

welcome to the lecture module on optics we have been discussing wave optics in the last few lectures we discussed about two important phenomena in wave optics namely interference and diffraction today we will discuss about polarization an important characteristic of light this is the last topic that we will have in this module of wave optics polarization is an important characteristic of light so we will discuss about polarization polarization of light in this lecture we will see what is polarization and the state of polarization of a light how a state of polarization of light is defined why do we need to know and define the state of polarization of light how to obtain polarized light polarized light by reflection this is one of the techniques obtaining polarized light by reflection at the brewster angle and then we will discuss about propagation of plane polarized light through one or more polarizers

so first as polarization of light

so polarization of light is a property of light it is a an important property of light light is an electromagnetic wave comprising of rapidly varying electric and magnetic fields and the electric and magnetic fields are perpendicular to each other and perpendicular to the direction of propagation we have studied this in electromagnetic theory em theory that the light comprises of electric and magnetic fields which are perpendicular to each other and perpendicular to the direction of propagation polarization of light refers to the direction of oscillation of the electric field of light it refers to the direction of oscillation of the electric field we will see what is what these statements mean light waves light is an electromagnetic wave i have shown here an electromagnetic wave propagating in the x direction we see the blue colored one here variation shows the electric field vector the variation of the electric field vector along x at any given instant and the red color shows variation of the magnetic field vector

so we can see that the magnetic field vector in this diagram is along the z axis here this is the x axis this is the y axis and is the z axis the electric field variation is in the y direction

so it's increasing and decreasing its varying sinusoidally and along with that the magnetic field also varies sinusoidally but in a perpendicular direction and both the electric field and the magnetic field are perpendicular to the direction of propagation electric field is along y direction magnetic field is along z direction and the propagation is along the x direction as we discussed polarization refers to the direction of oscillation of the electric field therefore in this particular diagram here the electromagnetic wave that we have shown now we forget about the red red coloured variations here the magnetic field if we look at only the electric field variation we can see that the electric field is varying in the y direction only it is confined to the x y plane the x y plane and therefore this is a y polarized wave this is a y polarized wave now lets look at this more carefully the state of polarization s

o p the state of polar now i have dropped the magnetic field variation and i have shown only the electric field variation here and the electric field is varying sinusoidally in this direction now if you look from the direction x here a wave which is coming towards you when you look in a direction perpendicular so in a if you look at this in a plane perpendicular to the direction of propagation

so that is what is shown here this is a plane perpend x is the direction of propagation and a plane perpendicular to the direction of propagation is this it is the y z plane the y z plane is perpendicular to the direction of propagation what we see is as the electric field varies in this direction the electric field is becoming positive it is becoming negative then positive negative and

so on because we know that the electric field can be represented as a

sinusoidal wave

so we can write for example the electric field e is equal to some amplitude e_0 into $\sin(\omega t - kx)$ or $\sin(kx - \omega t)$ k is the direction of propagation and in this case the direction of propagation is x

so $\sin(kx - \omega t)$ t is the time ω is the angular frequency

so this is equal to 2π into ν angular frequency ν is the frequency of the wave which is equal to c/λ where c is the velocity of light and λ is the wavelength of light

so this is an electromagnetic wave propagating in the plus x direction

so that is what is shown here that the state of polarization

so if we look at a projection here the electric field is varying in this fashion but in the projection we see that the electric field is becoming positive negative but all along this line y and therefore the projection on a plane perpendicular to the direction of propagation is a line and therefore such a electromagnetic wave is called linearly polarized wave the projection of the electric field on a plane perpendicular to the direction of propagation is a line hence the name linearly polarized wave the state of polarization of any wave is given by the projection of the locus of the tip of the electric field you can see it is the tip of the electric field the tip of the electric field is always lying on this line as it becomes maximum then reduces then becomes negative but

so it is the tip of the electric field is the projection of the locus of the tip of the electric field vector on a plane perpendicular to the direction of propagation one need not remember this definition but this explains any state of polarization in this course we will see primarily linearly polarized light but there are different states of polarization namely circularly polarized light elliptically polarized light and

