

welcome to the lecture module on optics in the last lecture we discussed about refraction at a plane interface and we also saw the condition under which the total internal reflection takes place today we will discuss refraction at a spherical interface and then we will extend it to refraction by lenses because lenses are widely used for various applications

so we will first discuss refraction at a spherical interface followed by refraction by lenses

so refraction at a spherical interface spherical surface and by lenses

so first here i am showing refraction at a spherical surface

so let me show the diagram first this is the interface a spherical interface between two media of refractive index n_1 and n_2 this is medium one this side and on the other side medium 2 and in this case i have considered n_2 greater than n_1

so here o is a point object whose image is formed in medium 2 at a position i

so there is a straight ray which is incident normally on the spherical interface which passes undeviated and a ray which is coming at an arbitrary angle α a small angle α is refracted because the dotted line here shows the normal to the interface i is therefore the angle of incidence and because n_2 is greater than n_1 the ray bends towards the normal and therefore the ray bends towards the normal gear it intersects the straight ray at the point i and therefore i is the image point of this object now this is the angle of incidence and of course i have shown a ray here a small fraction of light is also reflected because reflection is always present however first this fraction is small typically four to five percent if it is an air and glass interface but this fraction can be minimized by coating this surface by what are called anti reflection coatings and therefore in the subsequent diagrams we simply neglect this reflection and we are focusing only on the refracted ray here

so α , β and γ are the angles here as shown in the figure α is the angle subtended with the axis and β is the angle subtended here by the normal with the axis and γ is this is γ and this is r refracted angle r at the point of incidence m this is the object distance from the surface here at the point p to the object position is object distance u we will look at the sign convention later but right now u refers to the object distance and v refers to the image distance and R capital R is the radius of curvature of this surface C is the centre of curvature and r is the radius of curvature of the spherical surface now we assume a small aperture the condition of small aperture this i have already discussed in one of our earlier classes

so what it means basically is

so let me show it here

so small aperture here refers to

so when we have an optical system it may have several components or several surfaces but if this is the spherical surface by small aperture what i mean is if we put a block here that is an opaque stop in front of this with a small opening a small aperture and the rays of light which are entering through this aperture only are undergoing reflection or refraction or whatever

so we are considering a small aperture which means the rays

so let me show you the different color

so rays which are making

so if i have let us say a point object here o or a point source p here then rays which travel along the straight line rays which make small angles here will only be able to pass through this aperture

so small aperture means we are restricting to rays which are passing close to the axis and rays which make small angles only and that is nothing but the

paraxial approximation

so small aperture satisfies the paraxial approximation this is what we had discussed

so paraxial approximation

so rays which are close to the axis approximation is valid that means that i keep back the slide here and that means that angle alpha the angle alpha here actually this m is very close to this but only for clarity i have shown is a little away

so that the angles are clearly visible but the angle alpha is very small because the point m is very close because we are assuming small aperture therefore paraxial approximation is valid which is which means point m is close to p that means the angle alpha beta and gamma all the angles i and r because if this point comes here then the normal will be like this and i will be very small and then we have the approximation $\tan \alpha$ nearly equal to $\sin \alpha$ nearly equal to α when α is very small of course α is in radians $\tan \beta$ is nearly equal to $\sin \beta$ is nearly equal to β etcetera

so these things are valid

so this is of course i mentioned that the reflection of light can be minimized by using what are called anti reflection coatings we will not discuss that here because to understand anti reflection coatings we need to know wave optics and therefore we will discuss this at a later stage now we come back to the problem and here it is

so refraction at a spherical surface

so let us first focus on the angles here

so what we see is the angle i if you look at this triangle o m and c o m c then $\alpha + \beta = i$

so i is equal to $\alpha + \beta$ similarly if we look at this angle m c i this angle m this triangle here triangle m c i then we can see that $r + \gamma = \beta$ that is this angle angle of refraction r here and some angle here gamma which i have denoted gamma

so please see the the difference between r and gamma r is written like this r whereas in gamma we write like this and straight this is gamma and this is r

so i could have used some other symbol but just i had used gamma alpha beta gamma come together

