

so let me welcome all of you for the last lecture in the series of lectures on photoelectric effect we have spent a fair amount of time discussing the evidence for the wave nature of the electromagnetic waves through interference diffraction loss of reflection loss of refraction etcetera etcetera and we also pointed out how the great experiments of lenard and milikan of course the first experiment was performed by hertz they cannot be reconciled with the concept of wave nature for electromagnetic waves or light

so we are at an impasse if we accept the wave nature of light then we are not in a position to understand the results of photoelectric effect and on the other hand if we give up the wave explanation for the nature of light then we would be in trouble as to how to understand the processes of interference and diffraction

so at this point what we are going to do is to not ask how we will be able to reconcile the concept of a photon with that of the wave nature that will be done in a much much advanced course right now what we shall do is to suspend our belief or whatever evidence we have for the wave nature of light and merely try to understand how in a logical manner we can give an account of the results of millican and lenard milligan's experiments are of course the most important and we have spent a lot of time in other words as i mentioned in one of my earlier lectures we are not aiming for a complete description but we are aiming for a simple description obviously you will see that there is a contradiction with the wave nature but as i said that is for a future understanding at some future time

so we have to look at einstein's explanation we should remember that einstein is not the originator of the concept of a photon the

so called particle of light this concept was actually introduced by max planck and we have also discussed why planck had to introduce that concept by looking at the phenomenon of blackbody radiation blackbody radiation is not a part of your course but i was able to show you using the principle of equipartition of energy that if you accept the wave description of light then at any temperature my energy total energy of the wave would be infinite that is what we found out

so there is a quadratic divergence the picture i showed you and planck came up with the radical hypothesis that this can be understood the experimental results regarding black body radiation can be understood if you imagine light to be a stream of particles that is what he said but planck really did not believe in the concept of photon he simply thought that when electromagnetic waves interact with the atoms or molecules in the cavity because that is what you are going to look at when you study black body radiation it pretends to be a particle that is what planck believed

so the great step that einstein took when he gave his explanation was that he treated photon as representing reality in all its entirety

so he was not trying to give an effective picture he said that light can do behave like a stream of particles and i am going to discuss that at some length

so in that sense what einstein did is much bolder and more courageous than what planck did earlier for planck it was a convenient language whereas for einstein it was reality and that is the reason why we have spent a long long time before we write down some simple equations because i know that all of you have solved enough number of problems in photoelectric effect

so remember the famous experiment of lenard this is the millikan this is the experiment of milikan with sodium as the metal the target and you see the straight line that is coming

so this is the most surprising thing that it behaves linearly with the frequency the frequency is of course not entirely in the physical in the visible region because it is slightly above we are looking at sodium not a element or a metal like cesium but if you did that that is what you are going to see and this

is what that something this is something that cannot be understood in terms of the wave theory we have discussed it

so let us recall what are all the conclusions of the careful experiments of milliconvert millikan performed these experiments over a period of ten years it is not an experiment that was performed over a period of six months or eight months or a couple of years

so there is a lot of data the first consequence is that you need a minimum frequency for photo emission remember classically the energy of a wave is carried by its amplitude and not by its frequency frequency tells you how many times the wave is oscillating whereas amplitude tells you how much energy it is carrying there are two different things

so energy is like the intensity of the sound frequency is like the pitch so i can speak at a very high pitch at a very low intensity so there is not much energy but i have gone to a higher pitch or it could be the other way round i can speak at very very high intensity with very loud voice at a very low pitch musicians can do that they go to very very low pitch and they can sing with a very loud voice

so that is the distinction that we have

so this is a well established fact in all wave phenomena including sound but here we are saying that photo emission

so what are we saying

so let me write that down here

so i am repeating for the benefit of understanding whatever we have said so many times

so we are physicists

so we are not afraid of redundancy

so we will repeat it as many times as necessary what we are saying is that energy required for emission of electrons now the electrons are bound therefore minimum energy required minimum energy is necessary without that minimum energy i will call e_{minimum} if ν is less than ν_{minimum} means no emission this is a simple consequence of the conservation of energy you don't give enough energy you don't get enough you don't get electrons at all but what do experiments tell us experiments tell us that no emission takes place if $\nu < \nu_{\text{minimum}}$