so on

so this definition given here will help in identifying the state of polarization of light

so we will primarily discuss linearly polarized light now i have taken a wave an electromagnetic wave where the electric field vary is varying in the x z direction this is the z direction

so now electric field is varying in the z direction like this obviously the magnetic field is varying in the y direction but i have not shown the magnetic field

so if we look at the projection now in a plane perpendicular to the direction of propagation then the electric field will vary in the z direction and therefore this is a linearly polarized wave this wave is a linearly polarized wave but it is now z polarized wave if we look at this plane now the electric field is confined here to the x z plane x z plane the dotted plane here represents the x z plane and if i show the x that plane in two dimension now x z plane the electric field is varying like this basically i have flipped this up like this then what you see is this

so y direction is now into the board into the paper here and z is here and x is in this direction and what we see is the variation of the electric field and the electric field is confined to the x z plane and therefore linearly polarized light is also called plane polarized light the e field oscillations are confined to the x z plane in this wave here this is the two dimensional picture and then these are called plane polarized light and therefore linearly polarized or plane polarized mean the same thing now let us look at unpolarized light little bit carefully and then we appreciate the plane polarized light

so what i have shown is a beam of unpolarized light light from common sources such as sun electric bulb or a fluorescent lamp etcetera are unpolarized in

nature what is this unpolarized light

so for example what i have shown here is a torch a battery torch a beam of light coming out from a battery torch actually the beam comprises of large number of component waves these are the component waves these are waves which are emitted by different parts of the emitter this torch bulb has a filament and different parts of the filament give out different component waves these are all independent waves which are emitted by different parts of the filament and therefore what i have shown here is the component waves now the beam of light comprises of a large number of component waves the component waves are emitted by different atomic oscillators different atomic oscillators from different parts of the source electromagnetic radiation is emitted by atomic oscillators or dipoles oscillating dipoles this concept is probably a little advanced to our level here but they are small tiny oscillators which emit the electromagnetic radiation and different parts

so the component waves are emitted by different atomic oscillators and therefore what would happen is that let me show it here that a particular oscillator which is emitting would have probably the plane of polarization like this another oscillator which is oscillating like this would oscillate the wave emitted will have the electric field varying in this another oscillator which is oscillating could be oscillating in a different direction another oscillator oscillating in this direction may have a different plane of polarization and therefore if you look at the cross section here that is the x axis is here

so we are looking at the cross section

so here

so this is the y and this is the z axis and x is coming out then we will have linearly each one of them is linearly polarized but we will have vibrations varying like this some in the y direction some in the z direction some at various angles in other words the polarizations are random each of the component waves each of the component waves

so this is what is explained here the component waves are emitted by different atomic oscillators from different parts of the source they may have different plane of oscillations and therefore the combination forms a randomly polarized beam or a unpolarized light

so that is what i have been explaining here that different oscillators will have plane of polarization or the line at in different angles and therefore the net result is a randomly polarized light

so if you see the cross section again let me show it

so here then some of them may oscillate like this some of them may be in a plane like this

so this is the representation of randomly polarized light and this is also called unpolarized light

so the unpolarized light will have the plane of oscillations of different components in different directions and therefore it is sometimes called randomly polarized light or commonly called unpolarized light now the electric field if you look at the electric field vector here

so it is varying in this direction field is a electric field is a vector and therefore we can always resolve it along two perpendicular components

so if we have an electric field which is varying like this then we can resolve it into two components

so this component here

so one component here and one component

so this comprises of

so when it becomes reverse

so this component will come here and this component will be negative here and

therefore what we will have is the component is varying like this and it is varying like this

so this one electric field variation can be represented equivalently by components varying like this

so this is an electric field which has some component for example $y \cap e$ if i want to write the electric field e of this vector here then $y \cap y$ is the unit vector in the direction y here

