

welcome to the lecture module on optics in the last two lectures we have been discussing about the young's interference experiment we will take it a bit forward and today we will continue and see interference with coherent and incoherent sources

So the topic of today's talk is interference with coherent and incoherent waves we will quickly recall what we have studied in the last lectures

So we quickly recall that young's in young's experiment we have a source s here passing through an aperture here which is a small hole and there are two more apertures s_1 and s_2 here and the waves which are emanated from s_1 and s_2 interfere on a screen which is here depending on the path difference that is $r_2 - r_1$ we may have interference maxima and minima we have discussed this in detail

So s_1 and s_2 are point sources drawn from the same wave front please see there is a point source here and these two are drawn from the same wave front these blue curved circles represent the wave front here and as you can see the wave front reaches s_1 and s_2 simultaneously and s_1 and s_2 are drawn from the same wave front that means they are from the same phase front or s_1 and s_2 are in phase in phase means with the phase term here $\cos \omega t$ without loss of generality if I assume that this is x equal to 0 this is z equal to 0 then we have $a_1 \cos \omega t$ and ψ_2 is equal to $a_2 \cos \omega t$ the phase term is the same they are in phase now we see that we recall the conditions for bright and dark fringes that we have already derived in detail the conditions for bright and dark fringes at the point p here ψ_1 is equal to $a_1 \cos k r_1 - \omega t$ r_1 is this distance and ψ_2 that is disturbance due to the second source s_2 is $a_2 \cos k r_2 - \omega t$ and Δ therefore is the phase difference this is the phase term phase term

So the difference between them is simply k times $r_2 - r_1$ and $r_2 - r_1$ is the path difference we have also seen that whenever $r_2 - r_1$ is equal to plus minus $n \lambda$ where n is an integer it is the condition for bright fringes at points p where $r_2 - r_1$ is an integral multiple of the wavelength then that condition is that point would be bright and whenever it is $n + \frac{1}{2}$ times λ then we have the condition for dark fringes where n is equal to $0, 1, 2, \dots$

So on the plus sign here represents the fringes the maximas and minimas on one side and minus sign represents maximas and minimas on the other side at the point o where $r_1 = 0$ and $r_2 = 0$ are identical because this is a perpendicular bisector of s_1 and s_2 and therefore $r_1 = 0$ is equal to $r_2 = 0$ the path difference is 0 and that corresponds to a maxima which is called the zeroth order bright fringe we have discussed all this detail in detail in the previous lecture

So we now look at a different situation slightly different situation where the source s has a finite offset

So here is the new discussion that we want to make that the point the source s if the source s here

So let us look at the diagram first

So if the source s has a small offset it is not along this line here instead the source s now I am calling it as s' as a small offset it is a little bit upward here and then obviously we see that the distance s_1' and s_2' will be different because there is an offset in terms of the wave front what we note is that when the wave front look at the blue wave front here and the blue wave front reaches this plane the plane containing the apertures s_1 and s_2 we see that the wave front has reached the point s_1 but it has not reached the point s_2 the blue one

So the wave front has not reached the point s_2 it will reach the point s_2 at a later time

So the wave front here has reached s_1 but it has not reached it it will reach s_2 at a later time it means the wave front here is lagging it is lagging in phase at a later time it will reach here and therefore there is an initial phase difference of $\Delta \phi$ here between $\Delta \phi$ between the sources s_1 and s_2 please see that the wave front would reach here at a later time which means let me explain this

So which means if I had $\cos \omega t$ as the initial phase then we will get the same wave front at s_2 this is at s_1 at s_1 I have dropped the amplitude at s_2 the same wave front will come at a later time or when I have a phase term $\cos \omega t + \phi$ we have the phase at s_2 will be $\cos \omega t - \Delta t$ that is at that instant on the plane here one wave front the wave front has reached here at the point but at the second point here it has not reached now I have expanded enlarged it a little bit So this will be behind or at this instant at a particular instant when this is in

phase the phase front of the earlier travelled wave would be like this in other words this phase front will reach here at a later time or at a given instant the phase at s_1 and s_2 are related by this in other words we have the phase as s_2 as ωt minus $\omega \Delta t$ and this one I am calling as $\Delta \phi$ and therefore the phase difference between the two s_1 and s_2 will be equal to ωt minus $\omega \Delta t$ here