so this ν_{minimum} is different for different metals and we related that to the work function of the metal sodium cesium zinc lead etcetera etcetera we looked at all these elements and it was connected to the work function through the stopping potential i am not going to get into that

so we are saying no emission takes place if $\nu < \nu_{\text{minimum}}$

so in a similar manner no emission takes place if $\nu < \nu_{\text{minimum}}$ and since we are looking at the conservation of energy that means that my ν is somehow related to the energy this was the great observation of einstein on the other hand planck had already made the assumption had postulated that for any given frequency we can imagine energy to be coming with what photons of a certain definite frequency

so what did planck say if you have an energy density u whatever this quantity may be for a certain given frequency this can be written as $n \cdot h \nu$ h is the planck's constant and this was identified to be the number density this is identified with the number density

so what are we doing the left hand side of the expression comes from maxwell's theory which tells us that light is a wave phenomenon and we are equating that to an expression coming from particle theory namely each photon carries an energy $h \nu$ and there are n photons per unit volume therefore the total energy density is $n \cdot h \nu$

so logically speaking this is an incompatible relation

so what are we doing we are comparing apples and oranges the left hand side is the apple which is coming from the wave equation right hand side is the orange which is coming from the particle description something cannot be both a particle and a wave it is either a particle or a wave that is what common sense tells us but still planck did this in order to understand the black body radiation and einstein picked up the same idea and he said that we are going to use that in order to understand photoelectric effect

so please understand that this explanation goes against what we normally do it requires an enormous courage and of course a lot of work at a later stage to be able to do to be able to reconcile the both the pictures but we have to be aware of the fact this is not a simple assumption now there are additional things additional properties of my experiment i discussed only minimum frequency required for photo emission the next observation is that it is proportional to intensity beyond minimum frequency now there is no surprise about that if i keep this sheet of paper again here there is a minimum frequency that is required because of the work potential now after you cross the work potential as you keep on increasing the intensity you are increasing the number of photons therefore if you imagine that the emission is taking place because of the number of photons then of course your current keeps on increasing number of electrons keeps on increasing therefore this picture will be again consistent with what einstein said there is a linear relation that is also not surprising because greater the energy of the incoming radiation greater the energy of the outgoing electron

so there is a linear relation

so we have sort of been able to qualitatively appreciate that if we accept the photon picture we will be able to account for all these experimental observations and that is exactly what einstein did

so the einstein revolution to restate is the following thing radiation of a certain frequency ν is equivalent to a collection of quanta now the word quantum does not mean particle quantum means a unit that is what is a what is the quantum of such a such a thing it comes from the word quantity is that okay but we also tend to use the word particle because it is coming in discrete units this quantum is coming in discrete units

so we are saying that radiation of frequency ν is equivalent to a collection of quanta each of which carries an energy $h\nu$

so this is the einstein revolution where he took the photon concept very very seriously i am going to repeat in a more quantitative way whatever i wrote there earlier

so if you look at the screen you find two expressions from the classical expression you find that the energy density is nothing but $\epsilon_0 n^2 \omega^2$

so what am i doing i am writing a is equal to $\epsilon_0 n^2 \cos(k \cdot r - \omega t)$ my ω is nothing but $2\pi\nu$ my frequency that is what i am writing if i make use of the maxwell's expression the corresponding energy will be nothing but half $\epsilon_0 n^2 \omega^2$ there is something the two people have to remember here in writing this expression we have averaged over time over a period and that is justified because even in the visible frequency even in the visible range the frequencies of the order of 10 to the power of 14 hertz every second my light wave is oscillating 10 to the power of 14 times and we do not have that kind of a resolution that is what we have whereas the photoelectric effect the assumption that we are making is that u_p is equal to the photon density per unit volume that is what we are writing into $h\nu$

