

hello welcome to this lecture module on optics in the last lecture we discussed about reflection by spherical mirrors and in particular we discussed formation of images by a spherical mirror we also derived the mirror equation which tells us precisely the position of the image when the position of the object is given

so today we will move on and today we will discuss refraction of light to begin with we will first discuss refraction at a plane interface refraction of light from a transparent medium a transparent medium it has been historically known for a long time that when light is incident on a transparent medium such as glass or surface of water then a part of the beam is reflected back and a part of the beam is transmitted into the medium light incident on a reflecting surface here it is a transparent dielectric transparent medium a part of the light is reflected back we know that this satisfies the law of reflection that the incidence angle incident angle is equal to the emergent angle and a part of the beam is refracted or transmitted into the medium it was also known it was also observed that the angle of refraction was not equal to the angle of incidence while the angle of emergence here that is the reflected angle the angle subtended by the reflected beam was equal to the angle of incidence the refracted angle here that is the angle subtended by the refracted beam or the transmitted beam into the medium was not equal to the angle of incidence this was historically known for a long time

so this is partial reflection and partial transmission of a light beam a light beam incident a part of it is reflected and a part of it is transmitted

so this is called partial reflection now if we consider an air water interface in particular because water air interface is very commonly encountered in day to day life that a light beam incident on a water surface a part of it is reflected back and a part of it is transmitted into the medium as long as one is in the year one forty a.

the greek physicist ptolemy had tabulated the angle of incidence and the angle of refraction for different angles of incidence i_1 i_2 i_3 etcetera he had measured the angle of refraction and tabulated and given as a table if this is the angle of incidence then this will be the angle of refraction but he did not know any further about the relation between them however in 1621 w snell formulated the following that is he observed based on experimental observations he found that $\sin i$ by $\sin r$ is a constant for a given medium for refraction at an interface the angle of incidence \sin of the angle of incidence by \sin of the angle of refraction was a constant this is subsequently known as the snell's law

so the snell's law

so the snell's law is now given by $\sin i$ by $\sin r$ is equal to n_2/n_1 a constant when we have an interface between a medium one and medium two of refractive index n_1 and n_2 then if i is the angle of incidence and r is the angle of refraction then $\sin i$ by $\sin r$ is equal to n_2/n_1 where n_2/n_1 is called the refractive index of the second medium with respect to the refractive index of the first medium that is n_{21} is equal to n_2/n_1 the first and second that we denote in all subsequent discussions is the angle of incident where the ray is incident we call that as medium number one and there could be two three four and

so on

so the refractive index of the second medium with respect to the refractive index of the first medium

so n_{21} is equal to n_2/n_1 note that n_{21} is greater than one if n_2 is greater than n_1 if n_2 is greater than n_1 n_{21} is greater than 1 and if n_2 is less than n_1 n_{21} is less than 1 this has some important applications we will see this the medium with higher refractive index higher of

the two is called the denser medium and the medium with lower refractive index is called the rarer medium this denser medium density has nothing to do with the mass density which is density is equal to mass by volume

so this has nothing to do with that this is denser here refers to a higher refractive index of the medium and rarer refers to a lower refractive index is a relative term relatively lower refractive index its not any absolute value is just relative to the other one relative to the other denser relative to the the first medium here

so rarer medium is a medium with refractive index lower than the second medium or the other medium now let us see in particular refraction when light is entering from a rarer medium to a denser medium and when it is entering from a denser medium to a rarer medium

so note that angle of incidence i angle of refraction here i have considered air glass interface just as an example air is refractive index one glass is 1.5

so this is rarer to denser light entering from the rarer medium to the denser medium into the denser medium

so $\sin i$ by $\sin r$ is equal to n_2 over n_1 now n_2 over n_1 is greater than 1 because n_2 by n_1 n_2 by n_1 which is greater than 1 that implies $\sin r$ is less than $\sin i$ which implies r is less than i in other words the ray bends towards the normal