So phase difference

So ωt minus $\omega \Delta t$ will cancel

So we have $\Delta \phi$ and therefore we have $\Delta \phi$ that is why I have this term here

that I have shown that this the second wave has $k r_2$ minus ωt plus $\Delta \phi$ there is a phase lag which is $\Delta \phi$

So $\Delta \phi$ is the initial phase difference it is the initial phase difference between the two waves it's also shown here in a different way the blue wave front as we can see it has at this point the blue wave front has reached the red wave front will come at a later time although I have shown it in front blue has passed by and the red wave front has reached here and therefore the red wave front is behind the blue one and there is a phase lag of $\Delta \phi$ the phase lag is the same $\Delta \phi$ here and therefore the net phase difference is now not just $k r_2$ minus $k r_1$ but also there is a phase difference of $\Delta \phi$ all of this is because s is offset here therefore this propagation distance is different from this property this is smaller compared to this and therefore at the point O now therefore at the point O r_1 is equal to r_2

So this distance is the same r_1 is equal to r_2 but $\Delta \phi$ is not equal to zero at the point O r_2 is equal to r_1 here but $\Delta \phi$ remains and therefore at the point O $\Delta \phi$ is not equal to zero if $\Delta \phi$ is not equal to zero we will not have the zeroth order bright fringe here $\Delta \phi$ may be zero somewhere else the phase difference may be 0 somewhere else and therefore the 0 th order bright fringe will not appear at O because there is a phase difference at O let us look at this a bit more carefully

So let me put the condition for bright and dark fringes again here

So let us see at the point P at a general point P we know that ψ_1 is equal to $A \cos(k r_1 - \omega t)$ and ψ_2 is equal to $A \cos(k r_2 - \omega t + \Delta \phi)$ that is this I am showing only this and $\Delta \phi$ is the phase difference $k r_2$ minus $k r_1$ and r_2 minus r_1 is the path difference we have already seen this in detail and r_2 minus r_1 is equal to $\pm n \lambda$ is the condition for bright fringe and the condition for dark fringe here

So this we have already seen now with the $\Delta \phi$ not equal to 0

So let me show this slide now

So this slide we have seen and I have shown that there is a finite $\Delta \phi$ here and at this point $\Delta \phi$ is not equal to zero

So let me keep let me take this here therefore for $\Delta \phi$ is equal to 0 $k(r_2 - r_1)$ is equal to $\pm n \lambda$ please see that for this to be 0 this will be equal to $\pm n \lambda$ that is what I have written which means r_2 must be less than r_1 and therefore I have shown a point O' here where r_2 is less than r_1 and at this point O' the point O' is such that $s_1 + s_1 O'$ that is the total path here path length is equal to $s_1 + s_1 O'$ if it is equal to $s_2 + s_2 O'$ then we have path difference is such that $\Delta \phi$ is equal to zero

So the position of the zeroth order maxima or the central fringe will be shifted to a new position which is O' therefore the condition for maxima is $\Delta \phi$ is equal to $k(r_2 - r_1) + \Delta \phi = \pm n \lambda$ or $r_2 - r_1$ is equal to the path reference which is equal to I have taken the $\Delta \phi$ to the other side and we have divided by k everywhere and therefore k is $2\pi/\lambda$ therefore $\lambda/2\pi$ is here

So we have divided by k

So $n \lambda$ divided by k minus $\Delta \phi$ divided by k we have taken this and therefore we have the new condition now that the path difference $r_2 - r_1$ is equal to $n \lambda - \Delta \phi / k$ for the n th maxima there is an extra term now because of the finite $\Delta \phi$ please see that if $\Delta \phi$ is equal to 0 that is if this were on the perpendicular bisector here the original position S then we would have had $\Delta \phi$ is equal to 0 and the condition remains as path difference is equal to $n \lambda$ now there is an extra term which depends on the phase difference what is the effect of this on the fringe width let us see the effect of this on the fringe

width

So for the n th bright fringe here

So let's look at this let's look at the for the n th bright ring if $\Delta\phi$ is a constant then path difference $r_2 - r_1$.

