scientist and he had the courage to propose a model which was very baffling but it worked

so let us see what happens

so what i am going to do in the next 20 25 minutes available is to devote my time to the bohr model bohr model makes a simple assumption that the rather forward orbits are all circular

so let me start making assumptions some of them are assumptions some of them are postulates assumptions can be relaxed this is for simplicity the orbit of the electron about the nucleus is circular this assumption can be relaxed it can be made elliptic that was done by sommerfeld but that is the assumption that we are making to make

so that is perfectly fine there is no problem about it now comes the postulate very very important only some circular orbits are allowed if you look at gravitational force or if you look at classical problem what will i do i will write the equation $\frac{q_1 q_2}{4 \pi \epsilon_0 r^2}$ $\frac{m v^2}{r}$ and this r can vary continuously

so depending on whatever your energy is the r will keep on varying this one continuously this is for classical situation what does bohr say both say that this is discrete now we see why he wanted to be discrete if it is discrete then the corresponding spectral lines that are seen will also be discrete but we need a condition and here comes the most important condition the bohr quantization condition in other words i first observed that it should be discrete and then i give a rule on how we should be discrete

so what how do we discretize that circular orbit implies constant angular momentum

so $m v r$ equal to angular momentum for a circular orbit v is constant that is the speed r is the distance from the nucleus now bohr quantization says that $m v r = n \hbar$ where n is an integer very very important greater than zero that is important for us greater than zero

so what does it mean n equal to one two 3 4 etcetera

so all this time we were looking upon planck constant as relating energy to frequency now we look at it more carefully and we say that plank constant actually has the dimension of angular momentum i will make use of that and we say that the angular momentum in any orbit must be an integral multiple of \hbar

so integer multiple of \hbar and for those of you who have forgotten what \hbar is \hbar is h by two pi

so this was like a great stroke of genius which solved all the problems if you accept this then the rest of it is all very very simple algebra and let us see how it works about obviously bohr spent many many sleepless nights to come up with the model but let us see what is going to happen

so we have two equations one is coming from the classical equation

so now i will write $\frac{m v^2}{r} = \frac{k z e^2}{r^2}$ $n \hbar = m v r$ is equal to

so we can imagine a hydrogen like atom

so let me write it here hydrogen like atom

so that means the nucleus has a charge z but there is only one electron we have removed all the other electrons let us say this is nothing but $\frac{z e^2}{4 \pi \epsilon_0 r^2}$ the right hand side is coulomb the left hand side is the centripetal force valid for circular orbits now r and v are not independent of each other because what is the second formula that we are

going to use $m v_n r_n$ is equal to $n \hbar$ that is what we have all that we need to now do is to combine these two equations and see what are the allowed orbits

so what is our target combine 1 and 2 and get allowed orbits in fact allowed energies because that is the most important thing for us we have to get the allowed energies and we have to do that and let us do that

so i need to work it out that means i have to write the equation again

so the whole thing i am going to call as some constant k over r_n squared

so my k is $z e^2$ over $4 \pi \epsilon_0$ let me keep it and then i have $m v_n r_n$ is equal to $n \hbar$

so what do we do what i can do is to bring this r_n squared here there are many many ways of doing

so let us see which is the simplest way of solving this problem

so i can write $m^2 v_n^2 r_n^2$ over $m r_n$ is equal to constant this i think is a correct equation i multiplied by m and divided by m this becomes $m^2 v_n^2 r_n^2$ divided by $m r_n$ is what i am going to get and this is nothing but a constant and this constant is nothing but $z^2 e^4$ over $4 \pi^2 \epsilon_0^2$ but the numerator is nothing but $m^2 \hbar^2$ over $m r_n$

so now you see that depending on the orbit that you choose n equal to 1 2 3 etcetera etcetera you can fix what your radius is

so this tells me that the radius of the n th orbit is simply given by $n^2 \hbar^2$ over $k m$

so the radius of the n th orbit is increasing quadratically with n^2 if you go back and look at the Bohr formula it should strike a note in you because i have $1/n^2$