so this is the normal here normal to the interface

so this is the interface between the two media

so this is glass this is air this is called the interface and this line here is the normal to the interface and the angle with the normal to the interface is the angle of refraction here and the angle of refraction r is less than i in this case when light enters from rarer to denser light bends towards the normal and when it is the reverse that is when it enters from a denser medium to a rarer medium

so for example glass here and air here this is the interface then $\sin i$ by $\sin r$ is equal to n_2 over n_1 that is one by one point five which is less than one which implies $\sin r$ is greater than $\sin i$ and r is greater than i in other words the ray bends away from the normal

so the ray here the transmitted ray or the refracted ray bends away from the norm which means bends away from its original direction the dotted line here is here is the original direction

so it bends away from the normal whereas here the original direction is here light bends towards the normal the ray here we are referring to the refracted ray lets take now refraction through a glass slab that is now we will encounter two interfaces earlier we had discussed refraction at a single interface

so now we are discussing refraction at a two interfaces that is what happens if we consider a glass slab here a rectangular glass slab then it has one interface here and the second interface here at the first interface it is between air and glass at the second interface it is glass to air

so consider incidence of ray here making an angle θ_1 then at the first interface the ray bends towards the normal and therefore θ_2 here the refracted angle is θ_2 i have now used the notation θ when there are more than one interfaces or several interfaces it is convenient to use θ_1 θ_2 θ_3 and

so on rather than i and r because there will be more r s and the same r would become i for the next interface and

so on therefore it is convenient to use θ_1 θ_2 θ_3 and

so on

so i have used θ_1 here

so θ_1 and θ_2 is the angle of refraction earlier this was i and r but now we have a second interface where this ray that is the refracted ray at the second interface subtends an angle θ_2 which is the angle of incidence now as far as this interface is concerned and then θ_3 is the angle of refraction at the second interface now applying the snell's law at interface one and two at interface one here and interface two interface between two media $n_1 \sin \theta_1 = n_2 \sin \theta_2$ that the first one is n_{glass} by n_r that is n_2 by n_1

so we have noted i with the subscript n_{glass}

so n_1 is n_{air} and n_2 is n_{glass} t is the thickness of the glass slab l is the lateral shift we will talk about l in a minute

so $n_1 \sin \theta_1 = n_2 \sin \theta_2$ is equal to n_{glass} by n_r and at the second interface θ_2 is now the angle of incidence therefore $n_2 \sin \theta_2 = n_3 \sin \theta_3$ is equal to n_r by n_{glass} that is n_2 now there is the second third medium here that is air and therefore n_{air} by n_{glass}

so this simply gives you $\sin \theta_1 = \sin \theta_3$

so you can multiply the two equations and you see that $\sin \theta_2$ n_{glass} n_{glass} cancels here and air by n_{air} is one and therefore $\sin \theta_1 = \sin \theta_3$ or $\theta_1 = \theta_3$ what it means is when the ray passes through the glass block the ray which is coming out makes the same angle θ_1 which is equal to θ_3 here which is equal to θ_1 which means as far as the direction is concerned direction of the transmitted ray is concerned there is no deviation there is no deviation however there is a lateral shift as you can see here there is a lateral shift

so no deviation but a lateral shift of the ray and this lateral shift depends on the thickness of the glass block as we will see later now i extend it further

so we have now considered two interface but suppose i have now several interfaces

so we look at refraction through a multi layered structure now there are four layers one two three four and of course outside here air and outside here

so this is a stack a stack comprising of four layers of different refractive indices they are n_1 n_2 n_3 and n_4 and n_5 are different

so this is a stack comprising of four layers and therefore there are five refractive indices one from outside here and one here now if we apply the snell's law the cells law has to be applied at each interface depending on whether it is going from rarer to denser or denser to rarer the ray will bend away or towards the normal

so we can see here for example the ray here is bending away from the normal and again away from the normal

