[Music] [Applause] welcome to the lecture module on optics we are now discussing wave optics in the last lecture we discussed the huygens principle eigens christian hygienes introduced the wave picture for light propagation however as we have seen although he could explain the reflection and refraction of light at interfaces there were certain questions which he did not have answer or heightens wave theory did not have answer but as we discussed that in 1801 thomas young presented the young's interference experiment which was a convincing proof that light is a way

So today we will discuss the young's experiment in some more detail

So young's experiment young's interference experiment first of all interference in optics generally interference refers to two beam or two wave interference which result in some perceptible and sustained fringe pattern we will discuss about the fringe pattern what is this fringe pattern in the later part of the lecture

So we generally refer to two beam or two wave interference which results in a perceptible and sustained fringe pattern a fringe pattern refers to the light intensity pattern comprising of alternate bright and dark regions in the form of lines or rings for example let me show a typical fringe pattern

So here is a fringe pattern which is a linear fringe pattern

So this is the kind of pattern which we would get in a young's experiment or it could also be a circular fringe pattern these are of course computer generated fringes

So it could be a circular fringe pattern as in the case of newton rings and we will discuss the formation of these fringes in the next few minutes and formation of fringes can be explained in terms of superposition of the two waves

So first let us look at there are certain requirements although to get a sustained fringe pattern and let us look at the requirements to obtain a sustained fringe pattern

So here the requirements of interface what are the requirements of inter fans to be able to see a sustained fringe pattern the two waves interfering must have the same frequency or wavelength and there there should be a constant phase difference between the two waves this property is called coherence the two waves must be coherent

So there should be a constant phase difference between the two ways we will discuss these issues towards the end of the lecture or in the subsequent lecture and understand this more clearly

So let us come back to the young's schematic of the young's experimental setup

So here is the young's experimental setup

So first we see here that the experimental setup comprises of a source there is a small hole in this there is an opaque screen in which there is a small hole or aperture and then there is a second screen there is a second plate or screen or a cardboard where you have two small holes s one and s two two small apertures

So thats why its called young's two hole experiment

So there are two holes which act like point sources and then we have a screen here on which the interface pattern will be observed

So we will discuss the formation of interference pattern

So the the coordinate system that we consider is the z axis light is propagating in the z axis here

So this is the z axis and the obstacles here or the apertures here are on a plane which is perpendicular to the z axis and the screen is in the x y axis here

So x y axis the x y plane lies on the screen

So this is the experimental arrangement now if we see a longitudinal cross section along the x z plane

So x z plane

So this is the x and this is the z direction

So if we see a longitudinal cross section of the x z plane it would look like this

So here it is

So the source is here

So we are looking at a cross section

So source s is shown here there is a small hole which acts like a point source from which spherical wave fronts are emerging then there are two other small holes which act like point sources again

So the wave the spherical wave reaches here and as we know from the heights principle that each point on this wave acts as secondary source of secondary wavelets

So these two points or these two small apertures act like point sources and here is the screen

So again is shown here that the x axis is here and we have shown a cross section a longitudinal cross section the separation between the two holes here and the screen is capital d and the aperture the separation between the two holes here is small d and at any point p the resultant intensity can be determined by considering the superposition of waves superposition of waves occur at each point on the screen at each point first we will consider the superposition of waves on the x axis and then we will take a general point an arbitrary point anywhere on the screen

So first a point p is a point an arbitrary point on the x axis here on the x axis

So superposition of waves occur at each point p and d is the distance between the source s one and s two s one s s s one s two are small holes in an opaque screen ok

So we will now discuss the superposition of waves to determine the interference pattern