so that is y and this is $z \cap e$ plus $z \cap e$ where e_z is the z component each one of them is oscillating one is oscillating like this and the other one is oscillating like this therefore every component which is shown here whether it is this component or this component any randomly oriented components can be resolved along the x direction and y direction and the net unpolarized light can be equivalently represented here in the form of one y component and one z component this is an equivalent representation but actually the electric field is varying randomly in different directions

so that is what is explained here again that the electric field vectors of the randomly oriented polarizations are resolved into their components along y and z directions in the equivalent representation here unpolarized light comprises of two equal component both the components are equal in magnitude because there are random polarizations in all directions and therefore on an average we will have equal both the components are equal in two orthogonal directions comprises of equal components of the electric field light field of light in y and z directions

so therefore now onwards we will represent unpolarized in light in this fashion so the representation of polarized light

so in this here i am showing the representation summary of the discussions that we had

so far

so representation of polarized light assuming the following coordinate system here with the x axis as the direction of propagation

so y is here z is into the board and therefore we have y polarized light means is represented like this that is this is the direction of propagation and the electric field is oscillating in the direction y similarly z polarized light direction of propagation x and the electric field is perpendicular to the plane of the paper here that's why it is shown as a dot and unpolarized light will have both y component and the z component and therefore unpolarized wave is represented like this in two d what we have shown is in two dimension if you look at the cross section from this direction if you look in the x direction then you will see the cross section is a $x \cap y \cap z$ plane

so in the $y \cap z$ plane we see that

so this is x coming out of the paper and we have y polarized light will look like this and z polarized light will look like this its horizontal and this is vertical and an unpolarized wave can be represented by two arrows which are at this double sided arrow all the arrows as you can see are double sided arrow the double siding comes because electric field once become positive other time becomes negative that's why it is always represented by double sided arrows and this is the representation of polarized light including unpolarized or randomly polarized way now the next question would be how to obtain polarized light how to obtain polarized light the answer is straight forward the answer is here by passing unpolarized light through a polarizer now there are different types of polarizer polarizer is a device or a instrument or a component which polarizes that means if you launch a particular state of polarization then it will it can change the state of polarization to something else or it can polarize it can polarize an unpolarized light that is if you launch an unpolarized light then

the output of the polarizer will be plane polarized light there are different types of polarizers based on different working principle the simplest least expensive and most widely used are sheet polarizers or a polaroid sheet these are simple sheets i do not have a sheet here right now but these are small sheets which are widely used in laboratories now let us discuss a little bit about this simple polarizer tolerance sheet

so polaroid sheet or sheet polarizer

so what i have shown here is a sheet where i have shown some molecules

so the polaroid comprises of a sheet of certain long chain polymeric molecules these are polymers actually polymers are long chain molecules comprising of a number of atoms or molecules

so these are long chains

so long chain polymeric molecules which are almost aligned like a wire grid

so you can see the polymer molecules are all aligned here in this case there are techniques of making it aligned

so a polaroid sheet comprises of long chain polymeric molecules which are almost aligned like a wire grid wire grid is what you see here the grid here

so it is a grid

so it is aligned all of them in the form of as if it is a wire grid now the polarization component which is parallel to the long chain suffers loss now if light is incident like this unpolarized light is incident like this

so that is what is shown here unpolarized comprising of two orthogonal components here we have resolved one of the component along the molecule and another component perpendicular to the molecule perpendicular to these chains long chains the polarization component which is parallel to the long chain suffers loss or attenuation or it undergoes loss whereas the component which is perpendicular here hardly suffers any loss which means if you incident polarized light here then this component will pass through with very little loss but the other component will be highly absorbed or highly attenuated

so on the other side of the sheet what you will get is a polarized output

so in this figure the vertical component in the figure is attenuated that is is absorbed the horizontal component passes through the sheet with very little loss and therefore the horizontal axis is called the pass axis of the polarizer pass axis of the polarizer refers to that axis which allows polarization of light to pass through

so in this case the path axis is horizontal let me repeat again that the vertical polarization here undergoes loss but the horizontal polarization passes through the sheet and therefore the horizontal axis here is called the pass axis let us make it more clear lets look at the diagram again in a different way