so i have just taken some refractive indices but we have not given any values here

so if we apply the snell's law at each interface

so the first interface $n_1 \sin \theta_1 = n_2 \sin \theta_2$ is equal to n_2 by n_1 or we can cross multiply this $n_1 \sin \theta_1 = n_2 \sin \theta_2$ this is a more convenient form of snell's law $n_1 \sin \theta_1 = n_2 \sin \theta_2$ if we apply to the second interface here then it gives us $n_2 \sin \theta_2 = n_3 \sin \theta_3$ where θ_2 θ_3 the angles are indicated here θ_2 is the angle of refraction here which becomes the angle of incidence when the second interface is considered θ_3 is the angle of refraction here which becomes the angle of incidence for this interface and

so on

so we have $n_3 \sin \theta_3 = n_4 \sin \theta_4$ $n_4 \sin \theta_4 = n_5 \sin \theta_5$ is

equal to for the last medium θ_5 here is the angle of refraction which becomes the angle of incidence here for this interface and θ_6 here is the final angle of refraction if these are all equal it simply means $n_1 \sin \theta_1 = n_2 \sin \theta_2 = \dots = n_6 \sin \theta_6$ if the first and the last medium are the same in this case for example air this was a stack of four layers on both sides there is air a ray of light was incident here and the ray is emerging from here if the first and last medium are the same it may not be same it could be water in another problem this could be water for example but if they are the same then $\theta_1 = \theta_6$ which implies there is no deviation that is the final emergence angle θ_6 does not depend on the thickness and refractive index of the layers then what is the need this is quite interesting to see that the angle of deviation there is no deviation which is independent of the refractive index and thickness of the layers then what is the need to use such a multi layered structure there are large number of applications of multi layered structures in optics to understand this ray optics will not be able to help us to understand and design this applications we will have to go to wave optics however let us see something

so so we will come back to that a little later and here we summarize therefore the laws of refraction the laws of one and two the incident ray the reflected ray and the transmitted ray or the refracted ray lie in a plane perpendicular to the interface

so we can see here this is the incident ray there is a reflected ray and there is a refracted ray refracted or transmitted because the energy is partially transmitted into the medium and partially reflected from here and therefore all of them lie in one plane perpendicular to the interface second is the snell's law which is $\sin \theta_i / \sin \theta_r = n_2 / n_1$ or equal to $n_2 \sin \theta_1 = n_1 \sin \theta_2$ which is more conveniently written as $n_1 \sin \theta_1 = n_2 \sin \theta_2$ θ_1 or θ_2 could be the angle of incidence now we will see we will discuss some natural consequences of refraction of light

so some natural consequences of refraction of light the first one that i have shown here is apparent depth is illustrated here if we have a beaker of water or a container which has water and if there is a coin at the bottom i have taken a point p maybe a point source here at the bottom then it appears as if the depth where the point source is present is smaller compared to the actual depth that is illustrated here why this is happening

so you are observing from here try to show i and here is a point p it could be a point source

so therefore light is emitted it could be an object we know that when we discuss the image of an object then also we consider rays coming from a point on the object and this could be a point source as well

so point source rays which come out from here are refracted at the interface this for example is water and this is air

so it bends away from the normal at each of these interfaces we can mark the interfaces here

so this is the interface and if you draw the normal here then light bends away from the normal because refractive index here is smaller than refractive index here and it bends in this direction therefore it is a diverging beam

so what you have here is a diverging beam but all of them if you extrapolate here then they appear to come from a point p dash from a point p dash which is different from the actual point p in other words if we look from here it appears as if the apparent depth

so the point p dash the depth of the point p dash that is from the surface to the point p dash here is d_i i have denoted it by d_i the actual depth of course the point p is at the bottom of the container the actual depth is d and d

dash is the apparent depth d is the actual depth and d dash is the apparent depth

so the apparent depth in this case is smaller compared to the actual depth we can quantitatively determine how much it is smaller