So lets look at the superposition of waves here

So at an arbitrary point p these are the two point sources s one and s two at a point p here which the distance is r one from s 1 p is r 1 and s 2 p is r 2 and we can see that the point source s 1 can be represented by psi 1 equal to the disturbance due to s 1 the wave due to s 1 at the point p can be written as a 1 dash by r 1 cos k r 1 minus omega t and due to the point source at s 2 psi 2 is equal to a 2 dash by r 2 cos k r 2 omega t if we denote this a 1 dash by r 1 as a 1 and a 2 dash r 2 by r 2 as a 2 then we have the resultant at the point p is the super position which means we sum it psi one plus psi two psi resultant at the point p is equal to psi one plus psi two

So that is a one cos k r one omega minus omega t plus a two cos k r two minus omega t which can be opened up

So now we can open cos a minus b is equal to cos a cos b plus sin a sin b we can use that formula and

So therefore we have this term has cos k r 1 cos omega t plus sin k r 1 sin omega t and the second term a 2 is equal to cos k r 2 cos omega t plus sin k r 2 sin omega t using the trigonometric identity now the superposition of two ways

So let us calculate the resultant due to these two waves

So let me continue with the resultant calculation of the resultant

So the psi resultant is equal to cos omega t

So we have taken cos terms which are common

So in this expression here we had cos omega t here and cos omega t here

So we are taking common terms and sine omega t and sin omega t here

So this can be written as cos omega t into a 1 cos k r 1 plus a 2 cos k r 2 plus sin omega t into a 1 sin k r 1 plus a 2 sine k r now we set a 1 cos k r 1 plus a 2 cos k r 2 that is this term here as a cos phi and a 1 sin k r 1 plus a 2 sin k r 2 as a sin phi where phi is given by the phi the angle phi such that this holds good is given by tan phi is equal to a one sine k r one that is we simply divide this by this we see that dividing the second equation by the first equation we get tan phi is equal to a 1 sine k r 1 plus a 2 sine k r 2 divided by a 1 cos k r 1 plus a 2 cos k r and therefore the resultant psi now

So this term is a cos phi and this term is a sin phi

So the resultant is a cos omega t cos y plus a sine omega t sin phi in other words that is nothing but psi resultant is equal to a cos omega t minus phi at the point p now with a is equal to a square cos square phi plus a square sin square phi why we have to the power half why we have written this because a will come from this a will be determined from this square of this and square of this and square root a square cos square phi plus

So that is given by a is given by this and phi is determined by this equation

So we have the resultant disturbance or resultant function or the resultant wave at the point p as a cos omega t minus phi lets find out what would be the corresponding intensity distribution

So the intensity distribution is given by ah mod psi square and therefore we write that the intensity at the point p is equal to

So intensity at the point p is equal to mod psi resultant square which is equal to a square cos square omega t minus phi now cos square omega t minus phi because omega is a very large number

So we are determining the intensity omega is a very large number

So what is omega

So lets just discuss this here

So omega is equal to 2 pi into nu where nu is the frequency of like corresponding to light

So nu is typically of the order of 10 power 14 to 10 power 15 hertz for for light depending on whether we are at the blue end or whether we are at the red end

So this is a very large frequency and therefore cos omega t

So cos cos square

So we have cos square omega t minus phi

So this is what we have to determine this quantity is varying extremely rapidly with the time

So we have a cos square function

So cos square we know varies from 0 to 1 cos square it is therefore it varies from zero to one this is time axis

So with the time this is varying extremely rapidly very very rapidly and this time difference here is of the order of ten power minus fifteen seconds because the frequency is of the order of ten power fifteen hertz which means the corresponding period is two pi by t which is t is equal to two pi by f the frequency and that is of the order of ten power minus fifteen second which is extremely rapidly varying

So neither i nor any high speed detector can detect such high frequency variations and therefore what we will detect is an average value that is why we take an average value this has been probably discussed in an earlier chapter that cos square omega t minus phi is a rapidly varying function

So whenever we determine the intensity the rapidly varying function has to be average

So these brackets refer to time average time average

So the time average is equal to half because the maximum is one minimum is 0 it is varying rapidly between 1 and 0 and therefore on an average we see that the function varies the value of this cos square omega t time average is half using this we come back to the super position of waves