So this difference we have already calculated in terms of the x coordinate there

So it is $x_n - d$ by d for the n th fringe the n th fringe if x_n is the coordinate $x_n - d$ I have written $x_n - d$ just to distinguish that now we are dealing with a case where there is a finite $\Delta\phi$ that's all otherwise it is the same x_n as before

So for the n th fringe the condition we had earlier derived was $x_n - d$ by d equal to $n\lambda$ now because the position of the n th maxima has changed I am calling it as $x_n - d$ So that is equal to $n\lambda - c$ where c is that constant that is $\Delta\phi$ by 2π into λ

So please see that we had $n\lambda - c$ this constant I am calling as c if $\Delta\phi$ is a constant constant with time then this is a constant c and therefore $n\lambda - c$ where c is a constant for the $n + 1$ fringe that is for the next ring it will be $x_{n+1} - d$ by d is equal to $(n + 1)\lambda - c$ because c is a constant

So $-d$ is only to represent the next the case where there is a finite $\Delta\phi$ it is not derivative or anything and therefore therefore the fringe width β is equal to $x_{n+1} - x_n$

So from 2 and 1 if we subtract this then we have d by d into $(n + 1)\lambda - c$ minus $n\lambda + c$ which is equal to this is the fringe width which is equal to d by d into λ as before I have written here as before as before means the case when there was no phase shift no initial phase shift that is the case when the source S was on the perpendicular bisector there is no change in the fringe width even if there is a offset in the source but the fringe pattern is shifted the fringes are shifted the fringe width remains same which means the fringe pattern is shifted for example if we have all linear fringes bright dark bright dark the entire pattern is shifted otherwise it looks identical now that is the fringe there is a fringe shift each fringe is shifted but no change in β

So what does this mean let us calculate this first what is the fringe shift β in terms of the geometry

So let me calculate the fringe shift therefore the fringe shift here the fringe shift is given by $x_n - d$ is the new position of the n th order bright fringe in the presence of a constant phase difference $\Delta\phi$ between S_1 and S_2

So $x_n - d$ by d is equal to this condition remember that x_n into d by d was equal to $n\lambda$ when there was no phase shift when there was no $\Delta\phi$ or $\Delta\phi$ was equal to zero therefore for $n\lambda$ I am substituting $x_n - d$ by d the original position of the n th bright fringe now the n th bright fringe has changed to a new position $x_n - d$ but the original position x_n is here and then this into d by d is $n\lambda$ therefore we have written replace this by this minus c which means if I take the x_n to the other side $x_n - x_n - d$ that is the fringe shift that the shift of the n th fringe $x_n - x_n - d$ by d is equal to c the constant which is equal to $\Delta\phi$ by 2π into λ or the fringe shift Δx_n this is the fringe shift Δx_n is equal to $\Delta\phi$ by 2π into λ d by d and λ d by d is the fringe width and we have seen that the fringe width does not change whether $\Delta\phi$ is 0 or $\Delta\phi$ is a finite number and therefore we see that this side is independent of n whether it is the first fringe or fourth fringe or fifth fringe it does not matter it simply says the fringe shift is given by $\Delta\phi$ by 2π into β what does this mean this means that the fringe shift Δx now I drop the n because it is independent of n therefore the fringe shift Δx is equal to $\Delta\phi$ by 2π into β it depends on $\Delta\phi$ this is now completely understandable we can see that if we put $\Delta\phi$ here is equal to zero then Δx is equal to zero if $\Delta\phi$ is zero Δx is equal to 0 means there is no fringe shift and if $\Delta\phi$ is a finite number then there is a fringe shift for example if the phase shift is 2π $\Delta\phi$ is 2π then Δx is equal to β which means the bright fringes would shift by one that is n th fringe will take the position of the $n + 1$ spring or $n - 1$ depending on whether it is to that side or this side

So the fringes would shift by Δx is equal to $\Delta\phi$ by 2π into β let's see the geometry of the problem more carefully now

So we will see the geometry of the problem when the source is offset

So let me first show only the geometry here

So this is the geometry of the problem original position was S which is on the

perpendicular bisector here s_1 and s_2 symmetrically about this line s_0 now the source has shifted to a position s_{dash} and therefore we have the fringe shifted to a new position o_{dash} and the path difference total path reference will be 0 if this plus this equal to this plus this then the net path difference will be 0 and o_{dash} will be the new position of the central fringe corresponding to $\Delta = 0$ path difference equal to zero

