

a very good morning to all of you in the last class we discussed some problems in electrostatics we looked at forces on charges etcetera ah today what i am going to do is to discuss some problems in the field of magnetic using magnetic fields

so we will continue with our discussion of problems

so i will discuss today problems in magneto magnetic fields

so let us look at the first question consider a vector field given by  $f$  is equal to  $k x \hat{i}$  plus  $k y \hat{j}$  where  $k$  is a constant can this vector field represent a magnetic field

so we have a vector field given by  $f$  is equal to  $k x \hat{i}$  plus  $k y \hat{j}$  and the problem is to find out whether such a field can represent the magnetic field now what do we know about properties of magnetic fields we know that magnetic fields must satisfy an equation which is essentially says that  $\int \mathbf{b} \cdot d\mathbf{l}$  must be zero that is the flux of magnetic field from any closed surface must be zero

so we need to check whether this particular field satisfies this condition now as before i can take any arbitrary surface any arbitrary close surface and calculate the value of  $\int \mathbf{f} \cdot d\mathbf{a}$  and check whether it happens to be zero now for simplicity i would like to take a surface which will help me to evaluate this integrals easily

so here is the surface which i will take here are my coordinates this is  $x$ ,  $y$  and  $z$

so i let me take a cube here

so i have

so cube of side  $a$  and this is origin here

so this is these are now there are ah six surfaces here this is one surface here

so let me call this surfaces by names

so this one i call  $s_1$

so this is one this is two this is three and this is four at the bottom and then five is this one the the front surface this is five and the back surface is six six surfaces

so what i need to do is to calculate the value of  $\int \mathbf{b} \cdot d\mathbf{a}$  over this close surface

so i will have to integrate this over all these surfaces to check whether i satisfy this equation

so let us start with the first surface

so total  $\int \mathbf{b} \cdot d\mathbf{a}$  is equal to  $\int \mathbf{b} \cdot d\mathbf{a}$  over  $s_1$  plus  $\int \mathbf{b} \cdot d\mathbf{a}$  over  $s_2$  plus  $\int \mathbf{b} \cdot d\mathbf{a}$  over  $s_3$  plus  $\int \mathbf{b} \cdot d\mathbf{a}$  over  $s_4$  and similarly  $\int \mathbf{b} \cdot d\mathbf{a}$  over  $s_5$  and  $s_6$  ok

so i need to calculate each of these integrals and check whether the sum is equal to zero

so lets start with  $\int \mathbf{v} \cdot d\mathbf{a}$  over  $s_1$  integral sorry  $\int \mathbf{b} \cdot d\mathbf{a}$  over  $s_1$

so let me redraw the figure here

so remember these are the these are the this is the cube in which i am integrating

so this is  $s_1$  this is  $x$ ,  $y$  and  $z$

so what is  $d\mathbf{a}$  for this  $d\mathbf{a}$  will be equal to its an it is an area which is the normal to this area is pointed along the  $x$  direction this is the normal

so the area vector will be and this surface is parallel to the  $y$ ,  $z$  axis

so  $d\mathbf{a}$  will be  $dy dz \hat{i}$  and  $\mathbf{b}$  is given by  $k x \hat{i}$  plus  $k y \hat{j}$

so  $\mathbf{b} \cdot d\mathbf{a}$  on the surface

so this distance being  $a$  first of all this is equal to this is  $k x \hat{i}$  plus  $k y \hat{j}$