so lets just continue here and see this now the equivalent problem as shown here the point p was here a ray of light was at an angle i here it is angle of incidence at this interface comes out here with an angle of refraction r an angle of refraction r and the ray appears if i extrapolate it here that is if you were observing from here the ray appears to come from the point p dash and this angle here is r because the angle of refraction here is r and therefore this angle is r angle of incidence i is here and this angle is also i because this are two parallel parallel lines

so this is array which is normally incident which transmits through the medium of course its also partial transmission everywhere it is partial transmission

so d dash is the apparent depth d is the actual depth now for small angles i these are actually small angles because we are looking from here

so i can show the i which is here

so the i is here actually

so this is the i which is observing the point p now therefore the rays which enter i are those which make very small angles the ray which would come here would enter the eye a ray which is coming here is any ray which is making a larger angle does not enter i

so all the rays which enter your eye here is the ones which make very small angle therefore the approximation is very much valid for small angle i and r if i is small r is also small although r is a little larger than i but it is still quite small therefore we can write $\sin i$ nearly equal to $\tan i$ for small angles θ $\sin \theta$ is nearly equal to θ θ is nearly equal to $\tan \theta$

so $\sin \theta$ $\sin i$ is equal to $\tan i$ $\tan i$ from this triangle here triangle p q r $\tan i$ is q r here divided by p q this length p q and similarly $\sin r$ nearly equal to $\tan r$ is equal to q r divided by p dash q this length and therefore $\sin i$ by $\sin r$ is equal to n_2 by n_1 by snell's law $\sin i$ by $\sin r$ if we divide one by the other it is p dash q by p q which is nothing but d dash by d

so apparent depth divided by actual depth is equal to n_2 by n_1 n_2 is always the second medium n_1 is the first medium by notation usually n_1 is air and n_2 is air that is from where we observe is air as in our problem our problem this is the liquid where the point was at the bottom of the container and here it is air apparent depth is equal to actual n_2 is equal to one therefore actual depth

so we if we take apparent depth actual depth comes here actual depth divided by refractive index of the medium the apparent depth is equal to actual depth n_2 is one divided by n_1 which is the refractive index of the medium

so apparent depth is equal to actual depth divided by refractive index of the medium

so if we consider glass or water then we see that the apparent depth is smaller compared to the actual depth

so this ah everyone can observe this in practice we will take up some numerical to illustrate this point further as a second example we look at the inclination of the setting sun inclination of the setting sun apparent inclination or apparent position of the setting sun

so the diagram here is not to scale its just a schematic because what i have shown is the earth earth has an atmosphere around it up to few hundred kilometers earth has an atmosphere this is of course several thousand kilometers therefore this diagram is not to scale

so earth surrounded by an atmosphere and beyond that of course it is free space

and the stars and sun are in free space which are far away from the earth
so this distance is of course much much larger compared to the thickness here
or the width here of the atmosphere

so its not to scale but just a schematic illustration

so what is being illustrated is the following there is an observer here on the
surface of the earth the size of the observer is of course negligible compared
to the dimension again not to scale

so the observer here looks at the sun if this is the horizon then the observer
is looking at the sun he is his apparent position of the sun is here he is
seeing the sun which is above the horizon but the actual fact is that the sun is
below the horizon because when the sun is here i consider a typical ray
one ray just to illustrate the ray which is coming from the sun there are of
course large number of rays a bunch of rays which are coming but array which is
coming like this when it enters the atmosphere free space or vacuum has
refractive index n is equal to 1 exactly 1.