So s_{dash} there is the offset here is l we denote it by x that is o_{dash} is here x_{dash} x_{dash} is nothing but Δx because x position is 0 originally and the new position is x_{dash} therefore Δx is equal to x_{dash} the coordinate of the new position of the central maxima s_1 s_2 the separation here is d small d between the two sources and l let l be the separation between these and d is of course the distance to the screen from the two sources s_1 and s_2 and therefore now let us see that for the central fringe the path difference is equal to 0 if the condition this plus this is equal to this plus this the length here are equal which means i can take this s_2 o_{dash} to the other side or s_1 o_{dash} to this side and write s_2 o_{dash} minus s_1 o_{dash} s_2 o_{dash} minus s_1 o_{dash} this separation is equal to s_1 s_{dash} s_1 s_{dash} minus s_2 s_{dash} So that is nothing but s_2 o_{dash} is r_2 r_2 minus r_1 is equal to i am denoting this as q_1 and q_2 is equal to q_1 minus q_2 .

So r_2 minus r_1 we know the path difference between these in terms of the separation here this and d and that is x_{dash} d by d x_{dash} is the position here of o_{dash}

So r_2 minus r_1 is x_{dash} d by d similarly q_1 minus q_2 q_1 minus q_2 just like this q_1 minus q_2 will be equal to l the offset divided by l into d because it is from this triangle this part difference we had calculated from this part difference now we are calculating from this triangle and that is identically l by l into d this implies please see here this implies that x_{dash} by d

So d is common and therefore x_{dash} by d is equal to l by l x_{dash} by d x_{dash} by d is this angle θ the angle θ $\tan \theta$ is equal to x_{dash} by d $\tan \theta$ of course in is very very small here because the fringe shift is much smaller this is of the order of hundred centimeters one meter and the fringes are only moving a few millimeters or

So so this is x_{dash} by d is very small even if we do not make the approximation it is valid because x_{dash} by d is $\tan \theta$ and l by l is $\tan \theta_{dash}$ here

So i have shown θ_{dash} here this angle $\tan \theta_{dash}$ and they have to be equal So $\tan \theta$ is equal to θ_{dash} is equal to θ_{dash} which means if θ is equal to θ_{dash} that simply means that these are opposite angles which means that s_{dash} o_{dash} is a straight line passing through the point m here

So we have x_{dash} by d this divided by this is equal to the offset l divided by l

So s_{dash} o_{dash} is a straight line passing through m now let's see now the fringe shift now in terms of we see the fringe shift now in terms of the geometry that is the offset earlier we have seen the fringe shift in terms of the phase shift $\Delta \phi$ by 2π into β now we see we get an expression for the fringe shift in terms of the offset

So let's see that

So here fringe shift due to the source offset

So we have l as the source offset here and Δx is equal to x_{dash} minus 0 0 is the original position which is the fringe shift and we have just shown that x_{dash} by d is equal to l by l and therefore Δx x_{dash} is Δx the fringe shift Δx the fringe shift Δx is equal to d by l into l in other words if an offset is given if it is given to you that the source s is offset by a certain amount and the corresponding separations d and l are given then you can determine what is the fringe shift

So it is the fringe shift due to source offset by l take an example put some numbers typically d is about 100 centimeters this could be tens of maybe 10 20 centimeters i have taken l is equal to 10 centimeter for an offset of 1 mm just 1 mm offset here

So this is the perpendicular bisector an offset of 1 mm leads to a Δx a shift of 10 mm here 100 by 10 into 1 that is 10 mm almost 1 centimeter shift a typical idea to see how the friendship take place in other words if you are setting up the experiment if the source s is not exactly normal and on the perpendicular bisector here if there is a small shift then the central fringe will shift to this side the same thing would happen if instead of the source moving to this side if the source had shifted offset to this side then we would have got o_{dash} here that is the central fringe would have moved here the same thing would also happen

So let me show you the same thing would also happen if i have

So the source is right on this source is here but the s_1 and s_2 there is an offset

between s_1 and s_2 that is s_1 the perpendicular bisector is here now this is shifted a little bit to this side then also we will have a corresponding path shift because now the distance s to this and s to s_1

So s_2 s_1 here and s_2 s_2 will be different and correspondingly there will be phase shift of $\Delta\phi$ between the sources here the wave front which is reaching here will be at a different time the wave front will reach here therefore there is an initial phase shift of $\Delta\phi$ and correspondingly we will have the fringe shifted here which will be on the line here joining this