$\mathbf{j}$  at  $x$  is equal to  $a \hat{i}$  into  $d x d y d z$   
 so this is nothing but  $k a$  i have substitute  $x$  is equal to  $a$  and  $\mathbf{j} \cdot \hat{i} = 0$   
 so  $k a d y d z$   
 so  $\int_{S_1} \mathbf{b} \cdot d\mathbf{a}$  over  $S_1$  which is equal to  $\int k a d y d z$  over  $S_1$   
 one which is equal to  $k a$  times  $\int d y d z$  over  $S_1$  and  $\int d y d z$  over  $S_1$   
 one is nothing but the area of this flat surface which is nothing but a square  
 so i get  $k$  times a cube  
 so that's the flux of magnetic field crossing the surface  $S_1$  is  $k a$  cube for  
 this given magnetic field similarly i can calculate the flux through all other  
 surfaces  
 so for example let me just calculate one more surface which is  $S_2$   
 so let me calculate from  $S_2$   
 so  $\int_{S_2} \mathbf{b} \cdot d\mathbf{a}$  over  $S_2$   
 so again i draw the figure here  
 so  $S_2$  is the top surface  
 so this is the normal  $\hat{x} \hat{y} \hat{z}$   
 so what is  $\mathbf{a} \cdot d\mathbf{a}$  here  $d\mathbf{a}$  will be equal to now this surface is oriented the  
 area is along  $\hat{y}$  cap  
 so this is  $\hat{y}$  cap and it will be  $d x d z$   $d x d z$  is at  $\hat{y}$  cap and  $\mathbf{b}$  is equal to  $k x$   
 $\hat{i}$  cap plus  $k y$  cap  
 so  $\mathbf{v} \cdot d\mathbf{a}$  on surface  $S_2$  will be equal to on the surface  $S_2$   $y$  is equal  
 to  $a$   
 so and  $\hat{i} \cdot \hat{j} = 0$   $\hat{j} \cdot \hat{j} = 1$  and  $y$  is happens to be  $a$  because this distance  
 is  $a$   
 so this is nothing but  $k a d x d z$   
 so  $\int_{S_2} \mathbf{b} \cdot d\mathbf{a}$  is equal to  $\int k a d x d z$  over  $S_2$   
 which is equal to  $a$  times  $\int v x d z$  over  $S_2$  what are  $S_2$  this is  
 the area and  $d y d x d z$  set is nothing but area which happens to be  $a$  a square  
 so this is  $k a$  cube okay  
 so i have evaluated over surface  $S_1$  and  $S_2$   
 so i would like you to continue the discussion and evaluate all other integrals  
 so let me just give you the values here over  $S_3$   $\mathbf{v} \cdot d\mathbf{a}$  will be equal to  
 zero over  $S_4$   $\mathbf{b} \cdot d\mathbf{a}$  will be zero over  $S_5$   $\mathbf{b} \cdot d\mathbf{a}$  will be zero and  
 over  $S_x$   $\mathbf{v} \cdot d\mathbf{a}$  will be zero  
 so  $\int \mathbf{b} \cdot d\mathbf{a}$  happens to be equal to it has only contributions from two  
 surfaces  $S_1$  and  $S_2$  and that's why i get two  $k$  times a cube and this is not  
 equal to zero  
 so hence this hence  $\mathbf{f}$  cannot represent a magnetic field  
 so this particular vector field which i wrote in the question this particular  
 vector field cannot represent a magnetic field because  $\int \mathbf{v} \cdot d\mathbf{a}$  of  
 this vector field is not zero over a closed surface  
 so please remember just like not all vector fields can represent electric  
 fields not all vector fields will represent magnetic fields  
 so they have to satisfy certain properties to be able to represent a an  
 electric field or a magnetic field now let me look at another question here  
 so calculate the magnetic field produced by a finite current element as shown  
 so i have an element of current  
 so this current is  $i$  here and i want to calculate the magnetic field at some  
 point  $p$  here due to this current element  
 so the problem is i have a now a certain current element and i want to  
 calculate what is the magnetic field produced by this current element at this  
 point now obviously this current element cannot exist independently but many in  
 many circuits you will have many straight sections and for each section i can  
 actually individually calculate the magnetic field and