so unpolarized light passing through a polarizer here is a polaroid sheet and unpolarized light is incident as usual we have resolved it into two component one component parallel to the path axis and another component perpendicular to the path axis the component which is parallel to the path axis passes through the sheet the perpendicular component is blocked and therefore we get 50 percent of the light passing through because as we have already discussed unpolarized light can be thought of as comprising of two components one vertical component and another horizontal component each of them fifty percent fifty percent strength equal strength and therefore fifty percent of the electric field is attenuated and fifty percent passes which means if i had the input intensity i zero then what we will have on the other side is i zero by two because fifty percent light is blocked by the polarizer but on the other side we get a plane polarized light

so plane polarized light on the other side with the plane of polarization parallel to the path axis of the polarizer

so this is the way it works

so I_0 by 2 is the output intensity of course we have neglected absorption of the vertical component there is a little bit of absorption even for the vertical component even though it is passing through the axis but if otherwise actually in practice it is slightly less than I_0 by two but we neglect the absorption and say that if I_0 is the input then I_0 by 2 is the output of the on the other side now what if we rotate the polarizer

so instead of this we rotate the

so unpolarized light is coming here the polarizer we are rotating that is the path axis we are rotating

so what would happen because if the path axis for example if the path axis is like this then we can always resolve this randomly polarized light into one component parallel to the path axis and the other component perpendicular to the path axis as before the component parallel to the path axis will be on the other side but the perpendicular component will be blocked however now the output state of polarization

so let me show this here if i consider the path axis to be like this at an angle then what we do is the light which is coming here will be resolved one like this component and the other component will be perpendicular to this the perpendicular component would be blocked and then on the other side we will have light which is polarized like this

so the if i rotate the polarizer or if i rotate the polarizer then the plane of polarization at the output also rotates earlier we had the polarizer pass axis like this

so at the output we had polarization coming like this output polarization vertically polarized now we have rotated the path axis what is shown here is the path axis then the plane of polarization will be rotated but 50 percent of light will still come on the other side

so if we had I_0 here we still have I_0 by two independent of the rotation of the path axis what it means is if you launch unpolarized light passing through a polarizer then if you rotate what if we rotate the polarizer if you rotate the polarizer about an axis obviously the path axis rotates but there is no change in the intensity of light at the output we have already answered this question why unpolarized light passing through a polarizer this is one of the ways one of the ways is to use a polarizer sheet or a polaroid sheet but there is another important technique that is polarization by reflection

so let us look at the second technique which is polarization by reflection of light now let us first look at this recall reflection of light at a plane interface in ray optics earlier we had discussed reflection of light at an interface and we had also discussed the snell's law

so here there we had discussed in terms of ray here the ray represents the direction of propagation of the wave the wave is the electric and magnetic fields are perpendicular to the direction of propagation and this is the direction of propagation

so the wave is incident here the wave now as per snell's law we know that $\sin i$ by $\sin r$ r is this i is the angle of incidence that is the angle between the normal and the direction of incidence here i and r is the angle of refraction these are two different media of refractive index n_1 and n_2 and this is the interface we are looking at reflection of light at an interface now snell's law says that $\sin i$ by $\sin r$ is n_2 by n_1 which is also written as $n_2 \sin r = n_1 \sin i$ now let us start with here a small angle the first one here the black one

so i and then this is r this is the transmitted ray this is the reflected ray the reflected wave or ray here has the same angle i