So we get the intensity variation here is cos square omega t time average using this equal to half we have i is equal to a square by 2 a square divide into half

So a square by 2 if only the source s 1 was present for example

So if we had

So this is resultant due to both the sources s 1 and s 2. if only the source s 1 was present let us say s 2 was not there then we would have got intensity at the point p equal to mod psi 1 square which is equal to here of course we have taken all real quantities

So we even need not write mod

So a 1 square into cos square k r 1 minus omega t because there is only one source

So k r 1 minus omega t gives us the phase term and therefore we will get a one square by two the intensity due to source s one is a one square by two and similarly intensity at point p due to only s two that is if s one were not there then we would get i two is equal to a two square by two now lets continue further and see what is the resultant therefore

So the resultant here now is a square is equal to remember we had written a square cos square plus a square sin square

So this is a cos phi and therefore a square is equal to this square plus this square

So recall the expression we had written here

So a is equal to a square cos square phi plus a square sin square phi where a cos phi was this term and a sin phi was this term

So now we are using this to determine a

So a square is equal to square of this and therefore a square is equal to square of this plus square of this which gives a 1 square plus a 2 square

So we can square and add them this gives cos square k r 1 this q sin square k r 1 a 1 square therefore we will get 1 term as a one square another term as a two square and the term the mixed term as two a one a two cos k r one minus k r two that is two i a square we have just now shown that a square is equal to two i

So we have just now shown that i is equal to a square by two or a square is equal to two i

So a square is equal to two i is equal to two i one plus two i two plus two times a one is square root of two i one into a two is square root of two i two into cos delta we are calling this as delta where delta is equal to k r 2 minus r 1 that is the phase difference between the 2 waves at p this is the phase difference between the two waves because omega t is common

So we had one wave with cos k omega t minus kr1 and the other term with cos omega t minus

k r two therefore the phase difference is k r one minus k r two delta is equal to k r two minus r one is the phase difference between the two waves interfering at the point p now r two minus r one is the path difference r two is this distance s two p r one is this distance therefore r two minus r one is the path difference between the two waves path difference multiplied by the phase constant gives us the phase difference delta therefore using this as delta we have

So 2 cancels throughout we have i is equal to i 1 plus i 2 plus 2 root i 1 root i 2 cos delta this is called the interference equation the interference equation now we will determine the intensity distribution as a function of delta delta depends on r two minus r one which means at different positions p on the x axis we will have different path differences and therefore different phase differences and therefore different intensities

So we will determine the intensity distribution along the x axis

So the intensity distribution here on the screen here on the x axis is given by

So here we are

So this is the interface equation for delta is equal to 0 plus minus 2 pi plus minus 4 pi etcetera directly looking at the mathematical formula here

So delta is equal to this we have i is equal to i max because this cos delta is 1 and therefore this term is simply square root of i 1 plus i 2 the whole square i max and for delta is equal to plus minus pi plus minus 3 pi and

So on we will get i is equal to we will get a negative sign here cos delta being minus one and therefore we have i will be equal to minimum which is equal to square root of i one minus i two minus square root of i 2 the whole square

So this is the maximum intensity and minimum intensity depending on the value of delta which as we see that it will depend on the position of the point p because delta will have take different values depending on the position of the point p and therefore if a 1 is equal to a 2 as is the case here we can see that s 1 and s 2 are drawn from the same wave front if we look at the for if we recall the diagram here that s 1 and s 2 are drawn from the same wave front and therefore they have the same amplitude and they are in phase at this point we will discuss more about the phase and phase differences in a later lecture a 1 is equal to a 2 and therefore i 1 is equal to i 2 is equal to i 0 let us call it as i 0 then i max will be equal to 4 times i 0 because this will be i 0 this will be i 0

So it will be two times square root of i zero square gives us four times i zero for delta is equal to zero plus minus two pi and