so from there evaluate the total magnetic field from there  
 so i would like to evaluate this  
 so for this what i do is the following i take a small element of length here  
 so let me call this direction this as z axis and let me call this distance as r  
 from here  
 so i take a small element and join these two points  
 so this is the small current element which i will call  $idl$  and this distance  
 this vector i call  $s$  vector and let me call this angle  $\theta$   
 so i calculate what is the magnetic field produced by the small current element  
 at this point and integrate over all elements from this point to this point to  
 get the total magnetic field and magnetic field produced by the small current  
 element  $db$  will be equal to  $\mu_0$  by four pi  $idl$  cross  $s$  by  $s^3$  please  
 remember this we had ah discussed earlier in bios about law the magnetic field  
 produced by the small current element  $dl$  vector at this point will be  $idl$   
 cross  $s$  by  $s^3$  into  $\mu_0$  by four pi  
 so and the total magnetic field will be obtained by integrating now first thing  
 that we must notice is that  $dl$  cross  $s$   
 so  $dl$  vector is pointing upward  $s$  vector is pointing here  
 so  $dl$  cross  $s$  is pointing inwards  
 so the magnetic field produced by this is pointing into the paper and all  
 current elements from here to here all these current elements have a magnetic  
 field which are pointing into the paper here  
 so each current element in this length will produce a magnetic field which are  
 all pointing inwards and that helps me to sum up all the magnetic field  
 components  
 so the total magnetic field i will calculate by integrating from this point  
 so let me call this z one and coordinate of this is z two  
 so  $z_1$  is this distance and  $z_2$  is the distance of this point from this point  
 so this is actually a normal which i have drawn from here  
 so this is my current element i need to calculate the magnetic field here i  
 drop a perpendicular on this line that distance is small  $r$  and  $z_1$  is the  
 coordinate of the bottom end of the element  $z_2$  is the coordinate of the top  
 of the element and i have defined the angle  $\theta$   
 so now what is  $dl$  cross  $r$   $dl$  cross  $s$   
 so  $dl$  cross  $s$   $dl$  cross  $s$  which is equal to  
 so  $dl$  crosses if  $\theta$  is this angle i will have  $dl s \sin \theta$  now what is  
 $\theta$   $\theta$  is here if i call this  $\alpha$  then  $\sin \theta$  is nothing but  $\cos$   
 $\alpha$   
 so this triangle you can actually find out that  $\sin \theta$  is nothing but  $\cos$   
 $\alpha$  and  
 so i get  $dl$  cross  $s$  is equal to  $dl s \cos \alpha$  and  $s^2$  is equal to now  
 what is  $s$  the length of the this length from here to here  $s^2$  is  $r^2$   
 plus  $z^2$   
 so this has a coordinate i am assuming ah recorded  $z$   
 so  $s^2$  is equal to  $r^2$  plus  $z^2$   
 so  $db$  magnitude is nothing but  $\mu_0$  by four pi  
 so  $dl$  cross  $s$  which is  $dl s \cos \alpha$  divided by  $s^3$   
 so i have ah i re leave one  $s$  and the remaining i write as  $r^2$  plus  $z^2$   
 square  
 so this  $s$  cancels off and i have essentially  $\mu_0$  by four pi  $dl \cos$   
 $\alpha$  by  $r^2$  plus  $z^2$  and  $dl$  is nothing but let me write it as  $dz$   
 because  $dl$  is nothing but a small current element along the  $z$  direction  
 so i replace  $dl$  by  $dz$   
 so the total magnetic field will be obtained by integrating this this vector

so b magnitude total will be equal to  $\mu_0 i$  by  $4\pi$  integral  $dz \cos \alpha$  divided by  $r^2 + z^2$  from  $z_1$  to  $z_2$   $z_1$  is the coordinate of the bottom point of this element  $z_2$  is the top coordinate so from  $z_1$  to  $z_2$  we find integrate  $i$  will get this now  $i$  can use a small change of variables

so  $z$  is equal to  $r \tan \alpha$  if  $i$  write  $z$  is equal to  $r \tan \alpha$  you can see here  $z$  is this distance  $r$  is this distance  $\alpha$  is this angle

so  $z$  by  $r$  is  $\tan \alpha$

so  $z$  is equal to  $r \tan \alpha$

so and  $dz$  will be equal to  $r \sec^2 \alpha d\alpha$  and  $r^2 + z^2$  is nothing but  $r^2 (1 + \tan^2 \alpha)$  which is equal to  $r^2 \sec^2 \alpha$

so magnetic field vector magnitude is  $\mu_0 i$  by  $4\pi$  integral

so  $dz$  is  $r \sec^2 \alpha d\alpha$  by  $r^2 \sec^2 \alpha$

so  $\sec^2 \alpha$  cancels off and  $i$  get  $\mu_0 i$  by  $4\pi r$  integral  $\cos \alpha d\alpha$  which is equal to  $\mu_0 i$  by  $4\pi r$   $\sin \alpha$  two minus  $\sin \alpha$  one where  $\alpha_1$  and  $\alpha_2$  are the limits and

so let me write here

so  $\alpha_1$  is this angle and  $\alpha_2$  is this angle

so  $\alpha_2$  is this angle where the top portion of the element subtends at this point and  $\alpha_1$  is the angle subtended by the bottom portion of the current element with the horizontal line at this point  $p$  and  $i$  can ah from here  $i$  can immediately write the values of  $\sin \alpha_1$  and  $\sin \alpha_2$  in terms of  $z_1$  and  $z_2$

so you can see here ah  $\sin \alpha_1$  is this  $z_1$  divided by this distance and  $\sin \alpha_2$  is  $z_2$  divided by this distance

so  $i$  can also write this equation in a slightly different form

so magnitude would be vector which is equal to  $\mu_0 i$  by  $4\pi r$  into  $z_2$  by square root of  $z_2^2 + r^2$  minus  $z_1$  by square root of  $z_1^2 + r^2$