so i used alpha beta gamma

so the point is beta is equal to $r + \gamma$ and therefore we are interested in i and r because we want to apply snell's law and therefore we write $r = i - \alpha + \beta - \gamma$ then the second point because of the paraxial approximation which we just discussed $\alpha \approx \tan \alpha$ is equal to $\frac{m d}{o d}$ in this diagram if we see $\alpha \approx \tan \alpha = \frac{m d}{o d}$ but because the point m is close to the point p here that is it is close to the axis whether we write $o p$ is nearly equal to $o d$ because the point m is close to the axis and therefore this is nearly equal to $\frac{m d}{o p}$ that is we approximate $o d$ by $o p$ this is true for a paraxial approximation or when we consider small apertures

so exactly like that for this angle if you look at the triangle m d c $\tan \beta \approx \frac{m d}{c d}$ and as before we are approximating $c d$ by $c p$ because $c p$ is exactly the radius of curvature that is why we are a proc making this approximation and $\gamma \approx \tan \gamma$

so if you look at the triangle here m d i then $\gamma \approx \tan \gamma = \frac{m d}{i d}$ but i p is the image distance

so we are approximating it by $\frac{m d}{i p}$ and therefore the angle i is given by $i = \alpha + \beta$ which means $\alpha = \frac{m d}{o p}$ $\beta = \frac{m d}{c p}$

so i is equal to $m d$ by $o p$ plus $m d$ by $c p$ and angle r is equal to $\beta - \gamma$ β is here $m d$ by $c p$ minus $m d$ by $i p$ $m d$ by $c p$ minus $m d$ by $i p$

so i have denoted these as equation three and four now we apply snell's law because i we have r we have and therefore $\sin i$ by $\sin r$ is equal to n_2 by n_1 or $n_1 \sin i$ is equal to $n_2 \sin r$ but again we know that the angles i and r are very small and therefore for small i and r we can write $\sin i$ nearly equal to i $\sin r$ nearly equal to r which means $n_1 i$ is equal to $n_2 r$ this is nearly equal almost a very good approximation $n_1 i$ is equal to $n_2 r$ now i and r are given here therefore $n_1 i$ from equation three is equal to $n_2 r$ from equation four

so let me call this as equation number six now we continue further

so here and therefore if we

so let me keep this

so that we focus on this from the pre page we have $n_1 i$ is equal to $n_2 r$ into r and $m d$ is common throughout

so this $m d$ goes off and therefore we are left with n_1 by $o p$ plus n_2 by $i p$

so this part that is $m d$ has gone

so n_2 by $i p$ minus n_2 by $i p$ i am bringing to this side

so n_2 by $i p$ is equal to $c p$ was common and therefore we take this term to the other side to make it n_2 minus n_1 by $c p$ now here we have to look at the sign convention we are going to substitute for $o p$ $i p$ and $c p$ appropriately and what is the sign convention

so just let us very quickly recall the sign convention it is almost the same as what we have in the case of mirrors

so we have a refracting surface here and the point which is normal to the here to the axis here is the origin x equal to 0 x equal to y is equal to 0 and for light incidence from the left we are considering light incident from the left uniformly therefore x direction positive x direction is along this

so this is the positive direction which means from this point whatever distance is on the left is negative and whatever distance is on the right is positive and therefore the object distance is the same diagram object forming an image and for the objective distance $o p$ here will be equal to minus u because it is on the left of point p whereas $i p$ is the image distance which is positive $c p$ which is the radius of curvature which is positive in case we had a if we had a concave surface like this the object is here for this case by chance the image is also on the left hand side object is here

so a ray which is bending here the straight ray does not intersect with this but they appear to come from a point i here and therefore the image virtual image is formed at the point i in any case in this $p o i$ in this case we see that the object distance is also on the left of this point x equal to 0 y equal to 0 image distance is also on the left

so it is minus v and the radius of curvature is also on the left side because this is the concave surface its center of curvature c is on the left side and therefore all of them are negative whereas in this case we see that the object distance is negative but these are positive

so this needs to be taken care when we substitute in the expression because then only the result that we will get will remain valid whether we take a concave surface or a convex surface ok