so i am writing it again

so that it may settle in your mind what we want to do is to equate these two

expressions that is what we want to do

so i use classical mechanics in order to classical electromagnetic theory in order to get the energy density of the electromagnetic wave then i make use of the planck hypothesis in order to obtain information on the photon density how many photons are there per unit volume and using these two seemingly contradictory concepts what i wish to do is to understand the photoelectric effect i have practically explained it to you in words

so all that we have to do is to do a little bit of more work and then put it on a quantitative footing

so at this point we should remember that the planck hypothesis was a reluctant explanation he did not believe in the concept of photon of limited validity limited applicability but what einstein did was to take it seriously

so here is an important concept that we have to keep in mind

so what is the great difference philosophically speaking from the viewpoint of the way we look at nature when we say that light can be looked upon as a collection of particles when you speak of a wave we think of a continuous function a wave where varies continuously your field varies continuously in space and time that means energy at any given time is continuously distributed over all space whatever the region is

so for example let us consider the field produced by a point particle let us go step by step

so fields classical fields let us look at the classical fields let me look at the field produced by a point particle i say that let us say that this has a charge q this is at a distance r the position vector is \mathbf{r} in some coordinate system i say that my electric field is simply given by q by r squared r hat that is what i write when i write this expression it is understood that this r can be taken anywhere this r can be taken anywhere that means i can take a test charge q place it here here here anywhere that you want wherever it is i experience an electric field which is given by this formula therefore my field is a continuous function of position if it is a function of time it is also a continuous function of time and that is the reason you are able to use gauss's law etcetera etcetera when you make use of the integration of the electric field or when you write the potential for instance in terms of the electric field what do we do when we if you want to get the field in terms of the potential we take the gradient of the potential we take the derivative of the potential

so potential is continuous my field is a continuous function what maxwell's theory is telling me is that this electric field also carries an energy

so what is the other example that i can think of the other example that i can think of is two capacitor plates

so i put a charge q here i put a charge minus q here

so let me write a capacitor then in between the two plates there is a constant electric field and we say this capacitor stores that much electric field or that much what energy capacitor is a storage mechanism for energy electric energy and whenever we want we will discharge it and the current will start flowing when i complete the circuit that is your examples with rc circuit lc circuit lcr circuit and

so on and

so forth which you people have will do in your network analysis or electromagnetic theory it stores the electric field and again the electric field in this case is given by $\epsilon_0 E^2$ that is what you are going to get it is proportional to this therefore my electric field is again a continuous function that is my energy density is also a continuous function that means whenever i think of a field i am imagining a continuum that is the great contribution of maxwell that is what he said but when i speak of a particle

nature of light i am imagining a stream of particles and the very notion of a particle means there is discontinuity that means there is a discontinuity and there is necessarily gap between the two particles that is what we say for example when i say that my water is at a certain density or this pen is at a certain density i know that it is a crude approximation because if i look deep inside it using a microscope it is discrete because there is a lot of space between different atoms that is what we mean

so when i speak of particle nature what are we saying we are speaking of discontinuity in space and time therefore when i say that i am going to replace a continuum description of energy or momentum carried by a certain physical phenomenon by a discrete description of the same energy or momentum we are making a radical change continuity to discrete and this is something that bothered einstein a lot

so in one of my earlier lectures i advised all of you to go and look up the original paper of einstein it is eminently readable it is as easy to read as your twelfth standard textbook it is nicely very very very well nicely returned einstein raises this question as to how one can replace a continuum description by a discrete description and he justifies that to two simple words in his paper and that is what you see on your screen he says temporal scale that is the most important word he uses the word temporal skin

so we should be able to understand what we mean by the word temporal scale so let me spend some time on that

so wave nature seen over large temporal scales this is the fundamental observation of einstein

so as i told you my electromagnetic wave is coming and it has a frequency of 10^{14} hertz that is what i have and if i perform a double slit experiment for example and if i put the detectors

so what is your detector the human eye i keep on scanning and i see maxima and minima i know that my light does not respond have a response time of 10^{-14} seconds i don't have that