so  $i$  have just replaced  $\sin \alpha_2$  by  $z_2$  by square root of  $z_2^2 + r^2$  and  $\sin \alpha_1$  by  $z_1$  by square root of  $z_1^2 + r^2$

so that is the magnetic field produced at this point

so the field at the point where  $i$  am interested depends on the coordinates  $z_1$  and  $z_2$  and essentially these angles and that is a nice expression which is useful whenever  $i$  need to have a calculated magnetic field by straight segments of current elements and for each strike stage segment if  $i$  know the coordinates of the two ends  $i$  can use that to calculate the magnetic field now  $i$  want to use this expression to solve another problem which is the following a length  $l$  of wire carrying a current  $i$  is to be bent into a circle or a square each of montan in which case will the magnetic field at the center be greater that means from a given length  $i$  make a square and for the same length  $i$  make a circle and at the center  $i$  want to calculate the magnetic field the length of this square is the same as the length of the circle and if  $i$  flow a current  $i$  need to calculate the magnetic field at the center here and center here and compare the two magnetic fields

so  $i$  want to solve this problem now please note in the square case if  $i$  draw the square here  $i$  have to calculate the magnetic field here

so if  $i$  need to calculate the magnetic field here suppose  $i$  want to  $i$  draw the lines here this is the current flowing

so it this is a problem which is very similar to what we had before

so the total magnetic field at this point consists of magnetic field produced

by this current element plus the magnetic field produced by this current element plus the magnetic field produced by this current element plus the magnetic field produced by this current element we have already seen the magnetic field produced by one current element and also remember that all current elements produce magnetic fields in the same direction

so they are all all the magnetic fields are coming up from here

so you see the current is flowing like this

so the magnetic field is coming up here this current also produces magnetic field coming up

so all the magnetic fields produced by all the four current elements are in the same direction

so i can calculate the total magnetic field by simply adding the magnitudes of the magnetic field produced by each of them now if you look one of the current elements if i look at this current element because the total length is  $l$  of the wire each side has a length  $l$  by four

so this is an  $l$  by four length and if i draw the perpendicular

so this is this distance is  $l$  by eight and this being the center this is also  $l$  by eight

so if you recall we had just obtained this formula  $\mu_0 i$  by four  $\pi r$  into  $z$  two  $\pi$  square root of  $z$  two square plus  $r$  square minus  $z$  one by square root of  $z$  one square plus  $r$  square

so for this current element  $z$   $z$  one is equal to minus  $l$  by eight look at this is the perpendicular drawn here this this this is minus  $z$  by a minus  $l$  by eight and  $z$  two is equal to plus  $l$  by eight this is the coordinate of this point and  $r$  is equal to  $l$  by eight that is this distance

so this distance is  $l$  by eight this is  $l$  by eight this is  $l$  by eight

so if i want to calculate now  $z$  two by square root of  $z$  two square plus  $r$  square which is equal to  $l$  by eight by  $l$  by eight into square root two which is equal to one by square root of two  $z$  one by square root of  $z$  one square plus  $r$  square is equal to minus  $l$  by eight by  $l$  by eight times square root two which is equal to minus one by square root of two

so the total magnetic field because of one current element magnetic field due to one side of the square will be equal to  $\mu_0 i$  by four  $\pi$  into  $l$  by eight remember  $r$  is here that is  $l$  by eight and then i have one by root two plus one by root two which is equal to two  $\mu_0 i$  by  $\pi l$  into root two

so this is two by root two which is root two and i have two  $\mu_0 i$  by  $\pi l$  into root two

so the total magnetic field is four times this four into two  $\mu_0 i$  by  $\pi l$  root two which is equal to eight square root of two  $\mu_0 i$  by  $\pi l$

so let me call this  $v$  square

so that is the magnetic field produced by the square ah  $l$  is the total length of the wire which is producing this magnetic field

so the square current carrying square produces the magnetic field at the center given by this and which i have calculated by summing up the magnetic field produced by each of the current elements now i want to find out what is the magnetic field produced by circle

so thats my wire put in the circular form and if the radius is  $r$  the total length is  $l$  is equal to two  $\pi r$

so the radius of the circle will be  $l$  by two  $\pi$  now to calculate the magnetic field here i use ah again i can very simply do this bio sabot law

so here it is

so i take a a current element

so this is  $d\phi$  and this angle is  $\phi$  this  $r d\phi$  and the current is flowing like this