so coming back therefore applying the sign convention now we have come back here applying this is the equation applying the sign convention $o p$ is equal to minus u $c p$ is equal to r and the object image distance is v positive

so we substitute here and 1 by minus u plus n_2 by v is equal to n_2 minus n_1 by r or we can put it in the form n_2 by v minus n_1 by u is equal to n_2 minus n_1

by r now this is a very important equation in the sense that it gives a relation between object distance and image distance in terms of the refractive index and radius of curvature of the spherical surface given a spherical surface it means the radius of curvature and refractive indices of the materials are given then for any position of the object it will tell us what is the position of the image

so if we take an example it will become more clear

so let me take an example here

so let us take an example a very quick example here

so here is a spherical surface and an object is at a distance of 100 centimeter the radius of curvature of the spherical surface is given 25 centimeter here the material is given as glass with refractive index 1.5 and outside it is air with 1.

0

0

so the question is determine the position of the image when the point object is at a distance of 100 centimeter 50 centimeter and 25 centimeter its a simple problem basically substituting in the formula because the single interface for which we have just now derived this formula

so let us quickly pick up that for 100 centimeter here u is 100 centimeter and r is 25 centimeter r is positive for this convex surface here and refractive indices are given

so if we substitute in the expression we get this as v is equal to 150 150 centimeters real image in the glass medium it is positive 150 centimeter which means if this were 100 centimeter here the image would have formed somewhere here at 150 centimeter from the point p here 150 centimeter

so that would be the position of the image

so its a quick illustration of the application of this formula if i pick up similarly you can do for 50 centimeter but let me take quickly the third one that is for u is equal to minus 25 centimeter substitute back in the formula straight forward substitution and you get v is equal to minus 75 centimeter minus 75 centimeter which means v is also on this side and that that is where we get a virtual image

so the situation is what i had briefly explained earlier

so we have a spherical surface like this and here is the axis and in this case the object point is relatively close closer to the surface the centre of curvature is somewhere here the center of curvature is on this side see here but because and therefore the line joining the center of curvature

so let me take a ray like this one ray going like this normally incident will pass through the secondary i am picking any arbitrary ray which is like this will if i draw the center of curvature here

so the normal to the surface

so let me use a different color

so this is the normal to the surface which is the line joining the point of incidence to the center of curvature then we see that this will of course bend

so the ray will refract

so the ray refracts bends towards the normal however in this case it is still diverging its not coming to this and intersecting with this ray which means this appears to come from a point which is just extending this back to a point somewhere here

so this was the object distance

so this was 0 and therefore it forms a virtual image at this is the image distance

so this is the sorry this is the image distance and this is what we got answer as minus 75 centimeter for the third case when this was 25 centimeter we got this was 25 centimeter which means u is minus 25 centimeter we got the image

position as minus 75 which is a virtual object virtual image which is formed on the same side that is why we have this situation because the object is closer if the object were a little bit further off it would have been incident and it would have refracted and intersected with the axis somewhere here

so you would have got a positive image distance in that case

so depending on the position of the object we will have the position of the image for the same spherical surface

so that is why i have picked up those two simple examples

so lets proceed further and we consider

so here

so this lets ah after a single interface now lets go to refraction by a lens

so here we are refraction by a lens now lets look at the refraction first and then come back to the lens

so here is the lens is a biconvex lens this is refracting surface one this is refracting surface two this is of refractive index n_2 the lens medium is of refractive index n_2 and in this case i have taken on both the sides n_1 it could be n_1 and n_3 on the other side also but in a simple case we have considered outside there is a particular medium and the lens is of a particular medium usually a glass lens and it has refracting surface one and two there is a object here

so rays are emanated from the object and therefore i have shown three rays one straight ray which passes through this along the axis now what is the axis we will see in a minute and then two other rays i have shown they are refracting first they undergo refraction here and then they undergo refraction at the second surface there are two refractions here

so first refraction and second refraction to form the image now the lens is shown we have to remember that the lens has two spherical surfaces we had discussed this in the first class that these two surfaces are part of spheres two spheres here of radius of curvature r_1 and r_2 first surface radius of curvature r_1 its center of curvature c_1 here and the second surface which is here is part of this sphere here of radius of curvature r_2 with the centre of curvature here the object point we have considered is here and the image point is here