So the fringe will be shifted to a new position o dash if it is if this aperture is shifted downward then the fringe shift would occur here this kind of shift is called the lateral shift it is because the alignment there is an offset either in the source or in the double hole aperture then we have a corresponding shift in the central maxima and this is called lateral shift

So here we are the fringe shift will also occur let me come back the fringe shift will also occur if the aperture plate q q dash is offset with respect to offset with respect to this line s_0 this type of fringe shift is called lateral shift why i am discussing this is we will soon see another type of fringe shift its fringe shift because of let us say introducing a $\Delta\phi$ due to an external plate for example if we introduce a glass plate on one of the paths we will see that there will be a fringe shift

So this fringe shift is simply because of the offset not because of introducing any other thing the offset introduces a constant phase shift of $\Delta\phi$ and due to which there is a fringe shift which is called lateral shift lets therefore just let me summarize what we have seen

So summarizing the fringe shift issues the fringe shift Δx due to a constant phase difference $\Delta\phi$ if there is a constant phase difference why i am insisting on constant is next i will take up when the phase difference is changing with time

So the constant phase difference $\Delta\phi$ including the $\Delta\phi$ could be zero between the interfering waves then there can be sustained observable interference fringes when $\Delta\phi$ is zero or constant we will have a sustained observable interference pattern the phase difference is 0 means the interfering waves are in phase and if the phase difference is π we call it out of phase here i have shown this that waves with a constant $\Delta\phi$ among them are called coherent waves

So i have shown here $\Delta\phi$ is equal to 0 which means whenever the first wave is maximum second wave is also maximum

So this is ϕ or time

So the maximas are coinciding the minimas are going that is crests and troughs come at the same time at any given position if the phase difference $\Delta\phi$ is equal to π it is a constant but it is π then we have the crest of one wave corresponds to the trough of the other wave at that point at that point the phase variation what i have plotted is the phase variation of the two waves with time

So at any given point if the crest of one wave coincides with the trough of the other it means the two phase two waves are out of phase and the net amplitude we have already seen in another chapter that the superposition of waves the net amplitude will be the sum of the amplitude which will be zero and if there is a constant phase shift $\Delta\phi$ here which means the waves there is a constant phase shift all the three cases correspond to coherent wave whenever $\Delta\phi$ is a constant we will be able to see sustained

observable interference fringes this is the first point the second point that we have is the central fringe here and all other fringes let us look at this the central fringe and all other fringes would shift by an amount Δx which is proportional to $\Delta\phi$ but the fringe pattern and the fringe width does not change Δx is equal to we have derived this Δx is proportional to $\Delta\phi$ but the fringe width remains the same and the fringe pattern remains the same now therefore there is a question how to measure the fringe shift in practice particularly when $\Delta\phi$ is several times 2π in practice we see a fringe pattern as i showed earlier let us say bright and dark linear fringes the fringes are shifted because of a finite phase difference $\Delta\phi$ but how to measure the fringe shift because it looks identical and if the phase shift $\Delta\phi$ π $\Delta\phi$ is let us say 8π then four fringes will be shifted

So we do not know where is the central fringe now

So how to locate the position of the central fringe because they all look identical all the fringes look identical and the fringes have been shifted exactly by four fringe which means the pattern again looks the same how to locate the central fringe the answer here

is use of white light interference we will discuss this shortly but before we go to white light interference i want to come to the next question that is what if $\Delta\phi$ varies randomly with the time what if $\Delta\phi$ varies randomly with time it varies with time means $\Delta\phi$ is a function of time till now i had assumed a constant $\Delta\phi$ but now it is a function of time at any point and when do we have such situations where $\Delta\phi$ is varying with time we will discuss this in a minute if S_1 and S_2 are two independent sources or derived from an extended source then we will see that $\Delta\phi$ will be varying randomly with the time

So let me discuss this a little bit more

So let us look at light sources and waves coming out

So if i have a light source there is a light source here a bulb let us say and this is giving out light radiation we know that in any particular direction light comprises of waves which are travelling but these waves are not sinusoidal from end to end the electromagnetic wave is not sinusoidal end to end because it depends on the mechanism of generation of light for example if we take a sodium lamp let us say this is a sodium lamp in a lamp there are sodium atoms which are excited