so  $\Delta l$  can write  $d\mathbf{b}$

so the current is currently flowing like this this is the  $\Delta l$  this is the  $\mathbf{s}$  vector here

so you can see here this line  $d\mathbf{l}$  is always perpendicular to this  $\mathbf{s}$  vector here and

so I will get  $d\mathbf{b}$  is equal to  $\mu_0 i$  by  $4\pi$  into  $i$  into the length  $d\mathbf{l}$  cross  $\mathbf{r}$

so  $d\mathbf{l}$  is  $r d\phi$  by  $r^2$

so  $d\mathbf{l}$  cross  $\mathbf{r}$  by  $r^3$

so one of the  $r$  cancels off

so I will essentially get  $\mu_0 i d\phi$  by  $4\pi r$

so the total magnetic field will be equal to  $\mu_0 i$  by  $4\pi r$  integral  $d\phi$  from zero to  $2\pi$  which will give me  $\mu_0 i$  by  $4\pi r$  and I can replace  $r$  in terms of  $l$  by  $2\pi$

so I get  $\mu_0 i$  by  $2r$  is nothing but  $l$  by  $\pi$

so  $\mu_0 i \pi$  by  $l$

so if I call this circle

so  $b_{\text{circle}}$  is equal to  $\mu_0 i \pi$  by  $l$

so I have taken the same length of wire one put in the form of a square one wire put in the form of a square of length  $l$  and the same wire I have been to into a circle and I find these two magnetic fields

so let me write these two fields here

so so for the square  $b_{\text{square}}$  was equal to  $2\sqrt{2} \mu_0 i$  by  $\pi l$  and  $b_{\text{circle}}$  for the circle of the same length is  $\mu_0 i \pi$  by  $l$

so the ratio  $b_{\text{square}}^2$  by  $b_{\text{circle}}^2$  will be equal to  $8\sqrt{2} \mu_0 i \pi$  by  $l$  divided by  $\mu_0 i \pi$  by  $l$  and that is equal to

so I cancels off  $\mu_0$  cancels off here and  $l$  cancels off

so I get  $8\sqrt{2}$  by  $\pi^2$  which is approximately one point one five

so the if you take a certain length of wire and put it in the form of a square or in the form of a circle the magnetic field at the center of the square will be larger by a factor of one point one five compared to the magnetic field produced at the same point if the wire was bent into the form of a circle

so you have seen here how to use the Biot-Savart law to calculate the current element magnetic field produced by current element and use that to calculate the magnetic field produced by  $\Delta l$  by wire structures consisting of multiple current elements if you are given for example an  $n$  sided polygon with straight current elements you can actually calculate the magnetic field produced by each of those elements add them up but please remember magnetic fields are vector quantities and you have to ensure that all the fields you had are done in vector addition  
me go to another problem another interesting problem here if a finite wire is used to wind six turns around an insulating sphere of radius  $a$  such that each turn makes an angle of  $30^\circ$  with the adjacent turn and that all the turns intersect at diametrically opposite points on the surface of the sphere I will draw a figure to explain this little more carefully if a current  $i$  is passed through these turns find the magnitude of the magnetic field at the center of the sphere

so let me draw a figure to explain this problem a finite wire is used to wind six turns around an insulating sphere

so I take an insulating sphere of radius  $a$  and I wind turns like this

so I have one winding which goes like this the next winding comes here

so this angle is  $30^\circ$  another winding comes here at  $60^\circ$  another winding comes here at  $90^\circ$  then another winding comes here at  $120^\circ$  and now the winding comes here at  $150^\circ$