so there is the image point the object point and in this course we particularly look at thin film thin lenses thin lenses thin lenses means the separation a b here a to b this separation if i call that as thickness t this thickness is very small it is a thin lens under this approximation the distances o a is approximated to o p provided this thickness is small o a is nearly equal to o p nearly equal to o b

so this is an approximation which is followed in the case of thin lenses that is why we are considering here thin lenses in this course the axis is the line passing through the center of curvatures the two centre of curvatures line joining the two center of curvature c_1 and c_2

so that is the axis

so this is the diagram which indicates the spherical surfaces surface one surface two and radius of curvature two and radius of curvature are one here

so let us go further and to determine the image to determine the image position of the image we treat the lens because it has two spherical surfaces we have seen refraction at a single spherical surface now we will treat individually each of the spherical surfaces and we will look at refraction by the lens as successive refractions by the two surfaces

so that is what i am going to show in the next slide here

so here i have drawn these diagrams in advance

so that they are relatively clear

so we can see object undergoing refraction here at the first surface and then undergoing a second refraction here at the second surface to form the image point here if this surface the second surface was not there the refracted ray would have traveled here somewhere here and this is medium one medium two and medium one i have considered this n_2 greater than n_1 and this is the object distance and this is the image distance with the appropriate sign conventions when we go for the derivations we will see that now as i mentioned we will treat this as you can see exactly this surface is shown here and this surface is shown here

so we treat the net refraction here as successive case of successive refractions at interfaces 1 and interface 2.

why do we do that because we have already seen refraction at a single interface the case of refraction at a single interface first medium of refractive index n_1 second medium of refractive index n_2 and radius of curvature r_1 here then we have this equation that refractive index of the second medium divided by the image distance here minus refractive index of the first medium by the object distance is equal to the refractive index difference divided by radius of curvature of the spherical surface now the second refraction is as if it has nothing to do with this because the ray has already refracted here

so the ray has refracted and it is proceeding when it encounters the second medium here and therefore we show this as it is as if the entire medium on the left is of n_2 and the medium on the right is of n_1 in other words now this is the first medium and this is the second medium and therefore we write the same equation for refraction at this interface as the if the second interface were not there the object would have formed a image here i_1 at this point but because of the second refraction at the second interface the actual image is formed here otherwise it would have formed at i_1 here it is in the same line as i_1 .

therefore as far as this interface is concerned the ray is coming from here there is no object there but it is this i_1 acts as an virtual object the image i_1 acts as a virtual object for the second interface and therefore the distance from here to i_1 is the object distance in this case and the distance to i is the image distance

so the object distance image distance and r_2 is the radius of curvature therefore the formula is refractive index of the second medium second medium is on this side

so this is now n_2 refractive index of the second medium divided by the image distance refractive index divided by the image distance image distance is this one which is v

so what is shown here v from the center to i this is v

so refractive index of the second medium always we on the left is the first medium right is the second medium

so refractive index of the second medium divided by the image distance minus refractive index of the first medium first medium is now this one which is n_2 of refractive in x n_2 divided by the object distance object distance is now v_1 here v_1 object distance is equal to refractive index of the second medium minus refractive index of the first medium divided by radius of curvature

so equation one and two this equation applicable to the first interface this equation applicable to the second interface and therefore if we now add 1 and 2 please see 1 and 2 if we add this term is common and therefore this is with the negative sign therefore this term cancels off and we will have n_1 by v plus n_1 by v minus n_1 by u is equal to

so this we can flip this we can make n_2 minus n_1 with the negative sign and that is what we get here

so let me show the in the next slide here therefore adding equations one and two

so let us focus here adding equations 1 and 2 we get $n_1 \left(\frac{1}{v} - \frac{1}{u} \right) = n_2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$ we can take the n_1 to the other side and we can write this as $\frac{1}{v} - \frac{1}{u} = \frac{n_2}{n_1} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$ note that what is there on the right hand side is a constant this is a constant for a given length a lens is given means the refractive the radius of curvatures are fixed and the refractive index of the lens medium is fixed and of course depending on where you place n_1 is also fixed and therefore this is a constant this is the image distance this is the object distance