so it should have an important role to play

so let us remember that ok

so let me keep this term next to me

so i got the relation r_n is equal to $n^2 \hbar^2$ over $k m$ that is what i have got

so given this i can immediately find out what my velocity is

so how do i find that out i can calculate for example by plugging it back into the quantization equation what is that $m v_n r_n$ equal to $n \hbar$ therefore my corresponding speed in the n th orbit will given by $n \hbar$ over $m r_n$ which will be $k m$ over $m^2 \hbar^2$ that is what i have

so if i simplify this quantity what would this be this m will cancel and i am going to get n sorry $k \hbar$ over $n \hbar^2$ this is my velocity or the speed in the n th orbit and the square of the velocity will go like $1/n^2$

so v_n^2 is proportional to $1/n^2$ and r_n is proportional to n^2 this is something that we have to remember now if you grant me this i can now write the total energy what is the total energy total energy is kinetic plus potential

so this is in the n th orbit

so this is half $m v_n^2$ now i should be careful it is a centripetal force it is attractive and the potential is negative i have chosen the potential to be 0 at infinity and this will be whatever k i wrote divided by r_n this is my expression

so what is my k $z e^2$ over $4 \pi \epsilon_0$ let us not forget this now all that i have to do is to substitute the expression for this and this is nothing but half m

so let me show this expression for you $k \hbar$ over $n \hbar^2$ is what i have i have to do the square of this therefore i will square it and i will get $k^2 \hbar^2$ over $n^2 \hbar^4$ never mind we will set it right in a minute $n^2 \hbar^2$

to the power of four i could have cancelled it then and there that is what i am going to get and then the next expression will be minus k and i need the expression for r_n and r_n you people can check is $n^2 \frac{h^2}{4\pi^2 m k}$ if i have made a mistake i will find out in a minute

so this becomes $\frac{h^2}{4\pi^2 m k}$ because i have to take the reciprocal divided by n^2 $\frac{h^2}{4\pi^2 m k}$ dimensionally potential energy and kinetic energy are the same therefore they should have the same form

so let us take track of the situation there is a m there is an m there is a k squared there is a k squared there is an n squared there is an n squared there is an 1 over $\frac{h^2}{4\pi^2}$ squared there is an 1 over $\frac{h^2}{4\pi^2}$ squared the only difference between these two expression is a factor of half and minus 1.

so luckily we have done a correct calculation without making any mistake therefore energy in the nth orbit let me call it as E_n E_n is minus 1 over 2 m $\frac{h^2}{4\pi^2}$ k square $\frac{h^2}{4\pi^2}$ sorry $\frac{h^2}{4\pi^2}$ square this is not there

so let me rewrite it again minus half m k squared over n square $\frac{h^2}{4\pi^2}$ square that is what i have mass is a constant for a given atom k is a constant $\frac{h^2}{4\pi^2}$ which is equal to $\frac{h^2}{4\pi^2}$ is a constant therefore this whole thing can be written as some big constant divided by n squared with a minus sign

so let me fix the value of that constant explicitly therefore my c is nothing but 1 over 2 m over $\frac{h^2}{4\pi^2}$ square now i will substitute for k squared k square is nothing but z squared e to the power of 4 over 4 pi epsilon naught whole square

so this is the value of c

so let me repeat it for you my E_n is nothing but minus c by n square this is a formula that bohr derived after deep thought

so you must already be getting a hint as to how the red bar constant will crop up obviously this is sitting here and what is the dimension of this c this dimension of this a is nothing but of energy because n is a dimensionless number

so if i were to write down the energy levels as a function of n squared

so let us say i have n equal to 1 here then i have n equal to 2 n equal to 2 then there will be something closer closer closer closer

so on

so this is the spacing this gap is the largest this is smaller in fact when n becomes larger and larger the gap becomes smaller and smaller and it will go all the way up to n going to infinity

so these are my energy levels

so you can draw corresponding circular orbits where initially the distance is very large and after that it keeps on shrinking and if you go to very large values of n it is almost like classical orbit because the difference between one over n one and one over n two is very very small therefore it is almost continuous that is something that you find excellent now there is a third postulate of bohr very very important