So on and i minimum i one is equal to e to i two therefore i minimum will be equal to zero and the i the intensity here given by this for the case of i one is equal to i two that is a one is equal to a two will be i is equal to two i zero into one plus cos delta here one plus cos delta and this can be written as four i zero cos square delta by two

So the intensity distribution on the x axis depending on delta will be given by this expression four i zero cos square delta by two

So we can see what how does the intensity vary

So we can plot intensity as a function of delta

So if we plot the intensity variation as a function of delta here its shown here i of delta is equal to four i zero cos square delta by 2 and delta here is given by

So let us look at the diagram first

So delta is equal to k times r 2 minus r 1 and for this is delta versus intensity we can see that whenever it becomes an integral multiple of two pi intensity becomes maximum and whenever it is an integral multiple that is odd integral multiple of pi such as minus pi minus three pi pi three pi four we have the intensity going to minima

So this is the intensity variation as a function of delta now delta is equal to this therefore k being constant k is 2 pi by lambda therefore it is the path difference which will determine the intensity distribution

So let us calculate the path difference for a point p on the x axis here r two minus r one is nothing but s two p minus s one p s two p minus s one p

So if we see the coordinates here s two p

So this hypotenuse of this right angle triangle

So s 2 p square is equal to d square plus this square this is x coordinate x plus this small difference here

So this is d by 2 because d is the separation between this and therefore half of it

So o is on the perpendicular bisector between this s one and s two therefore each one is a half here

So d by two as shown here d by two and d by two

So we have s two p square is equal to d square plus x plus d by two square and s one p square is equal to same d square plus x minus d by 2 square and therefore s 2 p square minus s 1 p square is simply equal to 2 x d or s 2 p minus s 1 p is equal to 2 x d divided by s 2 p plus s 1 we have not made any x approximation

So far

So there is no approximation and we have got the expression for path difference

So we will discuss how the path difference determines the intensity but let us have a look at the practical situation now

So the path difference here r 2 minus r 1 is 2 x d by s 1 p plus s 2 p in a practical setup one can determine this accurately but in a practical setup we want to make a practical approximation typically the d the separation between the source plane and the screen here is 50 to 100 centimeter when we do experiments in a lab and the separation between s 1 and s 2 the 2 holes here are separated by a very small separation typically between 0.1 and 1 mm we will see some numericals and we will see that this d is generally about 0.3 mm 0.4 mm and

So on

So we want to get a feel for the numbers now

So the separation is of this order d is much larger

So this is 500 to 1000 millimeter and this is d is very small and the screen on which we see here is typically few millimeters to few centimeter distance only where the fringes are formed and because of various reasons as we will discuss a little later and therefore i have deliberately drawn this setup to get a feel earlier we have discussed showing this setup now i have shown tried to show in a real scale although this is not exactly to scale d is much larger compared to d and the x the point p that we are look the position of the point p is also much smaller compared to d

So noting that x comma d are much smaller than this then we can write s 1 p plus s 2 p that is s 1 p here and s 2 p here as 2 times t we are approximating s 1 p equal to d and s 2 p equal to d this approximate this is an approximation but it is a very good approximation later on we will see that if we put some numbers the error that we will make is extremely small much smaller than point zero one percent or

So and therefore this is a very good approximation in practice and using this approximation therefore we can write the path difference r two minus r one

So we are substituting two d for this two capital d is equal to x d by d in other words the path difference r 2 minus r 1 is proportional to x the position coordinate of p here p is a point on the x axis and we have already seen that the phase difference is proportional to r two minus r one because delta is equal to k into r two minus r one now let us see what is the corresponding fringe pattern how would how would we see the intensity distribution we are determining the intensity distribution and through estimation of the path difference and therefore the intensity maxima and minima are now given by

So recall lets determine the intensity maxima and minimum i is equal to i zero into cos square delta by two and the intensity maxima are given by phase differences delta is equal to 2 pi by lambda into r 2 minus r 1 equal to plus minus n times 2 pi where n is equal to 0 1 2 etcetera that is the path difference