θ and atmosphere here which has air and other gases has refractive index
slightly more than one maybe one point zero zero something but it is slightly
more than one and therefore the ray is entering from a rarer medium to a denser
medium and it continuously bends towards the normal we can stratify this for
example we can see that this is when it is entering

so this is the atmosphere

so the ray is entering like this ray is entering here there is a higher
refractive index therefore if i stratify this that is if i consider this into a
large number of layers then the ray which is incident here bends towards the
normal

so it bends towards normal it bends towards the normal it bends towards the
normal

so it slowly slowly because why slowly slowly because the refractive index here
is one and maybe here it is one point zero zero seven eight or something like
that but it is slightly larger and therefore the ray is continuously bending
towards the normal and therefore we see that however the observer who is here
when it reaches the observer then the ray is entering his eye like this

so it appears to him as if the ray is coming from some point here

so if this was the position of the actual sun then the sun appears to him as if
it is coming from a point which is here whereas the horizon is here

so this is the horizon

so that is the diagram

so let me put the pre drawn neat diagram here

so the ray we can see here starts bending and continuously towards the observer
and the observer finds it as if it is above the horizon

so this is what is known as inclination the apparent inclination of the setting
sun this is an illustration which shows that although the refractive index is
very small because the length of the atmosphere is of the order of hundred
kilometer or two hundred kilometers then it starts over that period it bends
significantly and there is a significant difference between the actual position
of the sun and the apparent position of the sun

so we have ah these two natural examples which i have illustrated

so we will take up some numericals to illustrate refraction of light and let me
take up some examples ok

so we come back and ah with these examples what we have we have got a better
appreciation of the refraction of light and there are numerous problems which
are possible particularly the multi layers are very important and let us take
some examples now to illustrate what we have studied

so here in narrow beam of light a narrow beam of light travels from medium 1

through three layers of different transparent media into medium five as shown in the figure

so see the figure a narrow beam of light travels from medium 1 here through medium 2 medium 3 medium 4 into medium 5 as shown in the figure rank the media in ascending order of their refractive indices

so we have to find out which is the medium with the least refractive index to which is the maximum one and therefore we have to ask we have to rank them or list them in ascending order from the lowest to the highest refractive index

so the data the figure shows angles here 45 degree 30 degree 40 degree fifty degree and thirty five degree here

so how do we go about this

so we use the snell's law we use the snell's law in the form that $n_1 \sin \theta_1$ is equal to $n_2 \sin \theta_2$ is equal to $n_3 \sin \theta_3$ etcetera or $n_i \sin \theta_i$ is a constant for any given medium

so we apply it to this and find out which one would be the lowest refractive index

so here

so use snell's law $n_i \sin \theta_i$ is equal to constant

so a medium which has the largest angle here which means $\sin \theta_i$ will be largest $n_i \sin \theta_i$ is constant therefore when the angle here that is θ_i this becomes θ_i here this is θ_r but θ_i here

so when the angle is the largest the refractive index must be the smallest because $\sin \theta_i$ increases with θ_i and therefore medium four must be having the smallest refractive index

so medium four the smallest one is medium four the largest angle

so medium four will have the smallest refractive

so let me rank it in that order in the ascending order of refractory index

so this will be 1 medium 4 and then the next angle that we see is 45 degree here the largest next largest angle is 45 therefore medium one will have the next higher refractive index

so two medium one medium one then we have forty and therefore three this is medium three

so medium three here and then after forty we have thirty five

so fourth will be medium five and finally the smallest angle that we have here is for medium two and therefore this will have the largest refractive index

so five medium two

so we have now ranked the various media here in ascending order of refractive index medium two where it makes the smallest angle will have the largest refractive index and medium four where it makes the largest angle here this is the rarest medium among all of them you can see that it is the rarest medium that is why it goes well above the well beyond the normal bending away from the normal making a large angle 50 degree here

so medium 4 1 3 5 2.

so as i mentioned that when we have several media then it is easier for us to write the snell's law in the form of $n_i \sin \theta_i$ is a constant for every one of the media

so this is like a quiz question very quickly we can identify without doing any mathematics just looking at the angle we can identify which are the ah media that have largest refractive index let me take another example here

so let us go to the next example a glass beaker of height 10 centimeter contains water of refractive index 1.