so bohr third postulate actually it is a second postulate let me correct it there was one assumption and there were two postulates the second postulate that is radiation is emitted when an electron jumps from a higher orbit to a lower orbit that is the most important thing

so the radiation is not emitted when an electron is in a given orbit radiation emits is emitted when it jumps from a higher orbit to a lower orbit the model does not tell you how and when and why the electron jumps from one orbit to the other orbit but it does tell you that radiation is emitted when it is making a transition when it is jumping from one orbit to another orbit and as a corollary of this postulate we can make another statement radiation is absorbed when the electron jumps from a lower orbit to a higher orbit

so what is going to happen suppose i send a continuous beam of electromagnetic

radiation in wavelength for almost all wavelengths the atom will get elastically scattered but the minute the wavelength corresponds to the energy corresponds to the difference between the two energy levels then the electron immediately absorbs it and goes up we have to understand that

so radiation is emitted when an electron jumps from a higher orbit to a lower orbit and what is the relation between the radiation emitted and the two energies

so you are going from e_n to e_m

so the energy emitted is $e_n - e_m$ that is the energy carried and this is nothing but what is given by planck $h\nu$ this is the bohr postulate the third postulate which is again quantified

so we are saying that when the electron goes from an energy e_n

so my energy e_n is that constant over n^2 my energy e_m is another constant over m^2 remember c is positive now this is my n th level this is my m th level there is a corresponding orbit my electron comes down from n to m

so obviously n is greater than m there is no question about that because these are negative numbers

so the energy carried by the photon or the radiation let me use the word photon because that is what he made use of that corresponding from n to m is given by $e_n - e_m$

so this should be $\frac{1}{m^2} - \frac{1}{n^2}$

so $e_n - e_m = hc \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$ which is $hc \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$ this is the energy carried as i told you we are going to equate this energy to $h\nu$ from n to m

so planck's hypothesis is playing a role in two places one is in giving a rule as to what the allowed values of orbital angular momentum are and second one is what is the energy carried by electromagnetic radiation that means we are making use of the conservation of energy combining it with the planck formula making use of the constraint on the allowed orbits and we are drawing a conclusion

so let me write that again

so $h\nu = hc \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$ is equal to that constant into $\left(\frac{1}{m^2} - \frac{1}{n^2} \right)$ but what is the relation between the frequency and wavelength we know that

so $\nu = \frac{c}{\lambda}$ is equal to $\frac{c}{\lambda}$ please do not confuse this capital c that big constant with the small c which is the speed of light

so i can write $hc \left(\frac{1}{m^2} - \frac{1}{n^2} \right) = \frac{hc}{\lambda}$ this is the big $c \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$ divided by h into $\left(\frac{1}{m^2} - \frac{1}{n^2} \right)$ and you people can verify this capital c divided by h has the dimension of inverse length and lo and behold we identify this with R but constant

so if the bohr model is a correct model then i should be able to identify that with the rydberg constant now i am going to plug in all the numbers

so my R constant equal to $\frac{hc}{h^2} \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$ where c is the speed of light and what is this quantity let me write down explicitly $\frac{1}{2} \frac{m_e e^4}{4\pi\epsilon_0 h^3}$ divided by h^2

so when rydberg gave his great number that was an unknown constant and if the bohr model is correct i should be able to get this mass of the electron is known prime constant is known put Z is equal to one this is electron charge that is known $4\pi\epsilon_0$ is known again h is known c is known if you work out you will see it agrees with rydberg constant to a great accuracy not with absolute accuracy but your great accuracy

so what i will do is i will stop at this point because this is the right time to stop in the next lecture i will discuss what the meaning of the accuracy of

this experiment is i will also discuss other experimental evidence for the board model there is an experiment called the frank hertz experiment i think which is there in your syllabus and then i will round off my discussion by relating the deep rawley model with the bohr model what do you mean by saying that the electron is going in a circular orbit both are good that it is a what did you argo it is a standing wave we will establish that connection and then we will go on to discuss not the structure of the atom but the properties of a nucleus which will come from the radioactivity and the structure of the nucleus itself fission fusion etc etc and that should start off cs through okay have a good day you

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