so this also gives a relation between image distance and object distance in terms of the parameters of the lens now for large distances

so let us look at this for large distances $\frac{1}{u} \rightarrow 0$ when u large distances object distances when the object is at infinity let us say that $\frac{1}{u} \rightarrow 0$ this means that we have $\frac{1}{v}$ is equal to a constant what is there on the right hand side is constant it has nothing to do with the u whatever be the position of u it does not depend on the position of u therefore for large distances we have $\frac{1}{v}$ is equal to a constant which is independent of u that is when the object is at a large distance it means the rays from the object are almost parallel to the axis but they all concentrate or they all converge to a point at a distance v and that point is called the focus the principal focus will discuss this in more detail in the next slide

so the when the image point is fixed for large values of u $\frac{1}{v}$ is constant image point is fixed independent of u and this is called the principal focus f we will show this in a diagram the corresponding image distance is called the focal length and therefore $\frac{1}{v}$ is equal to $\frac{1}{f}$ that constant the constant on the right hand side is denoted by $\frac{1}{f}$ is equal to $\frac{n_2}{n_1} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$ into this from 4 and 5 we have $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ basically what we have said is this is a constant which is equal to $\frac{1}{f}$ now what is that f f is the focal length f is the focus where parallel rays from a distant object focus on to converge on to the point f

so this we i will illustrate

so this is the important formula which is called the lens formula lens formula relates the object distance to image distance for any given lens of focal length f which depends on the parameters of the length that is radius of curvature and the relative refractive index difference now let us discuss this focal length a little bit more here

so here i am

so we are we will discuss the focal length thats a very important property of a given lens

so focal length

so this is the lens formula with $\frac{1}{f}$ is equal to this is what we called as focal length and $\frac{n_2}{n_1} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$ this is a biconvex lens r_1 is greater than zero and r_2 is less than zero because r_2 is having a center of curvature on this side therefore r_2 is less than zero for u tending to infinity this what we discussed rays from the object are almost parallel to the axis and the object the image distance v is equal to f which is the focal length

so the rays the parallel rays which come all of them converge to a point f because they are independent of the distance u all of them have the same image distance which we call as the focal point they converge to a point f and the distance between the lens and the focus the principal focus is called the focal length now this is true for a given lens and if we can note that here we have taken n_2 for example as glass and air then we have a certain value for the

focal length but if we immerse the lens in a liquid just as a case when the lens is immersed in a liquid of refractive index n_1 then one by f_1 f_1 is the focal length in the liquid is n_2 by n_1 instead of n_1 i have used n_1 that is refractive index of the liquid minus 1 divided by this now note that n_1 is greater than air n_1 if it is air outside it is one but the liquid has a refractive index greater than air

so n_1 is greater than n_{air} therefore the focal length in liquid is greater than the focal length in air because n_1 is greater than one and therefore this difference is now smaller therefore this quantity is smaller compared to the case of air this is smaller means one by f_1 is smaller or f_1 that is the focal length in the liquid is greater than the focal length in air there are several applications where the lens is immersed in a liquid to have a different focal length or the effectively the focal length changes and we know that the focal length in the liquid is larger than the focal length in air all right

so let us take ah go further and see the lens makers formula i want to now discuss this is a familiar or more common formula because most of the common applications for common applications of a lens n_1 is equal to n_{air} is equal to one when we use a lens normally the outside medium is air except in special cases when we have a liquid on the outside

so it is air and therefore refractive index is one and the refractive index of the lens is denoted by n because there is only one other refractive index

so there is no point in writing n_1 and n_2

so we n is the refractive index of the medium of the lens material of the lens and n_2 is equal to n refractive index and then we have 1 over f is equal to n minus 1 into 1 by r_1 minus 1 by r_2 this is called lens makers formula because when one would make a lens for a particular application to obtain a required focal length f the lens maker can choose a material and required values of the radius of curvature r_1 and r_2 r_1 may be equal to r_2 or may not be equal to r_2 but he can choose the radius of curvature to achieve the required focal length for a particular application hence this formula is traditionally called as the lens makers formula although the general formula is what we had already seen that one by f