33 up to 4 centimeter height from the bottom and then a transparent oil n equal to one point three one above water up to the top edge of the beaker

so this here i have tried to draw the diagram
so there is a glass beaker here the first of total height 10 centimeter and the first 4 centimeter is filled with water n is equal to 1.
33 and the next 6 centimeter is filled with that transparent oil of refractive index n is equal to 1.

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so what when viewed from above that from the top when viewed from the top what would be the apparent depth of a small coin located at the bottom of the beaker there is a small coin placed here at the bottom of the beaker what would be the apparent depth when viewed from above

so this is actual depth is 10 centimeter but will it appear as 10 centimeter deep or will the apparent depth is smaller or larger than the actual depth that is the question

so to determine the apparent depth of the bottom of the beaker actually a small coin is placed or it could be a point source it could be a point p at the bottom of the beaker but basically to estimate the apparent depth of the beaker

so let us comprehend this problem a little bit more carefully

so here is the beaker let me draw that again

so beaker and there is water up to a certain level

so 4 centimeters of water

so this is four centimeter and this is six centimeter six centimeter

so this is of refractive index one point three one this is one point three three slightly different refractive indices and viewed from above which means you are viewing from the top here

so viewing from here

so that means the i is here i am showing i a little bigger i just for convenience the point to see is when you view this a bunch of rays enter the eye there is a small cone

so a cone over which rays enter the eye

so rays enter over a small cone coming from the bottom if i have at the bottom a point p or there is a point p here a point source p then a bunch of rays which come out will enter the i over a small cone this angle here is very small this angle of the cone is very small however this as we will see will lead to an apparent depth and we are asked to determine the apparent depth of the coin in this ah mixture of in the in the liquid in the beaker containing two different liquids

so here i have drawn a a more neater figure to illustrate the problem more carefully

so here is the solution

so the first medium of refractive index n_1 height h_1

so i have not put any numbers we are handling it in general analytically and second medium is of refractive index n_2 and of height h_2 and the third medium which is here outside which is the air in this case ah let me call it as n_3 and here is the i the i is here

so we are viewing from here the i is here but i have right now take

so in the last diagram that i had made i showed that in practice this angle is very small when you are viewing from the top but it is not necessary that i should be viewing from the top i could be viewing from an angle

so even then a small cone of rays would pass through this

so i may be watching from here

so my eye could be here

so this is viewing from above but this is viewing at an angle

so this angle i may be watching at an angle of 40 degrees for example

so both cases are keeping both cases in mind

so here i have tried to analyze this problem

so here it is from the point p the point source here ray which is coming here is incident at an angle theta one

so the refracted angle in medium two is theta two and the refracted angle in medium three is theta three

so see the diagram

so if you are watching from here

so if the i is here

so you may not be able to see let me draw an i there

so here is the i then

so i is watching this point observing this point which is coming at an angle theta three from the geometry what we can see is if this angle is theta 3 this angle is theta 3 i have marked this distance as x_3 and therefore from the geometry we can see that h_1 h_2 are the thickness of this water column here and h is the apparent position of the point object p there is a point object here but i sees it as if the point object is located here in other words h is the apparent depth in this problem h is the apparent depth

so we have to determine what is h the apparent depth the actual depth is of course h_1 plus h_2 the total height from surface to the bottom h_1 plus h_2 but the apparent depth is h