so there are six windings  
 so each of them is carrying current  
 so here there are current flowing like this  
 so six windings each of them by rotated by 30 degrees  
 so this is horizontal next 30 degrees 60 degrees 90 degrees 120 degrees 150 degrees and then you have the the original one which is at 180 degrees  
 so the problem is to calculate the magnitude of the magnetic field at the center now what is actually happening is for example the horizontal winding will produce a magnetic field like this at the center the next winding which is at 30 degrees will produce a magnetic field at the same point slightly inclined because the two pointings have the same radius they will produce the same magnetic field but inclined now by 30 degrees the next one which is at 60 degrees will produce one inclined at 60 degrees the fourth one will be at 90 degrees it produces like this then the one which is at 120 degrees produces like this then the one which is at 150 degrees will produce like this  
 so you have 6 turns each of them producing a magnetic field which is slightly oriented with respect to these the earlier magnetic fields  
 so what you need to do is to calculate the total magnetic field the sum of the magnetic fields of all this and we have to remember as i mentioned that magnetic field is a vector quantity and i must use vectorially  
 so let me draw the vectors  
 so you will have one magnetic field like this next one 30 degrees next one is 60 degrees next one is 90 degrees then 120 150  
 so these are the six magnetic fields produced  
 so i can actually calculate the magnetic field along the vertical directions and the magnetic field along the horizontal direction and from those two i i can add up the total magnetic i can calculate  
 so now let me first talk about component of magnetic field along the vertical direction now for this of course i know i need to know what is the magnetic field produced by a current loop of radius a at the center and that magnetic field happens to be  $B$  is equal to  $\mu_0 n i$  by two a which has now calculated in the earlier problem the magnetic field produced by a current element like current a circular loop of current will be like this and it is pointing perpendicular to the the plane of this the circular loop and at the magnitude  $\mu_0 n i$  by two at the center  
 so each of these each of these loops produces a magnetic field of magnitude  $\mu_0 n i$  by two a at different orientations  
 so i need to calculate  
 so this this is thirty degrees this is another thirty degrees this is thirty degrees this is 30 degrees and this is 30 degrees  
 so what is the material component b vertical  
 so the first one is  $\mu_0 n i$  by two a plus the second one the magnitude of this vector is  $\mu_0 n i$  by two a its vertical component is  $\cos$  thirty the next one the magnitude is  $\mu_0 n i$  by two a the component is  $\cos$  sixty degrees that the fourth one  $\mu_0 n i$  by two a  $\cos$  ninety degrees plus  $\mu_0 n i$  by two a  $\cos$  one twenty degrees plus  $\mu_0 n i$  by two a  $\cos$  one fifty degree one two three four five six six coils produce different magnetic fields this is the vertical component of each of the magnetic field which i can add up and this happens to be  $\mu_0 n i$  by two a into this is one plus  $\cos$  thirty is  $\sqrt{3}$  by two plus  $\cos$  sixty is half  $\cos$  ninety is zero  $\cos$  one twenty is minus half and  $\cos$  one fifty is minus  $\sqrt{3}$  by two and this happens to be equal to  $\mu_0 n i$  by two a  
 so what is actually happening is the vertical component of this field is cancelling the vertical component of this field the vertical component of this

field is cancelling vertical component of this field there is no vertical component of this

so the total vertical component is nothing but the the magnetic field produced by the horizontal coil which is essentially a vertical and it is  $\mu_0 n i$  by  $2a$  i can similarly calculate the horizontal component component of magnetic field along the horizontal direction

so b h now instead of cosine i will have sine functions horizontal coordinates are all sign

so i will have  $\mu_0 n i$  by  $2a$

so  $a h \sin 0 + \mu_0 n i$  by  $2a \sin 30$  degrees plus  $\mu_0 n i$  by  $2a \sin 60$  plus  $\mu_0 n i$  by  $2a \sin 90$  plus  $\mu_0 n i$  by  $2a \sin 120$  degrees plus  $\mu_0 n i$  by  $2a \sin 150$  and you can simplify this this is  $\mu_0 n i$  by  $2a$  into  $0 + \frac{1}{2} + \frac{\sqrt{3}}{2} + \frac{1}{2} + 0 + \frac{\sqrt{3}}{2} + \frac{1}{2}$

so sine zero is zero this is half this is  $\frac{\sqrt{3}}{2}$  by  $2$  this is one this is  $\frac{\sqrt{3}}{2}$  by  $2$  and that is half which is equal to  $\mu_0 n i$  by  $2a$  into  $2 + \sqrt{3}$

so horizontal component is this one we have already calculated the vertical component as  $\mu_0 n i$  by  $2a$

so we can calculate the total magnetic field magnitude

so b magnitude is equal to  $b^2$  vertical square plus  $b^2$  horizontal square square root which is equal to  $\mu_0 n i$  by  $2a$  into  $1 + 2 + \sqrt{3}$  whole square whole this per half and half that comes out to be about one point nine three  $\mu_0 n i$  by  $a$

so in this problem if you have these kind of windings six windings at  $a h$  each inclined by thirty degrees you can actually calculate the total magnetic field at the center of the sphere and that happens to be about 1.