so reflected angle is equal to angle of incidence if i increase the angle

further we look at the blue line here then this is reflected here and the transmitted ray or transmitted wave is here if i further increase then to an angle i designate as i_b we will know that b is standing for brewster i_b then the reflected ray is here again the reflected ray makes an angle i_b recalling from the ray optics here and then this is r_b but an important observation which we did not discuss there is this that this angle i_b is an angle at which the angle between the reflected ray and the transmitted ray is 90° here it is 90° degrees and therefore we can write r_b is 90° minus i_b the r_b is 90° minus i_b and therefore from the figure you can see clearly here that this is i_b therefore this is 90° minus i_b and if this is r_b then r_b will be equal to 90° minus i_b from the figure and therefore $\sin i_b$ by $\sin r_b$ is equal to $\sin i_b$ by $\sin 90^\circ$ minus i_b which is $\cos i_b$ which is equal to $\tan i_b$ and therefore $\tan i_b$ is equal to n_2/n_1 and this is called the brewster's law and the angle of incident is i_b is known as the brewster angle what is special about brewster angle this is ok this comes from the ray optics also and we know that this is the brewster angle and at which the refractive index for example if this was air refractive index 1 and this is some glass refractive index say 1.5 then we know that $\tan i_b$ will be equal to n_2/n_1 which is equal to n_2 that is the refractive index of glass

so we can determine refractive index of glass if we know the brewster's angle but how to find what is special about the brewster angle

so let us discuss that reflection of unpolarized light at the brewster angle in the earlier diagram i did not show anything about the polarization now i am showing here in this diagram polarization of the light

so let us look at the diagram carefully there is unpolarized light which is incident at the brewster angle

so one component is we have resolved unpolarized light one component perpendicular to the plane of incidence that is out of the paper here and one component in the plane of incidence plane of incidence contains the normal the normal and the ray which is incident and the reflected

so this is the plane of incident and we have one component in the plane of incident and one component coming out of the paper that is perpendicular to the plane of what is observed is when light is incident at the brewster angle the reflected light here does not contain the component in the plane of incident it contains only the component which is perpendicular to the plane of incident which means it is completely polarized whereas the transmitted light contains both components the vertical component here as well as the in plane component and therefore it is called sometimes called partially polarized light because it contains both of them but the reflected light is perfectly plane polarized this is the example of course about the brewster angle but what is important for us is the reflected light does not have the in plane component the answer y is a little beyond the scope of our discussion but for the sake of completeness i will briefly explain why the reflected light does not have the transmitted component

so let me explain here

so here is the interface and light is incident like this and this is the transmitted light reflected light and we have the transmitted light when light is incident on any material or propagating in any medium

so this is a medium n_1 and this is a medium n_2

so light is a electromagnetic wave therefore it comprises of electric field varying like this the light is assumed that light is incident on a medium here the electric field is positive here and electric field is negative

so electric field positive negative when light enters the medium it the due to the electric field the medium comprises of atoms it is a matter which is made up

of atoms the or and molecules and if i look at individual atoms or individual molecules then the center when there is no electric field the centre of positive charge and centre of negative charge coincide at a point and the atom is neutral now when there is an electric field for example if you place this in an electric field you apply an electric field keeping this just a hypothetical situation place one atom between two plates and apply an electric field then the positive so if you apply an electric field that means if you apply positive here and negative here then the electric field is in this direction and the negative charge moves to the other side and positive charge moves to the towards the second electrode

so these are what i have shown are two electrodes and there is a atom with comprising of electrons and and the positively charged nucleus and then the charges separate because of the applied electric field and this such a entity here is called a dipole because now it is an entity which has a negative charge here and a positive charge here

so it is

so i am showing it like this its a dipole now this is a study dc field if i apply suppose i reverse the field then i will have the positive coming this side and negative charges moving to the other side i reverse again then i have positive negative here and positive here and

so on therefore when a time varying electric field is incident

so if you have a electric field which is changing with time as it propagates

so it is positive negative then this is equivalent to changing the field positive negative positive interchanging the positive negative

so in the medium this electric field of the light the varying electric field of the light induces what are called dipoles or induced dipoles

so induced dipoles this is beyond the scope of our discussion but just for the sake of completeness let me just describe it very briefly and the induced dipoles if i show the induced dipole here that is with the charge like this and changing at a later time like this because the electric field is varying at a different time and

so on

so this is at time t_1 this is at time t_2 and time t_3 and

so on its changing such a dipole emits

so this rapidly changing positive negative negative positive positive negative leads to emission of electromagnetic wave