So if i consider the ground state of sodium is here

So this is one is ground state and two this is energy axis

So i am plotting the ground state and the excited state sodium atoms are excited here by electric discharge and the excited sodium atoms come down that is they get de-excited and come down to a lower energy level and the energy difference here is given as a photon or an energy packet of energy $h\nu$ which is equal to the energy difference if i say that the energy here was E_2 and energy of the second first level was E_1 then $h\nu$ is equal to $E_2 - E_1$ we know this we have already studied this

So energy is given in terms of photon packets now these are not infinitely extended because infinitely extended means it will contain infinite energy and therefore these are finite wave trains which are emitted

So we have continuously excitation taking place and de-excitation taking place emission of photons coming out of the sodium lamp and these are therefore wave trains which are travelling off up to a finite duration it means if i take a several wave trains

So let me plot several wave trains

So one wave train is this and another wave train they they are emitted at different times there are continuously atoms getting excited atoms getting de-excited

So they are emitted continuously at different times which means the sine waves originate at different times all of them are of the same wavelength but they are emitted at different times and therefore if i look at

So these are the different waves which are travelling individual waves corresponding to photon

So all of them are of the same wavelength λ same λ but they are discontinuous

So if i look at two waves any two waves lets say there are two waves

So let me consider two waves one and two these two waves over this period you can see that there is a constant phase difference between them but after some time there is another wave which is here which has no phase relation with this

So this is time axis

So with the time we see that over a certain time period there is constant phase difference but if i look at the time here the phase difference between these two waves is different from the phase difference between these two waves this may be coming from the source S_1 and this may be coming from the source S_2 and therefore there is no phase difference the phase difference changes with time in other words $\Delta\phi$ is a function of time let me come back to this and now show the diagram of the let us look at the young's double slit experiment now this is the same sodium lamp which is giving out radiation here and we have the double slit here

So i am showing the double slit S_1 and S_2 this is an extended source this is not a point source this is an extended source extended source and therefore there are wave fronts where the wave fronts have no correlation what do i mean by that if i have a point source this should give spherical wave fronts like this and if my aperture is here S_1 and S_2 we see that this is S_1 and S_2 we have emphasized several times that the wave front reaches at the same time to S_1 and S_2 but if there are this not a point source but an extended source which comprises of number of point sources which are radiating independently then the wave which is entering here and the wave which is entering here have no phase relationship no fixed phase relationship it changes with the time which

means $\Delta\phi$ is a function of time when the source is extended source that is why in young's experiment what did we do we have a point source there was a first aperture here this was acting as a point source which was giving out spherical wavelets which are here and we placed the second aperture here with the two apertures the double hole or double slit is here before that there is a single hole which is like a point source we did not place an extended source directly in front of this or nor even this that you take two s_1 and s_2 two holes and we keep one bulb let me show the bulb like this here which is giving light and another bulb here in front of this

So this is giving

So what is the point if we have two independent sources or an extended source from which s_1 and s_2 are derived s_1 and s_2 are derived then $\Delta\phi$ will vary with the time at different times $\Delta\phi$ will vary because the light which is emitted from here has no phase relationship with the light which is emitted by the other source these are independent sources if there is an extended source different parts of the source independently give out light and therefore there is no phase relationship and phase varies with the time

So that is what I have stated here that if s_1 and s_2 are two independent sources or derived from an extended source then $\Delta\phi$ will be a function of $\Delta\phi$ will be equal to $\Delta\phi$ of t it is a function of time because there is no phase relationship between the two sources

So with this let us understand let us come back to the

So I have described both the sources and therefore what would be and therefore what would be the interference intensity

So we have derived the expression for intensity I is equal to $4I_0 \cos^2 \Delta$ by $2 \cos^2 \Delta$ by 2 Δ here is changing with time

So Δ where Δ is now a function of time this is a phase difference

So this Δ contains a path reference plus

So this Δ contains a path a phase term due to path reference this is fixed this is not changing but there is a second phase difference term $\Delta\phi$ which is a function of time and therefore Δ will be a function of time and this $\Delta\phi$ is varying randomly or rapidly with the time and therefore this function to see the intensity we have to take average of this rapidly varying \cos^2 function Δ is randomly or rapidly varying and therefore we have to write intensity I is equal to four times I_0 into time average of $\cos^2 \Delta$ by 2 and we have already discussed that this time average is nothing but half in the earlier class we have discussed that time average of a rapidly varying term is half \cos^2 term is half because \cos^2 varies between zero and one and therefore I is equal to two times I_0 I_0 is the intensity due to the individual sources I is the intensity at any given point

So now this is independent of the phase and therefore independent of the path difference what does this mean this means that if I see here

So each of these two sources s_1 and s_2 here the intensity of the source is I_0 here and I_0 here then on the screen on the screen at any point here we have simply two times I_0 in other words if I plot the intensity this is the x direction the screen

So if I plot the intensity along the x direction here I of x x direction here instead earlier we had very nice fringes here maxima minima maxima minima now we simply have we simply have

So let me take a different color show that this is two I_0 compare this with what we had earlier with x at x equal to 0 there was a maxima and then it was $4I_0$.