93 times  $\mu_0 n i$  by  $a$

so this problem tells me that i must i can actually use the magnetic field produced by small current elements and here it was a circle and i must be careful in calculating the total magnetic field because magnetic fields are vector quantities and when i add the vector quantities i have to be little careful ok now let me go to another interesting problem a long solid conducting cylinder of radius  $r$  has a cylindrical hole of radius  $a$  drilled out such that the axis of the hole is parallel to the axis of the cylinder if  $b$  is the distance between the two axis and a current  $i$  is passing through the remaining solid cylinder

so that the magnetic field is constant throughout the hole

so the problem is essentially like this i have a solid conductor cylindrical conductor and i have a hole drilled out

so the conductor now has only this is the only the conductor and

so this is this this is the hole and this this whole axis is parallel to the axis of the cylinder and this distance is given as  $b$  and a current  $i$  is passing through this entire structure i need to calculate the magnetic field inside this hole and show that it remains constant ok

so first thing is

so let me draw the figure again here

so i need to take a point here

so thats the center

so please remember there are the

so this the conductor is only this part this part is only the conductor now the problem to calculate the magnetic field like this may become complicated but i can use a very simple procedure which involves superposition

so what i can do is i calculate the magnetic field produced by a completely

solid cylinder without the hole i calculate the magnetic field produced by a cylinder of this diameter of this of the size at the same point and then subtract the two

so i first calculate the magnetic field at this point because of the solid cylinder without the hole then i calculate this magnetic field at the same point because of a cylinder of this radius at the same point and then subtract the second magnetic field from the first magnetic field to get the magnetic field at this point because of this the cylinder with the hole

so that is the procedure we are going to do

so for this first let me calculate the current density

so current density  $j$  is equal to current  $i$  is flowing through the entire area of this conductor which is now  $\pi r^2 - a^2$

so  $\pi r^2$  is the area of the cylinder  $a^2$  is a cylinder is the radius of the hole

so  $\pi r^2 - a^2$  is the effective area through which the current is flowing and

so the current density is  $i / (\pi r^2 - a^2)$

so now i am going to calculate what is the magnetic field produced at this point due to a solid conductor carrying this current density i will calculate then the magnetic field at this point due to a conductor of radius  $a$  carrying a current density  $j$  subtract the second one from the first and calculate the total magnetic field always keeping track that magnetic field is a vector quantity

so first things

so magnetic field due to solid conductor the hole of radius  $r$

so i need to calculate the

so this this distance

so this distance let me take it as  $ah$

so we can  $ah$

so this is now

so the problem is essentially i have a solid conductor of radius  $r$  and i take a point at distance  $r$  from here and the current is flowing parallel to the axis of the wire

so the magnetic field will be in this azimuthal direction

so i can actually use ampere's law

so i can use this flux integral  $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_{\text{enclosed}}$

so because the magnetic field is azimuthal as we have done before  $\oint \mathbf{B} \cdot d\mathbf{l}$  just gives me  $2\pi r B$  is equal to  $\mu_0 i_{\text{enclosed}}$  will be  $j$  times  $\pi r^2 - a^2$   $\pi r^2 - a^2$  is the area and the enclosed current is this thing

so  $B$  is equal to the magnitude of  $B$  will be equal to  $\mu_0 j r / 2$  now at this point the magnetic field will be directed like this

so let me call this

so this figure this this is the normal

so let me call this  $n_1$

so i can write the magnetic field vector as  $\mu_0 j r / 2 n_1$

so that is the magnetic field produced by a solid conductor of radius  $r$  at a distance  $r$  from the center now i must calculate what is the magnetic field produced by a cylinder of radius  $a$

so let me draw the same figure

so this was the the bigger cylinder of radius  $r$  and i am now

so this is the center now i take a cylinder of this radius  $a$  and calculate the magnetic field at the same point

so magnetic field due to solid cylinder of radius  $a$  at a distance  $s$  from its center

so this distance i am going to assume is  $s$

so if i go back to the original figure i call this distance as r this distance is s please remember r is the distance of this point where i am calculating the magnetic field from the axis of the big conductor s is the distance of that point from the axis of the hole and i am now calculating what is the field produced at this point at a distance s from the axis of the of a conductor of radius a

so just like before i can draw a an amperian loop like this and use this formula  $\oint \mathbf{B} \cdot d\mathbf{l}$  is equal to  $\mu_0 i_{\text{enclosed}}$

so  $2\pi s$  into b will be equal to  $\mu_0 j$  into  $\pi s^2$

so b will be equal to  $\mu_0 j s$  by two and now the normal will be in this direction

so if i go back to the original figure here this normal  $\mathbf{n}$  will be like this the magnetic field produced by the thick conductor is in this direction at the same point the magnetic field produced by conductor of radius a carrying current in the same direction is in this direction which i am calling as  $\mathbf{n}$