so this is the general formula this is valid in for all refractive indices but in the special case when n_1 is air then we use the lens makers formula which is the simpler where n is the refractive index of the medium the formula indicates to the choice of r_1 and r_2 to obtain a desired focal length now we proceed further for a symmetric biconvex lens symmetric means both the radius of curvatures are the same which is r_1 is equal to r_2 of course r_2 is with a negative sign and therefore r_1 is equal to minus r_2 equal to r

so it is a symmetric biconvex lens then we substitute here in the formula we have one over f is equal to n minus one into one by r minus minus r naught which is which is equal to two by r into n minus one

so note that n is the material of the lens which is greater than air n is greater than one therefore f the focal length is greater than zero which is positive

so that is called a converging lens a converging lens has a focal length which is positive

so we will see

so what about diverging lengths

so let us see converging and diverging lenses

so here it is converging and diverging lenses for a symmetric biconvex lens just now we showed that one by f is equal to two by r into n minus one or f is equal to r by two into one by n minus one

so here is the converging lens that's a biconvex lens a symmetric by convex lens

it need not be symmetric but the formula i have for a symmetric i have considered as a special case when r_1 is equal to r_2

so we have f positive for a symmetric biconcave lens that is is a biconcave here is a biconcave lens

so we can see that this is r_1 the first surface second surface is r_2 r_1 one has a radius of curvature on this side

so the center of curvature is here therefore radius of curvature is negative r_2 has a radius of curvature on the other side and therefore this has a positive radius of curvature

so r_1 is equal to minus r_2 both r_1 and r_2 magnitude is equal to r because it is a symmetric lens but r_1 is negative and r_2 is positive and therefore r_2 is equal to r gives f is equal to minus r by two

so this r is now a magnitude only

so this r because negative sign has been taken into account

so this is positive only f is equal to minus r by two into n minus one because n is greater than 1 f is less than 0 in other words the focal length is negative

so we can see here that if we have a by concave lens the focal length is on this side and therefore f is negative f is positive in the case of a bi-convex lens

so in this case the rays diverge away as if they are coming from a point f the principal focus is here on this side and therefore this is a diverging lens whereas this is a converging lens a biconvex lens is a converging lens whereas a biconcave lens is a diverging lens note something interesting that for n is equal to consider this consider the biconvex lens for n is equal to 1.

f is equal to r n is equal to one point five this is one point five minus one that is point five multiplied by two is one therefore f focal length is equal to radius of curvature whereas for n is equal to two focal length is equal to r by two if you put n is equal to two here then this this whole thing is one and therefore f is equal to r by two

so it clearly indicates that not only it depends on the radius of curvature the focal length but also it depends on the refractive index of the material

so in one case the focal length is r in the other case focal length is r by two this is like in the case of concave mirrors we have seen earlier in the case of mirror that the focal length is r by two but in the case of a lens focal length need not be r by two

so in any problems do not jump to the conclusion that okay focal length is r by two thats not correct for the case of a lens it depends on the refractive index of the medium and therefore you have to substitute in the formula one by f is equal to $\frac{n_2}{n_1} \frac{r_1 r_2}{r_2 - r_1}$ into one by r_1 minus one by r_2 and find out the focal length now several situations here various situations which one encounters that r_1 is greater than zero there is a by convex lens r_1 greater than zero r_2 less than zero the normal biconvex lens which i have been discussing there are lenses which are used for special purposes where both of them have a convex surface here

so r_1 is greater than zero the radius of curvature is on this side and r_2 also has a radius of curvature on this side r_1 may not be equal to r_2 but both of them are convex surfaces and therefore r_1 greater than zero r_2 greater than zero both of them could be concave surfaces in which case r_1 is less than zero and r_2 is also less than zero and we could also have plano convex lens or plano concave lens this is a plano convex lens therefore this is r_1 is greater than zero radius of curvature is here and this is a plane surface therefore the radius of curvature is infinity r_2 is infinity but r_1 is greater than zero now finally in this situation all the while we have been discussing the case when n_2 is greater than n_1 but what if n_1 is greater