so what i see is a highly time averaged thing

so you can imagine for example a stream of particles which are coming and let us say the changes in their positions are of the order of 10^{-14} seconds going up and down or whatever whereas the resolution of your eye or your detector is of the order of a millisecond or of the order of a fraction of a second then there is a pretension there is an approximation that you are going to do that is it is as if you have a continuous distribution of matter that is what we are going to say simply because of the temporal fluctuation

so whenever we are going to average over such time scale over orders of 10^8 or 10^{10} or 10^{12} even discrete appears to be continuous just as everything around us appear to be continuous although they are deep down made of molecules by the way when einstein wrote this there was no direct evidence for the molecule or nature of matter that came from his brownian motion paper but this is the fundamental observation that einstein made he said that your evidence for diffraction interference everything comes because you have averaged over 10^{14} oscillations or 10^{12} oscillations whereas the emission of an electron is taking place over a much much smaller time scale in fact i gave you an estimate and i told you it is of the order of 10^{-9} seconds that is what i told you at that scale perhaps it is not a continuum description perhaps the particular nature of light can be seen and therefore einstein goes on to propose that we need two different phenomena whatever takes very very small intervals of time you would need the particle nature of light and whatever is averaged over

large time scales you can sort of approximate the discrete nature by a continuum nature

so einstein says that it is not too unrealistic for us to assume that light is made up of a large number of particles this is the justification

so i am going to repeat whatever i told you

so the two people can read it on the screen what einstein did was to essentially make two simple but radical assumptions

so let me read whatever is on the screen two simple but radical assumptions the first assumption is that the incident radiation of frequency ν can be looked upon as a stream of photon gas with each photon carrying an energy $h\nu$ i have explained that at great detail now that is a qualitative assumption but in order to understand the milliken experiment which was done very very carefully we need more quantitative assumptions and actually i am going to spend quite some time discussing those quantitative assumptions

so let me make the assumption electrons in the metal escape to free space by transfer of energy from an individual photon

so i should explain that to you

so what are we saying

so you have a metal here and you have a photon stream which is coming here and the electrons are getting ejected

so this is my light as einstein would imagine and these are my electrons that are coming now what is happening there is a transfer of the energy from light to the electrons through photons now the big question that einstein is asking himself is how many photons are required in order to eject an electron are you people understanding that that's what i am trying to say

so what is the assumption that we are making

so you have a work function ϕ this has the unit of energy

so let us say it is some 3 electron volts now what is the experiment telling me the experiment is telling me that new minimum required in order to eject the electron is given by what ϕ by h that is what is telling me from the viewpoint of the particle nature it is perfectly possible the two electrons of frequency new minimum by two they could have gone hit the electron and the electron would have come out

so imagine there is an electron the first photon goes and hits it it transfers its energy the second photon goes and hits it it transfers some more energy both of them add up to this work function whatever that energy is and it comes out that means it is possible that more than one electron can actually be absorbed by the more than one photon can be actually absorbed by the electron in order to come out but that is against the experimental result in that case there would have been no new minimum even if the frequency was one third some electrons would have been produced by the absorption of three photons if it is one tenth some electrons would have been uh produced by the absorption of 10 electrons if you work out the photon density will be 10 to the power of 12 10 to the power of 13 10 to the power of 14

so out of 10 to the power of 14 if some thousand or ten thousand or even the ten million are absorbed it really doesn't matter in order to produce one tenth of them you would have been able to see electron

so what einstein is saying is that for every electron that is coming out of the metal there is exactly one photon that is absorbed of course in this absorption process einstein is invoking a time honored law which can never be probably violated in nature and that is the conservation of energy

so let us go back to your screen whatever is there

so let us look at what i have typed on my computer the incident radiation of frequency ν can be looked upon as a stream of photon gas electrons get emitted

by absorption of a single photon because that is what the experiment is telling me the third one is very very important energy is strictly conserved in the process that is very very important for us maximum kinetic energy corresponds to complete absorption of photon this is very important for us

so let me explain that to you and that is important again i have a metal and i have my radiation coming and my electron is coming i argued that corresponding to every single electron that is coming i need a single photon that is an experimental result but now there is a converse is it necessary that my electron should absorb all the energy in the photon is it not possible that my electron absorbs only a part of the energy how is it like it is like collection of two particles