So we used to have intensity variation like this

So this is when $\Delta\phi$ is not

So $\Delta\phi$ is a constant constant and this is the case when $\Delta\phi$ varies rapidly with the time which means there is uniform intensity in other words we will not be able to see any fringes there will be no sustained fringes fringe pattern interference takes place but the interference intensity distribution is varying

So rapidly that we will not be able to see any fringe pattern and therefore the summary is if we have incoherent sources

So in the last lecture we had seen that there are two requirements for interference that the first requirement is that the sources should be coherent or they must have a constant phase difference between them and there is a second requirement

So we have already shown that if the phase difference $\Delta\phi$ is changing with time

then we cannot have sustained interference fringes

So that is if we have if the sources are incoherent is $\Delta\phi$ phase difference between them varies with time randomly then we do not have any interference the second requirement which we had pointed out is that the wavelength of the interfering sources must be the same

So we take up this the second issue now and wavelength of interfering sources must be the same

So let us look at the this issue

So let me take up two wavelengths one blue and red

So lets let me plot this with the time here here i have

So i am now looking at interference

So interference with two wavelengths interference between between two different wavelengths is it possible that's what i want to see two different wavelengths λ s let me write this interference between two different is it possible i am not saying that it is possible i am saying i am discussing interference between two different wavelengths

So lets look at the wavelengths here

So the a the red wavelength starts like this

So i am showing the amplitude variation with the time

So this is with the time amplitude variation and the blue wavelength it has a smaller wavelength

So time or x

So we know that the maxima to maxima that is this is the wavelength λ the face here if i write this as an amplitude $a_1 \sin \omega_1 t$ and for the blue one

So this is red and for the blue one this will vary more rapidly

So we know that blue varies more rapidly because the wavelength is smaller it will vary like this if the face here is 0 then this maxima here the red maxima here corresponds to a phase of $\pi/2$ and the 0 here corresponds to a phase of π because when phase ωt ωt is the phase this is the phase ϕ and ϕ equal to zero the total amplitude ψ one is equal to zero and when ϕ is equal to $\pi/2$ the amplitude is maximum a_1 the displacement ψ is maximum again at π it is zero and

So on

So the phase points if i were to plot this as five then this would be three $\pi/2$ point and this would be correspondingly the two π point for the blue if we see because for the blue one we have ψ_2 is equal to $a_2 \sin \omega_2 t$ the amplitudes can be same or different sine $\omega_2 t$ now ω_2 is 2π times the frequency ω_2 is the angular frequency ω_2 is equal to 2π times the frequency f_2 of the blue line and therefore because the frequency corresponding to the blue wavelength is larger because we know that the wavelength is smaller blue is about 400 to 450 nanometer and red is around 650 nanometer therefore the frequency is higher and therefore this is oscillating rapidly this is varying rapidly because this number is larger in the same time this varies rapidly which means the point $\pi/2$ corresponds to this phase and in terms of phase this will correspond to three $\pi/2$ this is π and two π

So what is happening there is a phase difference between the two waves and the phase difference is continuously changing with the time if we look at any point if we look at any point x

So if i plot as a function of x if i look at any point any position x then between the blue

So blue is varying rapidly like this and red is varying slowly

So red is the face of the red is varying slowly they of course both of them are travelling with the same speed if i consider this as vacuum or free space they are travelling with the same speed but the face if we take any plane will be continuously changing because this is varying rapidly that is varying slowly

So if we see the phase ϕ ϕ will be continuously changing if i say that $\Delta\phi$ is a function is the phase difference between ϕ_{blue} minus ϕ_{red} then it is changing with the time