than n_2 if n_1 is greater than n_2 that is if the outside medium as a refractive index greater than n_2 the situation will change a convex lens can become a diverging lens and a concave lens can become a converging lens earlier I had shown that in diverging and converging lenses a convex lens is a converging lens and a biconcave lens is a diverging lens converging and diverging lenses but all the while we had assumed that refractive index of the lens is greater than that of the surroundings but in the reverse case when the refractive index of the lens is smaller than that of the surroundings it is possible that if this is immersed in a liquid of refractive index greater than that of glass then it is possible to have this situation and in this case the convex lens can become a diverging lens and a concave lens can become a converging lens right what if the next question what if I have been considering light incident from the left side of the lens with the radius of curvature r_1 and r_2 what if light incident from the right side

so will it have the same focal length

so let's see

so here what if light is incident from right side

so now r_1 and r_2 this is the lens and we have

so let me block this for a minute

so the case is here light incident from here and getting focused at a point here

so this is f_1 ah I had initially written f_2 that is why it but it is f_1 and f_1

so the question is is this distance f_1 is the same as f_2 we had considered when light parallel light was incident from on this side and focus to a point to the principal focus here and we call this focal length as f now if light were to be incident from here whether it will focus to a point here and whether the focal length on this side is the same as the focal length on that side now light is incident on the surface with the radius of curvature r_2 therefore I can equivalently rotate this and put the lens like this

so that light is still incident on the left side but now it is encountering r_2 first r_2 here the light which was incident is encountering the surface here with radius of curvature r_2

so the same situation

so I have just flipped it and put it r_2 first and r_1 here and therefore f_1 over f_2 now f_1 over f_2

so this is not f_2 it is f_1

so f_1 over f_2 is equal to n_2 minus one by n_1 minus one divided by one by r_2 minus one by r_1 earlier we had the formula one by r_1 minus one by r_2 but now because r_2 has become r_1 and r_1 has become r_2 in this case because we have flipped the lens

so it is 1 by r_2 minus 1 by r_1

so what is this this is nothing but f_1 and therefore f_1 whether in this case or in this case the distance is the same the focal length is the same whether light is incident from this side or it is incident from this side although r_1 and r_2 are different only therefore

so long as n_1 is the same n_1 on this side and n_1 on this side is the same it is worthwhile to check what happens if it is n_1 n_2 and n_3 n_1 n_2 and n_3 but right now I am considering the case where on both the sides we have n_1 and this is n_2 and

so long as n_1 is the same on both sides of the lens even though r_1 is not equal to r_2 f_1 is equal to f_2 f_1 I have used because the focal length on this side is negative and focal length on this side is positive if we look at it but of course when the light is coming from this side this direction is

positive therefore the focal length f_1 continues to be positive this is not negative but anyhow for this case we have shown because we are all the while we will consider light incident from here and therefore it will have a focal length f_1 to this side and a focal length f_2 to that side and f_2 is positive and f_1 is negative thus a lens has two principle foci

so let me discuss this a little bit more

so here the principal foci and focal length of a lens

so here is the lens light incident from here from the left side all all light rays are incident from the left side here and it is focused to a point f_2 here with a focal length f_2 whereas f_1 is the first principal focus rays which come from the first principle focus here f_1 will be rendered parallel because if these if light were to travel from here that is from right to left then it would have focused to this principle focus point f_1 and this is what we saw in the earlier slide the focal length is called f_1 now in this case light is travelling from here but the parallel light is focused at the principal focus f_2 and the focal length is f_2 whereas light rays emanating from the first principle focus f_1 will be rendered parallel

so f_1 magnitude is equal to f_2

so f_1 is the first principal focus because when we go from here we encounter the first principle focus first surface first principle focus first focal length when we go further we encounter the second surface second refracting surface second principle focus and second focal length

so f_1 is the first principal focus f_1 is the first focal length f_2 is the second principal focus and f_2 is the second focal length this one and f_1 and f_2 are equidistant from the lens because we have just now showed that f_1 is equal to f_2 in magnitude therefore the principle foci f_1 and f_2 are the principal foci which are equidistant from the lens normally when we refer to the focus of a lens when we normally talk about a lens of focal length f we are referring to the second focal length f_2 because that is the one which we encounter later it is beyond the lens and it is the second focal length that we are referring to and the focus of the lens also we are referring to capital f_2 that is the second principle focus