so i have a particle here i have a particle here

so this particle comes hits it and goes away

so in the final state both of them are moving part of the energy is taken by this particle part of the energy is taken by this particle such a thing is possible but then depending on how much energy is transferred to the particle this follows particle energy becomes smaller and smaller

so i have a b a b as the energy of b becomes larger and larger the final energy of a becomes smaller and smaller

so i am telling you that when b acquires maximum energy a has lost all its energy

so that is very important otherwise you cannot understand the stopping potential

so mr einstein tells us maximum kinetic energy

so let me read that carefully again for you which is written on my computer maximum kinetic energy corresponds to complete absorption of photon now if you sort of look at them very very carefully you see there is a fundamental difference in the approach of planck and einstein planck made the assumption of photon he explained black body radiation what do you do with it what do you do with that photon but here einstein is opening up a new world he's saying that oh some photons may not give their energy completely that means i must be able to see it experimentally

so it is opening up a new world of further experimental evidence is that okay in fact that is what i am going to do at the end therefore this is significantly a better approach and an improvement over what the planck hypothesis is mean it is by no means to discredit or disrespect the planck hypothesis but in this context we have to understand that this was really lot of insight that was involved in it

so i simply wrote down the points because i am sure you people will solve hundreds and hundreds of problems per permuting stopping energy stopping potential maximum energy incoming frequency number of electrons ejected number of photons in the density

so let me not get into that

so let me read it out suppose the work function of an electron is ϕ then ϕ is the minimum energy required to amine electron by irradiation electrons completely absorb photon for their emission that is the maximum energy electron and each photon carries an energy that is what planck is saying

so if i combined all of them the minimum energy is nothing but ϕ by h that is the minimum frequency no greater than ϕ/h the current increases with intensity i already showed you and energy is strictly conserved in the process

so these are the points that we made and what do we get if you plug it all that is contained in these equations and i am going to write them down again for you

so the incoming energy is $h\nu$ i am going to make use of conservation of energy

what is the outgoing energy outgoing energy is that of an electron

so this is photon this is the electron

so i am writing e_f is equal to e electron what am i going to write oh i am sorry that is not what i should write i should write the total energy the total energy of my in the final state is maximum kinetic energy plus ϕ naught

so what is the assumption that i am making here the incident energy is entirely due to the photon

so let me write that incident energy initial energy of the electron is zero

so when i am writing it to be zero what do i mean by that by that i mean that its energy is negligible

so you people can figure out what the energy of the electron is

so the initial total energy is given by $h\nu$ the final total energy is the maximum kinetic energy of the electron because it has absorbed the photon completely but in doing

so some work had to be done in order to liberate some energy had to be given and that is your ϕ naught and that is what we are writing

so we have to write these two equations and this ϕ naught is what we call $h\nu_0$ because ν_0 is the minimum energy that is required therefore what do we write we combine these two equations and write h into ν minus ν_0 $h\nu - h\nu_0$ is equal to e kinetic maximum that is what i am writing $h\nu - h\nu_0 = e_{\text{max}}$ because initial total energy is the title total final energy and this was nothing but the result of miracle because he looked at this stopping power of the electron that corresponds to the maximum energy of the photon therefore now you see this very very simple explanation is able to completely describe all the experiments performed by lennard milliken and earlier by halo walk and hertz

so in a sense this completes the description or the discussion of photoelectric effect but it does not end there we have to do a few more things and let us see what we have to do where is the energy of the photon that is a very very important question that we have to ask where is the energy of the photon

so what are we asking if photon is a particle what is its mass that is a very very important question that we have to answer

so let me do a very naive calculation which is completely wrong electromagnetic radiation moves with the speed c

so what is mc^2 this must be 3×10^8 meters per second

so that is what i have if my photon has a mass m what will newton tell us

so newton would tell us that my energy of the photon should be given by $\frac{1}{2} mc^2$ that is what newton would give us where m is the mass of the photon