So we can see from the expression here

So we have a sine $\omega_1 t$ and a let me assume the same ampli or $a_1 \sin \omega_1 t$ and $a_2 \sin \omega_2 t$ this is the phase term of the first wave this is the phase term of the second wave at any given point

So i can without loss of generality i can write this as x is equal to zero point and

therefore that's why i have not written $\omega t - kx$ that term i have not written because i am looking at a particular point and what is the phase difference the phase difference between them the $\Delta\phi$

So the $\Delta\phi$ will be equal to $\omega_2 t - \omega_1 t$ or $\omega_2 - \omega_1$ into t

So this will be continuously varying with time

So $\omega_2 - \omega_1$

So $\Delta\phi$ we can see that $\Delta\phi$ is a function of time and it is varying very rapidly remember that ω_1 and ω_2 are light frequencies which are very big numbers these numbers are very big numbers because the light frequency f_1 and f_2 of blue and red are of the order of approximately 10^{14} hertz 2×10^{14} hertz and 5×10^{14} hertz

So on therefore this is a huge number multiplied by time which means $\Delta\phi$ is varying very rapidly and therefore the phase is varying very rapidly what this means is interference is possible

So i simply write the conclusion here interference is not possible is not possible possible between two different wavelengths between two different wavelengths that is why we had written different wavelengths that is why we had written this was the second requirement for interference when we started interference we had given two requirements one is that the sources should be coherent or there has to be a constant phase difference between them and the second is interfering waves must be of the same wavelength now i want to take up a interference with a source of multiple wavelengths suppose a source we just said that interference is not possible with between two wavelengths between two wavelengths but if i have a source interference with the source emitting multiple wavelengths for example we pick up here consider three different wavelengths 400 nanometer 500 nanometer and 600 nanometer one is near blue color this is very close to green and this is orange length

So recall that blue interferes only with blue in other words blue will not interfere with green we have just now discussed that the two different wavelengths do not interfere but blue will interfere with blue and therefore if we see the interference in the double hole arrangement the due to blue color we will have fringes with fringe width β is equal to 0.4 nanometer i have assumed a distance d equal to one meter and the separation between s_1 and s_2 as one millimeter typical numbers which we use in practice and therefore the fringe width is given by $d \sin \theta = \lambda$ substitute for λ is equal to 400 nanometer we get the fringe width 0.4 mm if we substitute 500 nanometer we get

So for the green we get 0.5 mm and for the orange color we have 0.6 millimeter how would the fringes look like

So lets see the diagram

So the fringes look like this

So what am i showing i am showing the interference due to blue interference due to green and interference due to red if only red were present then we would have got the intensity varying like this this is the x direction that is on the screen varying like this if i had only blue color i would have got this but with the multiple wavelengths such as white light if i have white light then it would have all colors from blue to red at the other end from violet to red at the other end and then we would have got interference due to each color here how would it look like but we see that at o here at o all of them have the maxima at the same point o but the maxima of blue is here the minima of red is here and after sometime we see that when blue is minimum red is maximum

So they are at the maximas are occurring at different positions what will be the net sum the intensities the amplitudes will will add in such a way that please see that the maxima and minima occur at different points for different wavelengths except the central maxima is the same position for all the colors

So what to expect the net effect

So here i have the net effect which is shown here which is white light interference pattern

So its a qualitative representation

So i have qualitatively drawn this that because all all the colors are having the same position of maxima at this point all the colors will be bright i will add up to have a white light a bright white fringe and that is what we have i have simply plotted intensity as a function of x

So this will be the central bright white maximum and then because most of these are

passing through minima here there will be a dip in the intensity total intensity there will be a dip then we have the blue maxima orange green maxima and red maxima and therefore we have slight variations bluish color reddish color and finally of course all of them vary in such a way that we simply have uniform illumination this is the young's first experiment when he did with sunlight what we see this is the first interference experiment of young he saw one bright fringe some colors and then uniform illumination it is then that he switched to the sodium lamp or the spirit lamp with the sodium salt sprinkled over the spirit lamp to see multiple fringes

So what is the conclusion the conclusion is that if you perform the experiment with white light you will be able to locate the central fringe where the path difference is 0 and the phase difference is 0 for all the wavelengths and therefore the final conclusion is thus the central fringe can be easily identified and the fringe shift Δx due to a phase change $\Delta \phi$ can be measured accurately using white light interference i had posed this question earlier how how therefore to determine the shift in the central fringe the answer is from white light interference thanks you