so emission of e m wave of the same frequency e m wave of the same frequency same frequency the important point is if the dipole is here now this is at different times but it is the same dipole if the dipole is here becoming plus minus minus plus plus minus and

so on then this gives out radiation

so let me show the different color

so this gives out radiation in the transverse direction

so the electromagnetic wave it emits electromagnetic wave in all directions

so what i have shown are the field lines these are a matter of details but the important point that we need to know is there is no the fields are propagating in the transverse direction there is no field propagating along the axis of the dipole there is no electric field variation or the electric field variation is of the same frequency as that of light there is no field along the dipole now how is this related to the problem at hand

so let me put back the slide here when electric field variation is incident here or let me take a different slide let me draw that again one last time because this is not

so we have the electric field variations in one case the electric field is

varying like this in the other case the electric field is varying in a direction perpendicular therefore the medium here this is one medium of refractive index n_1 another medium of refractive index n_2 when the transmitted wave

so this is the transmitted wave this polarization at which is incident on the medium

so we have seen that at Brewster angle this is 90° this is 19° this is the incident light this is the reflected light and this is the transmitted light at Brewster angle this is 90° degree angle between the reflected and the transmitted light and therefore this variation here the dipole which is oscillating like this recall again the earlier when the dipole is oscillating like this there is no radiation or no electromagnetic wave propagates in a along the axis of the dipole exactly like that when the dipole is oscillating because of this field when the dipole is oscillating like this then there cannot be any radiation in this direction in the direction here there is no radiation in this direction because it's along the axis because this angle is 90° degree however the dipole which is oscillating like this does give wave propagating in this direction and therefore this polarization is reflected back but the other polarization is transmitted this polarization can only be transmitted there is no reflection this wherever you have not followed it does not matter because it is a slightly advanced concept but the important point that we need to know is this that the reflected light contains component only perpendicular to the plane of oscillation one additional point which I want to discuss before I take up the last topic in this is the electric field and intensity electric field and intensity of light consider light propagating this direction and it's polarized in this direction that is y polarized wave

so this is y polarized this is x is the direction of propagation the electric field can be written as $E_y = E_0 \sin(kx - \omega t)$ \hat{j} is a unit vector in the y direction sometimes we denote it as \hat{j} instead of \hat{y} E_0 is the amplitude and $\sin(kx - \omega t)$ this is called the phase term phase term and this is the amplitude we are recalling what we have studied under electromagnetic amplitude and first if you need to know the intensity then you take $\text{mod } E^2$ gives you the intensity which in this case will be equal to $\text{mod } E_0^2 \sin^2(kx - \omega t)$ the whole square now ω is $2\pi \nu$ angular frequency of light this is a new is very very big number therefore

so ν is of the order of ten to the power of fourteen or ten to the power of fifteen hertz for light and therefore this is an extremely rapidly varying function and therefore the mod square gives you an average you have to take an average of this and this will be equal to $\text{mod } E_0^2 \langle \sin^2(kx - \omega t) \rangle$ half that is $\text{mod } E_0^2 \langle \sin^2(kx - \omega t) \rangle$ if you take the mod square and take the time average this comes out to be half we have discussed this in the case of interference in the case of intensity measurement if we take an x polarized wave then we represent $E_x = E_0 \cos(kx - \omega t) \hat{x}$ that is the horizontal polarization

so we have this horizontal polarization and this the direction of propagation is x $E_x = E_0 \cos(kx - \omega t) \hat{x}$ and again the intensity in this case will be equal to $\text{mod } E_0^2 \langle \cos^2(kx - \omega t) \rangle$ half as before if we consider light at an angle here light polarized at an angle now like this it has two components and therefore the electric field can be rep this is x direction the electric field E is represented by $E_x = E_0 \cos \theta \hat{x} + E_0 \sin \theta \hat{z}$ the amplitude is E_0 therefore this comprehends of one y component and one z component and therefore the electric field can be represented by $E_x = E_0 \cos \theta \hat{x} + E_0 \sin \theta \hat{z}$ what is θ θ is the angle here θ angle between y