so newton would tell us mass of a photon is fixed speed of a photon is fixed therefore all photons should come with the same energy but planck and einstein are telling us that the energy of a photon can change depending on its frequency now we see that there is a mismatch between what the newtonian expression for energy is and what the planck einstein expression for energy yes although both of them are using what the same particle description

so unless we answer this question then we are not doing a proper job the word particle would be a meaningless thing and there is an answer to this what is the answer to this the answer lies in relativity

so remember i told you that einstein wrote three great papers in 1905 it was photoelectric effect it was brownian motion and it was relativity

so it is remarkable that relativity should save us from this problem but it is not going to save us through a simple expression it requires an insight it requires a certain amount of smartness and let me tell you what it is

so you people have all heard of mass energy equivalence

so what does einstein tell us according to relativity my total energy of a

particle is given by $m_0 c^2 / \sqrt{1 - v^2/c^2}$ and this is not equal to $\frac{1}{2} m_0 v^2$ we know that this is not equal to $\frac{1}{2} m_0 v^2$

so this is my expression for the energy but still it does not help me because if i put v equal to c what will happen to this if i put v equal to c denominator becomes 0 this tells me e is equal to infinity

so maybe i was hasty in telling you that einstein or relativity will save us it seems to be a contradiction

so we are infatual newton tells us that all photons should come with the same energy and this expression for energy is telling us what all photons have what energy infinite energy we seem to be in trouble but that is not the way we should approach that we should do a little bit of more careful analysis

so let us see what we have to do the way to go around is to look back at the complete maxwell's equation

so according to maxwell's equation my energy density is given by $\epsilon_0 E^2$ and my momentum density

so imagine a plane wave which is coming and i ask how much momentum is there per unit volume

so i will denote it by π this is my momentum density that is given by u/c this is dimensionally correct

so both this equation comes from maxwell

so what einstein would say or what we would say with hindsight is that if you are going to give a particle description for the photon you should not only be consistent with the energy density description you should also be consistent with what the momentum density description this is my momentum density after all in the photoelectric effect not only did the electron absorb energy it also absorbed momentum that is why it is moving at a certain velocity different velocities and that momentum transfer is because of this π we should be able to write that

so now what i will do i will write u equal to the number density into $h\nu$ and i will write π is equal to number density u sorry π is equal to u/c

so what will it tell me this will tell me that the momentum carried by each photon

so i will denote it by e_γ e_γ is $h\nu$ and p_γ momentum carried by each photon is $h\nu/c$ that is what this relation tells me

so when i try to establish a consistency this is photoelectric effect where momentum was not very important for us but consistency demands that i should associate in energy this i have to worry about the energy sorry momentum and i have to energy worry about the p now in order to reconcile that what we have to do is to go back and rewrite the energy momentum relations in a slightly different language that is the achievement

so let me do that in your next slide

so what is the next slide i am going to write it two expressions all of you are familiar with this

so my energy density is given by $m_0 c^2 / \sqrt{1 - v^2/c^2}$ and momentum this is the relativistic expression for momentum is $m_0 v / \sqrt{1 - v^2/c^2}$

so i want to make two observations at this particular point these expressions are meaningless when i put v equal to c because i will get energy and momentum divergent these expressions are trivial if i put m_0 equal to 0 because classically there is no particle if there is no mass is that right

so put m_0 equal to 0 implies e equal to p equal to 0 v equal to c implies e equal to p equal to infinity both of them are meaningless but now i will ask what happens if i go to m_0 going to 0 and we go to c now what is happening

the numerator is going to 0 the denominator is going to 0

so probably there is a consistent way of understanding this that is what we have to look at

so in other words we are going to look at the limit m naught going to 0 c going to infinity we ask are there non-trivial solutions just as maxwell obtained non-trivial solutions in free space equations were obtained in the presence of what currents and charge densities but the solutions were obtained even in the absence of currents and charge densities that is how we got the wave solution we are going to look at that and the trick is to eliminate v

so let us do that

so i am going to write that again e gamma is equal to $h \nu$ and p gamma equal to $h \nu$ by c this is what maxwell plus blank einstein are telling us