so let me call this  $\mathbf{B}_2$  is equal to  $\mu_0 \mathbf{j} s$  by two into  $\mathbf{n}$  now let me draw a figure here to ah explain better

so this is x y axis

so this is the point where i am trying to calculate this distance is r and this distance is s ok

so this is the perpendicular here

so i have the magnetic field produced by the big conductor is along this direction and the magnetic field produced by the other the the the hole with conductor corresponding to the hole is actually in this direction

so let me draw  $\mathbf{B}_1$  into cap and i call this angle theta and this angle phi

so this line is perpendicular to this line this line is perpendicular to this line

so this angle is also theta this line is perpendicular to this line this is the normal and this line is permanent this line

so this is phi

so total magnetic field b is equal to  $\mu_0 i$  should have named this as let me call this  $B_1$  that is due to the thick conductor of radius capital r

so b is equal to  $B_1$  minus  $B_2$  this remember what i have done is  $B_1$  is the field produced by the thick conductor without the hole  $B_2$  is the field produced by the smaller conductor here of radius a at the same point

so if i remove this conductor i have to remove the component of magnetic field produced by that portion of the conductor which is  $B_2$

so  $B_1$  minus  $B_2$  is my ah magnetic field produced at this point

so which is nothing but  $\mu_0 j$  by two into now  $B_1$  is r and one cap minus  $B_2$  has s  $\mathbf{n}$  cap minus s and two cap

so the total magnetic field is this is the magnetic field produced by the solid conductor without the hole this is the magnetic field produced by a conductor of radius a corresponding to the hole

so if i remove that component that magnetic field portion i will get the magnetic field produced by the conductor with the hole

so now let me write in terms of components

so this is  $\mu_0 j$  by two now r times  $\mathbf{n}_1$  cap

so if you look at  $\mathbf{n}_1$  cap it has a x component of y component

so x component is minus sin theta i cap plus cos theta j cap this vector has a component along this direction x component which is negative

so minus sin theta i cap plus a positive component of y which is plus cos theta j cap and then minus s times  $\mathbf{n}_2$  cap

so because i am writing this is minus  $\mathbf{n}_2$

so i have a plus sign here

so this will be plus s into

so if you have  $\sin \phi \hat{i} + \cos \phi \hat{j}$  this is  $n^{-1}$  this is  $n^{-2}$

so I have taken the minus sign minus  $n^{-2}$  I have written

so this is nothing but equal to  $\mu_0 j / 2$  into  $\hat{i}$  into  $r \sin \theta + s \sin \phi + j$  into  $r \cos \theta + s \cos \phi$  ok

so I have combined the  $\hat{i}$  terms and the  $\hat{j}$  terms and I get these two

so you can see here  $r \sin \theta$  this is also  $\theta$

so  $r \sin \theta$  is this length and  $s \sin \phi$  is also this length

so these two cancel off  $r \cos \theta$  is this distance  $s \cos \phi$  is this distance

so  $r \cos \theta + s \cos \phi$  is the distance from here to here which is nothing but  $b$  the distance of the center of the hole from the center of the axis of the conductor

so I get a very interesting expression that  $b$  is equal to  $\mu_0 i \mu_0 j / 2$  into  $j$  which if I write in terms of current  $\mu_0 i b / 2 \pi r^2 - a^2$  to  $j$

so that is the magnetic field produced by the conductor and there is a hole here and inside also the conductor is carrying current like this that's a hole

so the magnetic field inside at any point is given by this and as you can see here it is a constant all along the  $y$  direction

so this  $y$  is defined here ok now let me come to a last problem involving electromagnetic waves the electric field of a plane electromagnetic wave propagating in free space is given by  $E = 10 \hat{i} + 15 \hat{j} \sin 4 \pi \times 10^6 t - z$  volts per meter  $c$  is the speed of light in free space and  $z$  is in meters

so what is the wavelength of the wave what is the wavelength of the wave and calculate the corresponding magnetic field

so let me give you the answers ah I would like you to work it out please remember electromagnetic fields the you have to find out what direction is propagating and

so let me give you the answers

so the wavelength is  $0.5$  micrometers

and the  $B$  field will be given by  $15 \hat{i} - 10 \hat{j} / c \sin 4 \pi \times 10^6 t - z$

so that is the magnetic field

so we will stop here

so what we have done today is to discuss some problems in magnetic fields and which has exposed how we can employ techniques like Biot-Savart law or Ampere's law to calculate fields produced by various configurations and I hope with the understanding of basic concepts in electromagnetics you will be able to solve many many problems in your career thank you very much you