so we can see this more carefully let me draw it more carefully
so here

so this is the y direction this is the electric field and therefore this angle is theta and therefore it has one component along y along y which is if this is E_0 then this is $E_0 \cos \theta$ the other component which is along z will be $E_0 \sin \theta$ and therefore this is $E_0 \sin \theta$

so that is what i have shown it has one component that is this component and this two components

so this component is $E_0 \cos \theta$ and $E_0 \sin \theta$ if you take mod square that is intensity is equal to mod E square will again come out to be $E_0^2 \cos^2 \theta + E_0^2 \sin^2 \theta$

so we have this into this is amplitude variation into $\sin(kx - \omega t)$ this phase term is always there what i have written here is the amplitude because now it is at an angle theta and therefore we get E_0^2 into half as before we get E_0^2 into half what is the implication what it means is the intensity of light intensity of light does not depend on the state of polarization whether it is y when light is passing through a medium not passing through a polarizer passing through a medium the intensity does not depend on the state of polarization whether it is this or this or this all of them give the same E_0^2 by half and therefore in the remaining discussion the phase term simply gives you a factor half otherwise there is no change phase term simply gives you a factor half and therefore in the remaining discussion in problem in problems we can drop the phase term we can and discuss only the amplitude variation in determining the intensity when we determine relative intensity that is input by output or output by input when we want to calculate the half factor will get cancelled and therefore we can simply look at the amplitude variation why i discuss this will become clear when i take up the following problem now let us take up the problem of propagation of a linearly polarized light through a polarizer

so what is shown here is plane polarized light at an angle incident at an angle to the path axis

so the path axis is here along y the incident plane polarized light the it its a linearly polarized light is making an angle theta with the path axis then the path axis will allow only one component of this

so this is what we have discussed here $E_0 \cos \theta$ the electric field here is $E_0 \cos \theta$ plus $E_0 \sin \theta$ y cap a component here

so i discussed this just now that the component along y is $E_0 \cos \theta$ and the component along that is $E_0 \sin \theta$ therefore the electric field can be written as a y component plus z component we have dropped the phase term as i discussed we do not we are we have dropped the phase term which is common everywhere and therefore the path axis is along y which means the y component will be allowed to pass through but the z component will be completely blocked by this polarizer and therefore the electric field here E^2 will comprise of $E_0^2 \cos^2 \theta$ that is the first component only the second component is along z component is along z axis therefore it is blocked

so we have $E_0 \cos \theta$ therefore the intensity here will be mod E two square which is equal to $E_0^2 \cos^2 \theta$ what about the original intensity the input intensity I_0 is mod of E_0^2 E_0^2 is the electric field here which is $E_0^2 \cos^2 \theta + E_0^2 \sin^2 \theta$ which is simply I_0 is equal to E_0^2 intensity here at the input is E_0^2 intensity at the output after the polarizer is $E_0^2 \cos^2 \theta$ and therefore I_0 the intensity at the output is equal to intensity at the input into $\cos^2 \theta$ this important relation is called malus law malus law where theta is the angle between the pass axis and

the polarization the plane of polarization of the input light now with this we now take up the second problem namely unpolarized light passing through two crossed polarizers now unpolarized light passing through two crossed polarizers so the diagram shows here there is unpolarized light which is incident on the first polarizer which has a pass axis like this and the second one has a pass axis perpendicular to this therefore such an arrangement is called crossed polarizers crossed means crossed polarizer means that polarize the at pass axis are perpendicular to each other the two polarizers have pass axis perpendicular to each other and if light is unpolarized light is incident at the input then i_1 is equal to i_0 the i_1 is here i_2 is here and i_3 is the intensity here the intensity at the input if it is i_0 then after passing through the polarizer the intensity will be i_0 by 2 we have already discussed that unpolarized light passing through a polarizer 50 percent of the intensity will be lost

so the output intensity here if i designate it as i_2 then it is i_0 by two now when light continues here this is a polarized light y polarized light but the path axis here is along z it is perpendicular and therefore this polarization will be completely attenuated or absorbed or blocked by this polarizer and we get no light what if we rotate any one of the polarizers