So we can see here 2 pi 2 pi cancels lambda goes here and we have r 2 minus r 1 is equal to plus minus n lambda n is called the order of the maxima similarly the intensity minima are given by phase difference is equal to plus minus n plus half into 2 pi

So we can put numbers n here

So n equal to 0 1 2

So if you put n equal to 0 we get pi delta equal to pi if we put n is equal to 1

So this is 3 by 2 2 2 cancels

So it is 3 pi and

So on

So with n is equal to 0 1 2 etcetera and or the path reference r 2 minus r 1 is equal to plus minus n plus half lambda the plus sign gives position of maxima on one side of the point delta is equal to zero

So let me keep the diagram here at the point o r one will be equal to r two and therefore delta is equal to 0 path reference is equal to 0 on one side the path difference is positive and on the other side the path reference is negative because when r 2 for a point p here r 2 will be smaller than r 1 and therefore path difference is negative and

accordingly we will have the phase difference negative on this side phase difference positive on this side therefore the conditions here the plus sign in this expression here gives position of maxima on one side of the point delta is equal to zero while the negative sign gives on the other side of the point o is equal to of the point o or delta is equal to zero

So let us see the position of maxima and minima we have seen the condition for maxima and minima now let us see the position x for maxima and minima

So position of maxima and minima

So at the point o

So lets see the diagram here at the point o which is on the perpendicular bisector here s one o is equal to s two o s one o is equal to s two o means r one is equal to r two therefore we have the phase difference equal to zero that corresponds to n equal to zero and this is a point of maximum intensity because n is equal to 0 corresponds to 0 phase difference and that is a point of maximum intensity and it is called the zeroth order maxima we will get maxima and minima here because we have already seen the intensity varies with the delta and delta is sinusoidally that is the cost square variation and delta is proportional to x or x is proportional to delta and therefore we have the same cos square variation with x and at the point o we have phase difference is equal to 0 and that corresponds to the condition here

So the phase difference is equal to 0 means n is equal to 0 we have the condition for maxima and therefore this point will be maximum in fact this is called the central maxima it is the central maxima central maxima is defined as the point at which the path difference is zero it is not it may not be at the point o always depending on the the correct definition would be the central maxima corresponds to the point of 0 phase difference or zero path difference we will see later on that due to insertion of a glass slide or due to some phase change the central maxima may not be at o it may appear at a different point therefore the central maxima is defined as the point at which the phase difference is zero now the next maxima

So here the next maxima occurs when r two minus r one is equal to n lambda that is 1 lambda which means r 2 minus r 1 we have already derived the expression for the

So we have just now derived the expression that r 2 minus r 1 is equal to x d by d and when r 2 minus r 1 becomes equal to lambda that is n equal to 1 we call that as x 1 the position of the first maxima is x one d by d is equal to lambda or the position of the first order maxima x one is equal to d by d into lambda exactly in general for different values of n

So x n the position of the nth order maxima is given by x n is equal to n times d by d into lambda and therefore we can determine the separation between adjacent maxima and therefore the separation between adjacent maxima is x n plus 1 minus x n which is denoted as beta beta is equal to n plus 1 d by d lambda minus n d by d lambda equal to d by d into lambda we will discuss about this beta in subsequently but note that the beta is proportional to d and it is inversely proportional to the small d which means at any given wavelength and therefore the separation between the maxima will be large if d is small and the separation will again be large if d is large that is why although it is multiplied by lambda which is a very small number

So this is maybe 600 nanometer 500 nanometer which is a very small number however if it is multiplied by a large ratio that is by making d small and d large we can have significant separation beta we will discuss this with more numbers but let me take a typical number here d is equal to hundred centimeter d small d is equal to point three millimeter and lambda is equal to six hundred nanometer is about the orange color or in light then we can calculate beta which is the separation between adjacent maxima separation between adjacent maxima as two millimeter which can be seen by i because its a two millimeter separate the intensity peaks are separated by two millimeter

So similarly the position of minima are given by r two minus r one is equal to x m by d