so that means if i want to get a consistent description of the particle nature of light i should not only worry about e gamma equal to $h \nu$ we have already accommodated that we also worry about how to incorporate this expression that the momentum carried by each photon should be $h \nu$ by c because we had two expressions one for the energy density another for the momentum density and there was a natural relation between them what was the natural relation we wrote u equal to πc this is my energy density this is my momentum density for a monochromatic plane wave we have to reconcile these two expressions with the expressions for energy and momentum that come for a particle that is what we have to do

so in order to do that i will make a fundamental observation

so we are now looking at particles relativistic i need two expressions the first expression is for the energy which i will write m naught c squared over root of one minus v squared by c square and the next expression is for the momentum when i write m naught v over root of one minus v squared by c square momentum is of course a vector but imagine that it is moving along only one direction

so i have not drawn the vector sign otherwise if you want i can put a vector sign over that no big deal now there are two important features that we notice both of them are problematic but if we combine both of them perhaps we can get rid of the situation that we are in what is the thing the thing is that if m naught equal to 0 e equal to p equal to 0 which is observed that means there is nothing to do there is no physical system on the other hand if v equal to c we get k equal to p equal to infinity which is observed because we don't see any particle with infinite energy

so what are we saying you can't see a particle with zero energy it doesn't exist at all no energy no momentum by energy i mean actually the rest energy included remember m naught c squared is there not just the kinetic energy

so both m equal to zero and v equal to c are two extreme limits which are meaningless now what we want to do is to reconcile the two by taking both of them together and try to get a consistent relation and the way we do that is to eliminate m naught between e and p and ask if there are non trivial solutions

so let me repeat that

so a is equal to m naught c squared over 1 minus v squared by c squared you will use it in your mass defect formula my p is m naught v over root one minus v squared by c square

so what will i do i will take the square of e i will take the square of p i will get e squared equal to m naught squared c to the power of 4 1 minus v squared by c square my p square will be m naught square v squared over one minus v square by c square a simple calculation will tell you i am not going to work out and that is you can write e squared equal to m naught square c square c to the power of 4 plus p naught square c square is a very very simple expression

so you can check that E^2 is nothing but $p^2 c^2 + m^2 c^4$

so what is happening here both E and p were proportional to m now the relation between E and p I should not write $p = \frac{E}{c}$ I am very sorry about that let me rewrite the expression again my E^2 is equal to $p^2 c^2 + m^2 c^4$

so here if you look at the relation between energy and p it is not a homogeneous relation E^2 is proportional to $p^2 c^2$ and this is an inhomogeneous term in homogeneous term and that means if I put $m = 0$ there are still non trivial solutions

so if I put $m = 0$ I will get $E = pc$ which is exactly what Maxwell is saying through the relation between the energy density and the momentum density

so what are we saying we are saying that there are two kinds of particles there are particles with rest mass that is $m \neq 0$ these particles can never move with the speed of light if you try to make them move with the speed of light their energy goes to infinity therefore they can never move but on the other hand there are particles which are all the time moving with the speed of light but what is their rest mass their rest mass is equal to 0 this corresponds the response is equal to 0 therefore there is no contradiction we have put $m = 0$ and $v = c$ this equation however does not tell us how the energy changes and how the momentum changes that depends on the physical system and Maxwell is telling us it changes because of the frequency that is the input coming from the Maxwell equation therefore we see that it makes perfect sense to speak of the particle nature of light provided we combine that with the concept of relativity and we all know that when the speed of a particle becomes larger and larger when we are looking at higher speeds we cannot use Newtonian mechanics but we have to use Einsteinian mechanics and this is what we have

so this is a very very important thing for us now what I am going to do is to give additional experimental evidences evidence I cannot discuss it in great detail but that should restore your faith in the concept of a photon and let us see what it is if Einstein is correct all these should be valid

so first let us look at scattering of light light is all the time getting scattered when the sun's light reaches us it gets scattered by the atmosphere in fact much of it gets reflected from the ionosphere because of the ultraviolet