so lets keep this constant and what if we rotate this of course as we rotate then when the pass axis changes and finally when it pass axis becomes parallel to y we get full light coming through this when it is perpendicular to y then no light when it is perpendicular parallel to y then all the light passes through the second polarizer as well what if the path axis makes an angle θ that's what we want to know what if we rotate any one of the polar whether it could be this or that but it is easier to imagine rotating the second polarizer now if we introduce a third polarizer what would be the output let me discuss this problem

so introducing a third polarizer please see the diagram unpolarized light incident on the first polarizer whose path axis is along the y direction the third polarizer the second polarizer as i showed in the earlier problem there was no third polarizer here we call this as the first and the second this is the third polarizer that we have introduced when this polarizer was not there we had pass axis like this path axis perpendicular and therefore output is 0 there is no output no light because we are passing through two crossed polarizers now if i introduce a second a third polarizer polarizer one polarizer two and third polarizer with path axis at an angle θ with the y axis then let us look at the output intensity let us estimate the output intensity we start with the input the intensity i_1 is equal to i_0 the intensity i_2 is fifty percent because this is a polarizer

so now we have only the vertical component that is y polarized light with the intensity i_0 by two if this passes through a polarizer which makes an angle θ with the plane of polarization plane of polarization is here y and therefore there is an angle θ therefore the intensity here must be i_3 equal to i_0 by 2 input intensity into $\cos^2 \theta$ that is the Malus's law Malus law says that if the angle is θ then we have i_2 intensity here will be $i_1 \cos^2 \theta$ and the same thing we apply here here the intensity is i_0 by two therefore the intensity here is i_0 by two into $\cos^2 \theta$ now we again apply Malus law now the polarization makes an angle θ with the y axis because beyond this the polarization is tilted making an angle θ with the y axis when we come here the angle between the plane of polarization and the path axis is this one or this one which is $90^\circ - \theta$ $90^\circ - \theta$ and therefore the intensity here i_4 on the other side will be equal to

so let me write

so it will be equal to $i_3 \cos^2 (90^\circ - \theta)$ i_3 is the intensity here intensity here will be equal to $i_3 \cos^2$ this angle which is 90°

minus theta

so that's what I have written here that I found the intensity will be equal to $I_0 \cos^2 \theta$ by two $\cos^2 \theta$ this is $I_0 \cos^2 \theta$ and that is $I_0 \cos^2 \theta$ into $\sin^2 \theta$ whole square $\cos^2 \theta$ minus theta is $\sin^2 \theta$ and therefore this is which is $I_0 \sin^2 \theta$ by two and which is equal to $I_0 \sin^2 2\theta$ by four and which is equal to $I_0 \sin^2 2\theta$ by four. theta is the angle between the pass axis and the plane of polarization of the third polarizer which is introduced between the first and second crossed polarizers between the two crossed polarizers we had introduced a third polarizer which now brings a finite intensity at the output before we introduce the third polarizer there was no output now when we introduce the third polarizer between the two we get a finite output I_4 is maximum when theta is equal to 45 degree that is when the angle here of the introduced polarizer makes 45 degrees to the y axis then we have maximum light coming out at the output which is equal to $I_0/8$ that is 1/8 of the intensity

so I_4 is maximum and I_4 is 0 when theta is equal to 0 which means when theta is equal to 0 we have this which means the third polarizer and second polarizer are crossed therefore output is 0 if theta is equal to 90 which means this rotates like this and then this polarizer and the third polarizer are at 90 degree although this is parallel to this and again the output is 0 output is 0 here itself and therefore output here is 0 that is what the mathematics shows here that I_4 is 0 when theta is equal to 0 and theta is equal to 90 degree there can be several numericals based on the physics the simple calculations that I have discussed here you can have different polarizers at different angles two polarizers three polarizers and

so on the picture if it is clear then all these numericals can be worked out so here I stop the discussion on polarization wave optics and the optics module thank you