so looking at the geometry x_3 by h x_3 by h is $\tan \theta_3$ or h is equal to x_3 by $\tan \theta_3$ now from the geometry we can also see x_3 here is equal to x_1 plus x_2 because this is parallel to the normal here this is also a normal and therefore x_2 plus x_1 is x_3 and therefore h is equal to x_1 plus x_2 by $\tan \theta_3$ however x_1 here is equal to this height is h_1 and therefore x_1 by h_1 is equal to $\tan \theta_1$ x_2 by h_2 is equal to $\tan \theta_2$ therefore x_1 is equal to $h_1 \tan \theta_1$ and x_2 is equal to $h_2 \tan \theta_2$ and therefore h is equal to x_1 by $\tan \theta_3$ plus x_2 by $\tan \theta_3$ that is $h_1 \tan \theta_1$ by $\tan \theta_3$ plus $h_2 \tan \theta_2$ by $\tan \theta_3$ note that we have not made any approximation here there is no approximation involved and therefore this is valid for any angle theta which is at which the observer is looking at the coin and therefore if we know that the observer is looking at an angle theta 3 i can calculate theta 2 by using the snell's law because the refractive index of the media are given n_1 and n_2 and n_3 and therefore i can calculate theta two if i know i can calculate theta one and therefore i can i know $\tan \theta_1$ $\tan \theta_2$ and $\tan \theta_3$ and therefore h the apparent height h is equal to apparent depth h is equal to $h_1 \tan \theta_1$ by $\tan \theta_3$ plus $h_2 \tan \theta_2$ by $\tan \theta_3$ i can exactly calculate this for any theta 3 theta 1 and theta 2 can be determined using snell's law however in this problem it is said viewing from above viewings from above means we are viewing right from above which means as i indicated in the problem a small cone here

so we are looking at from above which means a small cone of angles which we are considering which enters the eye and therefore viewing from above implies the angles theta 3 theta 2 theta 1 are all small angles and therefore $\tan \theta_3$ is nearly equal to $\sin \theta_3$ $\tan \theta_2$ nearly equal to $\sin \theta_2$ and $\tan \theta_1$ nearly equal to $\sin \theta_1$ this approximation is very much valid for all small angles and if we apply that and substitute here that instead of $\tan \theta_1$ if you substitute $\sin \theta_1$ by $\sin \theta_3$ $\sin \theta_2$ by $\sin \theta_3$ then we will get h is equal to h_1 by n_1

so we can simply substitute that and see

so h is equal to h_1 into $\tan \theta_1$ which is approximated by $\sin \theta_1$ $\sin \theta_1$ by $\sin \theta_3$ plus h_2 into $\sin \theta_2$ by $\sin \theta_3$ $\sin \theta_2$ by $\sin \theta_3$

so we know that $n_1 \sin \theta_1 = n_2 \sin \theta_2 = n_3 \sin \theta_3$ and therefore $\frac{\sin \theta_1}{\sin \theta_3} = \frac{n_3}{n_1}$ therefore this expression here is $h_1 \sin \theta_1 = h_2 \sin \theta_3$ so this comes here and so this is n_3 by n_1 .

so h is equal to $h_1 \sin \theta_1 = h_2 \sin \theta_3$ and because our surface here is air we are looking through the liquid medium here therefore it is understood that it is air here therefore n_3 is equal to 1 with n_3 is equal to 1 we have h is equal to $h_1 \sin \theta_1 = h_2 \sin \theta_3$

so that is what is written here h is equal to $h_1 \sin \theta_1 = h_2 \sin \theta_3$ with n_3 is equal to 1 and therefore we are asked to find out the depth

so what is the answer therefore answer the apparent depth h is equal to $h_1 \sin \theta_1 = h_2 \sin \theta_3$

33 plus six centimeter divided by refractive index one point three one and which comes out to be seven point five nine centimeter

so the apparent depth of the coin is seven point five nine centimeters there are several variations of the problem which is possible

so n_1 can be may be larger than n_2 n_2 may be larger than n_1 and

so on several combinations are possible there is also an interesting extension

so here it is

so let me just discuss an interesting extension a particular

so here is a point P a point P could be a point source P which is observed by an observer here

so the actual height that he would see or the point P is located at some depth here

so some depth d now we introduce a block

so if someone introduces a glass block of certain thickness t thickness t and refractive index n

so this was this was here now a block is introduced a thick a glass slab is introduced of thickness t and refractive index n