so in the scattering of light there is what is called as Rayleigh scattering where the initial frequency is equal to the final frequency the incoming frequency is same as the final frequency nothing is going to happen only the direction of the propagation changes but the energy does not change or the frequency does not change but if you believe in the Einstein hypothesis there is another possibility what is the possibility your radiation was coming inside part of the energy can go to the electron but part of the electron can a photon can get scattered that means if I look at the scattered photon or scattered light I do not just look at the electron some of the scattered light must have less frequency than the incoming one that is what Einstein is saying such a phenomena exists that is the interesting thing it existed even during the time of Einstein and that is called Stokes law that is called Stokes law what does Stokes law tell you it tells you that there is a new here there is an electron which is coming and this new is partially absorbed I am showing it schematically

so this is my initial energy this is my final frequency ν_f is less than ν_i because only part of the energy has been transferred to the electron therefore $\lambda_f > \lambda_i$ Stokes had observed this relation which is not easy to understand from the wave view point of light but here it is a very natural thing and Einstein said you see there is another experimental evidence

for the particle nature of light namely this is the stokes law this is what is called as a stokes law

so we have discussed an additional experimental evidence which we did not think of earlier that is why i said that einstein's explanation of photoelectric effect is more robust than the planck hypothesis now if that be true i should be able to perform an experiment where both momentum and energy can be monitored can be monitored and that is called compton scattering

so imagine there is a photon coming here there is an electron coming here and a photon goes out and an electron goes out gamma gamma electron electron you can look upon this as a perfectly elastic collision between two particles where the total energy and the total momentum is conserved that is called compton scattering and i believe quantum scattering was first seen sometime around 1911 or 1912 and if you make use of the full relation $e = h\nu$ $p = \frac{h\nu}{c}$ you should be able to understand the last one that we are interested in is what is called as the raman scattering and i will leave you people after this einstein assumed that in photoelectric effect only one photon is absorbed in photoelectric effect but there is no physical principle which tells us that only one photon should be absorbed nobody told me that electron should interact with only one photon at a time

so what should i say the correct way of formulating this principle is to say that the probability for the interaction between one photon and one electron is much much larger compared to the probability of an electron interacting with two photons or there is also the other possibility that the electron itself gives the energy to the photon in a certain situation

so what happens in the case of raman effect is actually where the electron gets scattered and actually it can acquire a higher energy all this time we assumed that my photon is all the time transferring energy to the electron but electron can also transfer to all can also transfer energy to photon in that case what should happen the scattered electron should have a higher frequency these are called anti-stock lines these are called anti-stokes lines and that is the famous raman effect which can again be understood as a consequence of the photon hypothesis

so to conclude what we have done is to give a careful attention to the experimental facts and realize that it cannot be understood in terms of the wave description but we also said that the wave description has a very robust experimental foundation in terms of interference and diffraction then we said however interference and diffraction phenomena that we observe involves very large time scales compared to the fundamental time scale of the oscillation whereas in photoelectric emission very top small time scales are allowed and we said the particle nature is not a very unreasonable thing that is what einstein said and we were able to explain the photoelectric effect and like any good model this model exposes itself it opens up additional avenues to verify that hypothesis and we showed three particular phenomena one is the stokes law where the scattered radiation can come with a higher wavelength or a lower frequency it transfers a part of the energy and a part of the momentum or compton scattering that takes place between a free electron and a free photon whereas here the electrons are all bound there you have to worry about the momentum of the electron and the momentum of the photon that is completely consistent with the particle description and finally i also mentioned in the passing raman effect where actually the outgoing radiation can have a higher frequency where the photon is in fact gaining energy i have not done complete justice to the last part

so don't worry too much about it if you are not following but the fact is that what einstein did is not simply a figment of imagination or some kind of a slate

of hand there is a lot of thought that has gone into it

so if this indeed be the case that what is classically a wave can exhibit wave like phenomenon a natural question arises maybe what is classically a particle can also exhibit a wave like phenomenon and this was the great intuition great insight of prince louis d brawley and that we are going to take up from the next class

so now let us stop have a good time you

Prutor@MITK