So now instead of n i have used m because this is for minima m standing for minima positions of minima x m by d is equal to x m into d by d is equal to plus minus m plus half lambda you could as well use n but i have just used m to make a difference between the intensity maxima positions of the maxima and minima

So m is equal to 0 1 2 etcetera thus x m that is the position of the maxima minima is given by m plus half d by d into lambda there is no central minima because the central the 0th order is maxima as we know as we have just seen and therefore when m is equal to 0 there is a path difference which is lambda by 2 and therefore m equal to 0 gives

position of the first minima m equal to 1 gives the position of the second minima and
So on
So in this formula we have m is the position of the minima but m equal to 0 gives
position of the first minima and
So on now we have determined
So what we have determined
So far is the intensity distribution on the x axis
So on the x axis we have a cos square intensity variation at different points on the x
axis with maxima at the point o and maxima and minima on both sides of the point o but we
want to see in general what would be the intensity distribution on the entire screen
So this is ok on the x axis we have determined how to determine the intensity
distribution on the screen
So we have to consider an arbitrary point q anywhere on the x y plane and therefore let
us do that and find out what is the intensity distribution on the plane now we have got
intensity distribution on the line along the line along the x axis but now we want to
determine the intensity distribution on the whole screen
So let us consider an arbitrary point q
So let me show the diagram again for
So here
So this is the young's double hole interference experimental setup but now instead of
taking a point on the x axis here i am taking an arbitrary point q
So s one s two a point q here
So the point q will have coordinate x y and zero the point s one the coordinate of point
one please see that it is x y z
So the o is x equal to zero y equal to zero and z equal to zero therefore the
corresponding coordinates of s one and s two are s 1 d by 2 as before because its the
total separation is d
So it is d by 2 here and minus d this is in the reverse direction therefore it is minus d
the separation is d therefore the coordinate z coordinate is minus t similarly s two is
minus d by two because it is in the lower x axis in the x axis below zero and therefore
the coordinate here is minus d by 2 0 and minus t and the path difference we have to
determine the path difference r 2 minus r 1 which is shown here s 2 q minus s 1 q
So once we know the coordinates of the points we know how to determine the distance
between the two points
So we will determine the path reference
So let us determine the path difference at any arbitrary point q
So here
So the path difference at the point q
So lets here
So is s 2 q minus s 1 q let us call it as delta say delta is given by this one
So here
So this is s two q is x
So it is x two minus x one whole square plus y two minus y one whole square plus z two
minus z one whole square if the two points have coordinates x one y one z one and x two y
two z
So that is the formula which has been used here
So we have x plus d by two the whole square plus y square plus d square because although
it is minus d but it is square therefore it is d square is eq to the power half
So this is s 2 q and this is s 1 q
So s 2 q minus s 1 q is equal to delta
So we take this to the other side and write and square it
So that we have this x plus d by 2 whole square plus y square plus d square we have
squared this term this has gone to the other side is equal to delta plus this to the
power half and whole square this can be simplified
So i would not derive the simplification steps we can easily simplify this and to get an
expression that d square minus delta square into x square minus delta square into y
square is equal to this term here for a fixed value of delta that is delta is what is
delta is the path difference for a fixed value of delta that is if you choose a point q
then it has a path difference delta
So for a fixed value of delta the above equation is of the form
So we can divide by this on this side then we will have it is of the form x square by a