so where will the point P now appear to be or what is the shift

so this is going to shift

so it may be shifting here or it is it may be shifted here because of the introduction of this because there is a refraction which is taking place and therefore we are asked to find out determine the shift in the

so this is the object now this point P is the object

so i can also call this as O

so this may be shifted to a point O' i am just showing it here O'

so determine the shift

so shift is this

so what is the shift

so this shift i can call this as s or Δh or Δd whatever s the shift determine the shift determine the apparent shift apparent shift the object is of course there but it is appearing

so determine the apparent shift of the object shift of the object it just occurred to me

so i had not drawn any pre drawn any diagram

so we can just determine the shift

so let us take

so this is d now if i draw a line here we can simply extend the earlier problem

so let me draw a fresh

so so the object is here O we have introduced a glass block here of certain

thickness

so let us say this is t this was the original position and therefore let me call this

so this is let this be l let this distance be l because i am interested only in determining the shift

so shift o and o'

so let this be l and

so this is the shift to be determined this is s to be detailed i can think of just extending the previous problem where we had apparent depth

so l and l'

so if i call this as l'

so this new position as l' we see earlier we had h actual height and h' as the apparent depth

so i am now calling it as l and l' this height does not matter because it remains the same there is no change taking place and therefore if i determine $l - l'$ that will give me the shift the shift is simply $l - l'$

so to determine l' what we had done was earlier if you remember that we had t of a thickness t of refractive index n_1 and the remaining part here that is $l - t$

so the total length is l

so i can write this as $l' = t \cdot n_1 + (l - t)$ that is this and we know this refractive index is one here because this is air this is air this is where a glass slab has been introduced a glass slab of thickness t has been introduced therefore $l - t$ divided by one refractive index is one and therefore this is what we have which means l' therefore $l - l'$

so i bring l' here

so there is a t also

so t goes to the other side is equal to $t \cdot n_1 + (l - t)$ i have simply applied the result the earlier result we said that h' that apparent depth was equal to $h \cdot n_1 + h_2 \cdot n_2$

so in this case h_1 is this thickness t by n_1 and the remaining length which is $l - t$ i had called the total length as l therefore $l - t$ is equal to which has which is of refractive index one because that is air and therefore $l - l'$ is e is nothing but s

so the shift s is equal to $t \cdot (1 - \frac{1}{n_1})$ is very interesting indeed see that it does not matter where you introduce the slab the slab could be introduced here the slab could be introduced here anywhere the slab could be introduced the shift simply depends on the thickness of the slab and the refractive index of the slab

so its just a variation of the other problem which i had discussed earlier there are several possibilities and i would suggest that you work out a number of problems to get a better feel of this subject a second part which i would also recommend you to take it as an exercise is to determine the lateral shift i had not worked it out here

so we have a ray of light incident like this it bends towards the normal if this refractive index n_1 is greater than the refractive index here and then it again bends away from the normal if the media are the same the angle that it subtends here θ_1 is the same as the angle it subtends here which means we had said there is no deviation t is the thickness of the glass slab and n_1 is the refractive index of the medium

so what we have got we have already discussed this that we have lateral shift

so what we get is a lateral shift here

so which i think i denoted as l

so find an expression for l find an expression for l for the lateral shift we can extend this problem by taking two slabs

so the like we had done for the apparent depth for the lateral shift also suppose you had two slabs one after another

so this is of thickness t_1 and this is of thickness t_2 and refractive index n_1 and n_2 and outside of course it is n_0 or air you can say this is one and this is one which means again as we have already discussed there is no deviation however the lateral shift

so let me show the lateral shift here

so this is bending towards the normal maybe it will bend a little bit away but finally it will come out in such a way that it will be parallel to this in other words there will be no deviation we have discussed this several times only there will be a lateral shift l now the lateral shift will depend on n_1 and t_1 and n_2 and t_2

so determine the lateral shift follow a procedure which is almost similar to what i have outlined here and determine an expression to obtain an expression for the lateral shift

so there can be several ah similar problems thank you you