so here it is f_2 and the focal length f_2 then what is the importance of f_1 because light is incident from here

so what is the importance of f_1 as we can see the importance is illustrated right here any ray which is coming from f_1 will be rendered parallel

so where do we need this we need this in determining the images formed by lenses

so that will be the next topic that is imaging formation of images by a lens

so formation of images by lx

so let me briefly discuss formation of images we have discussed formation of images in detail in the case of mirrors

so now we are discussing formation of images by a lens laterally extended i have already discussed formation of image of a point object but now we are considering laterally extended object that is a line object here of dimension a b a b is the object f_1 is the first principal focus f_2 is the second principal focus

so let us focus on the diagram here

so a parallel ray which is coming from the object passes through the second principle focus a ray which passes through the center of the lens here will pass undeviated and it will intersect the ray which is coming from the focus and that will be the image point of a

so image point of a is marked as a dash or the image of the extended object a b is a dash b dash here now a third ray which is passing through the first

principal focus will be rendered parallel there are situations many cases we are not able to get two

so these two rays sometimes we are not able to draw particularly in the case of ah by concave lenses and then we have to use this fact that a ray which comes from the principal focus will be rendered parallel a parallel ray ray parallel to the axis will pass through the principal focus but a ray which is passing or comes from the principal focus will be rendered parallel the intersection gives us the position of the object now let us quickly see this because we are familiar with the formation of images

so look at the triangle abp and triangle $a'b'p'$

so abp here and $a'b'p'$

so this triangle and this triangle these are equivalent triangles because this opposite angles are equal 90° degree

so all three angles are equal and therefore we have ab by bp ab by bp which is actually $\tan \theta$ ab by bp is equal to $a'b'$ by $b'p'$ $a'b'$ by $b'p'$ $a'b'$ by $p'b'$ $a'b'$ by $p'b'$

so this is $\tan \theta$ actually or $a'b'$ by $b'p'$

so i am shifting this ah here

so $a'b'$ by $a'b$ is equal to $b'p'$ by bp now applying the sign convention we know what it is

so we are interested in finding $a'b'$ by ab because we are interested in the lateral magnification just as in the case of a mirror we are interested in the lateral magnification m is equal to size of the image by size of the object size of the image by size of the object that is we are interested in $a'b'$ by ab $a'b'$ by ab

so $a'b'$ by ab is equal to $b'p'$ by bp

so substituting this is h h' as per sign convention this is negative and this is positive any distance above this above the axis is positive and therefore we substitute for $a'b'$ $-h'$ and ab h is equal to v object distance which is positive and image distance which is positive and object distance that is bp which is the object distance is negative minus u

so that is what we have substituted here or m is equal to h' by h is equal to v by u now very quickly if we see the formation of image for the case of biconcave lens

so i am do not have to discuss but you can see this very clearly that here is the object ab now a parallel ray here incident will be diverging it is a diverging lens appear to come from the first principal focus here here f_2 is coming from here a ray which is which would have gone which is which would have gone to this principle focus here would be rendered parallel because if a ray were to start from here then it would have been rendered parallel and that is why this ray would be rendered parallel and array which is passing through the center of the lens will go undeviated all the three rays 1 2 3 do not intersect anywhere here rays coming from a do not intersect anywhere on the other side of the lens but note that they appear to come from a point a' here where they intersect they appear to come if we extend these backward then they appear to come from a point a' and therefore $a'b'$ is the image of ab due to a biconcave lens very quickly if you look at the triangles abp and $a'b'p'$ they are equivalent triangles and therefore $a'b'$ by ab is equal to $b'p'$ by bp that is h' by h $a'b'$ by h h' here which is positive its above the axis by h this h' is equal to minus v the image distance and minus u

so minus v by minus u which is equal to v by u or the lateral magnification m is equal to v by u as before as before means the same formula that we got in the case of a convex lens because we have followed the sign convention in the next

class we will take up some examples and proceed on to the topic of power of a lens when a lens is converging or diverging there is a power associated with what is the converging power and what is the diverging power this we will take up in the next lecture you

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