square minus y square by b square b square is simply delta square divided by all of this
So delta square delta square cancels
So this in the denominator is b square is equal to one
So this is an equation of parabola note that these are positive constant see d is much larger than delta delta takes values of lambda 2 lambda 3 lambda a few lambdas and d takes values which are of the order of millimeter or point two millimeter point five millimeter this is of the order of micrometer typical values of d
So we can put what is d d is
So d is i have already said that this is typically from this to 1 mm what is delta delta is the path reference
So we are interested in finding the fringes where path difference can be lambda 2 lambda 3 lambda of course intermediate values also but they are a few lambdas
So few lambda this is few mm lambda is 600 nanometer which implies it is 0.6 micrometer
So few lambda is therefore of the order of micrometers micrometers and here it is of the order of millimeter d is of the order of millimeter that is a factor of ten power three
So micrometer is ten power minus six millimeter is ten power minus three meters and therefore d is much larger than delta much larger than delta and therefore this quantity here d square minus delta square here is always a positive quantity and therefore x square by a square a positive quantity minus y square by b square which is the equation of a hyperbola where a and b are constant for different values of delta different if we put delta is a constant but if i take different values of delta we get different hyperbole that is the locus of all points with the constant path reference are hyperbolic
So let me explain this
So this is the equation of an hyperbola for a given value of delta because this is of this form a and b are constants fixed constants for a fixed value of delta because d is fixed for the given experimental setup capital d the separation is also fixed it is only delta which will vary from point to point as we change the point q but for a fixed value of delta if we assume delta is equal to lambda for example delta is equal to lambda then this is a fixed constant and we have a particular hyperbola
So let me draw this hyperbola here
So this is the x axis and this is y axis
So we will have for one particular value of delta we will have hyperbola like this
So this is for a particular value of delta now delta 1 let me go if i take a different value of delta then i will get another curve here
So delta
So depending on the value of delta we will get a family of hyperbole here
So what we will have
So this is delta delta 2 delta three for different values of delta we will get different hyperbole for example if delta one is equal to lambda then we know that the path difference lambda would correspond to a bright point its a bright point or intensity maxima and therefore if for delta one equal to lambda if this were the curve then it will simply tell that this is a locus of all points which are bright points
So this is bright points
So all the points along this are bright because delta 1 is equal to lambda path difference is equal to lambda if this curve here
So delta 2 here is equal to let us say 3 by 2 lambda 3 by 2 lambda then this points will correspond to dark points this is the locus of all dark points this is the locus of all bright points in other words what we will see are bright and dark hyperbole alternatively because delta will continuously increase in this direction therefore alternatively we will get bright and dark hyperbole and these are nothing but the fringes
So what i have shown here is therefore
So let me put again for different values of delta we get different hyperbole the locus of all points with constant path differences are hyperbole this means that we get we get bright and dark fringes
So let me show you a some fringe system here
So interference fringes
So let us discuss the interference fingers as i have discussed if delta is equal to n lambda then the hyperbola would comprise of all points whenever delta is an integral multiple of lambda then the hyperbola that particular hyperbola would comprise of all points with the intensity maxima and if delta is n plus half lambda then that hyperbola would comprise of all points with intensity minima this implies we will see alternate

bright and dark hyperbole on a screen these are called the interference fringes
So the first time we are introducing interference fringes and the pattern on the screen is called a fringe pattern
So i had shown earlier that i did not show a hyperbolic pattern here but
So what we have is a linear fringe pattern
So we can see alternately bright and dark fringes i will show a hyperbolic fringe pattern at a later time
So this it could be now if the locus of all points with a constant path difference in this case the case that i had discussed it happens to be hyperbole constant part difference but in a particular setup in a particular setup if the locus of all the points with the constant path difference happens to be circles then we will get circular fringes and we will get circular fringes like this
So the circular fringes that i had showed you here is because the locus of constant path reference are circles in this case and if they the bright fringes correspond to n lambda whenever path difference is n lambda and the dark fringes correspond to path difference n plus half lambda
So that is how we get the fringe system in a interference setup
So we have discussed the formation of interference fringes here and now we come back to the interference fringes in the young setup in the young's experimental setup
So this formula we have derived we have just now showed that let me put this here
So we just now showed this that d square
So this is the equation of the hyperbola interference fringes in young's experiment now or x is equal to
So we take this to the other side and then we will have delta square multiplied by this and then divide by this here and take square root to get x is equal to delta divided by this into this in practice i already discussed that y is much smaller than d y is few millimeter why is the distance on the screen
So this is the screen x y axis and we are talking of this area which has dimensions of few millimeters to few centimeters only and therefore whereas d is of the order of hundred centimeters and therefore y is much smaller than d why a few millimeter to centimeter and d is of the order of hundred centimeter and therefore y square can be neglected in comparison to d square in the case of the young's experimental arrangement
So we had just seen that i had showed you a practical arrangement where the distances are very large
So we can see here d is much larger compared to x and y the x and y which are here on the screen is much smaller compared to this and therefore we can neglect y square compared to this d square this is millimeter square this is hundred centimeter square and therefore we can write x is equal to approximately equal to is a very good approximation but approximately equal to this now for a given delta for a fixed value of delta right hand side is a constant right hand side is a constant what it means is that the that is x is equal to constant for each fixed value of delta this implies locus of constant path difference are straight lines parallel to the y axis x equal to constant x equal to constant are straight lines unlike this hyperbole this is hyperbola is the exact solution but if y x and y are much smaller than d then this will turn out to be a straight line
So x equal to constant for each fixed value of delta this is a very valid approximation here there is no approximation but here there is a valid approximation and these therefore this leads to straight line interference fringes
So the interference fringes in young's experiment are straight line interference fringes
So that is what i had shown earlier that we get straight line interference fringes
So the fringe system would look like this
So along the x axis x equal to constant the locus of constant path difference are x equal to con they are straight lines parallel to the y axis or perpendicular to the x axis
So this is the type of fringe system that we will see and the separation between these bright bright lines is called the fringe width beta beta is the separation between the bright points now i have shown here in this diagram the point p which is on the x axis and an arbitrary point q here if p is a bright line bright point or intensity maxima then all along that line intensity will be maximum intensity will be maximum because we have just seen that x equal to constant corresponds to the locus of all points with bright or dark depending on the delta value therefore if we determine the separation between the intensity maxima along the x axis then we it is nothing but the fringe width that is the separation between the two bright fringes the separation between two bright fringes is

called the fringe width which is also nothing but the separation between two adjacent maxima

So this is what we had discussed earlier and here i come to the last part that is fringe width the separation between the adjacent bright or dark fringes is called the fringe width recall that beta is equal to here d by d into lambda this was the separation between maxima on the x axis just now we have derived that x is equal to this expression for x is this but i have mentioned i have explained that delta the path difference delta is much smaller than d which is much smaller than d and therefore this term here this delta can be neglected with respect to this d but this d itself is negligible the d square here is negligible compared to the d square here again let me repeat that this is of the order of hundred centimeter

So hundred centimeter square whereas this is of the order of one millimeter square extremely small compared to this and therefore this term can be neglected and similarly with respect to d delta e is negligible almost one thousand times smaller and therefore we can neglect this which means we get x is equal to delta into d by d x is equal to delta into d by t for delta is equal to 0 that is 0 part difference we get the position x equal to 0 which is a constant x equal to constant gives us the fringes the constant in this case is 0 for delta is equal to 0 x is equal to 0 which is equal to constant the fringe is given by x equal to constant and x equal to value the value of x being 0 which means it is the y axis is the locus of all bright points that is the central fringe y axis is the locus of all bright points when the path difference is 0 delta is equal to 0 path difference is 0 and that is called the central fringe

So in this case y axis the bright fringe in along the y axis is the central fringe if delta is equal to lambda substituting this delta is equal to lambda then we get x1 the position of the next bright fringe as lambda into d by d if delta is equal to two lambda path difference equal to n times lambda we get the position x two is equal to two lambda two lambda into d by d that is the second bright fringe and therefore the fringe width therefore the fringe width is given by x two minus x one is equal to lambda into d by d as before

So this is the fringe width that this is the difference between the maxima on the x axis which we had determined earlier but it is the same difference that we get for the fringe width

So to determine the fringe width one can as well consider the points along the x axis we will put some numbers and discuss this more carefully in the next lecture thanks you