

a very good morning to all of you we have come to the last lecture in this ah topic on electromagnetism in the last lecture i started discussing about electromagnetic waves and how the introduction of the displacement current term in the maxwell's equations leads to the generation or the prediction of waves called electromagnetic waves

so today what we are going to do is i am going to show you that the electromagnetic waves which i wrote down last time are consistent with the maxwell's equations now ordinary what we will do normally is that we will take those maxwell's equations and derive what is called as a differential equation from those equations and then by solving the differential equations we will get that there are waves present in the predicted by these equations and those are called electromagnetic waves now because differential equations are ah beyond the scope of this course here i will be assuming a solution of electromagnetic waves and showing that those solutions are consistent with the maxwell's equations we have been looking at throughout this course of lectures on electromechanism

so let me recall electromagnetic waves are nothing but waves of electric and magnetic fields

so we had drawn a figure last time for example if i if i were to draw the figure same figure again here i will have i will show the electric field like this wave like this and these are the electric field vectors which i am drawing here this is the z direction this is the x direction and this is the y direction

so the electric field is pointing in the x direction and the corresponding magnetic field i had drawn like this

so the magnetic field lines are magnetic field vectors are perpendicular to the electric field vectors

so as the figure shows when the wave is propagating in the plus z direction the electric field and the magnetic fields are perpendicular to the propagation direction this is the magnetic field here the electric field and magnetic field are perpendicular to the propagation direction of the electromagnetic wave the electric and magnetic fields are perpendicular to each other and both are perpendicular to the propagation direction of the electromagnetic wave and also they are in phase as you can see here when the electric field is zero magnetic field is zero when the electric field magnitude increases the magnetic field magnitude increases and these are called sinusoidal waves as i had defined last time sinusoidal waves because the space and time dependence of these waves as i will write on an equation are sine waves sinusoidal waves and

so they the electric and magnetic fields are in phase and as i mentioned last time one has to be very clear in interpreting this figure these lines only represent the magnitude and direction of the electric field and the magnetic field at different points along the axis of the figure here

so there is nothing there is no movement like this there is no displacement it is simply the electric and magnetic fields which are varying with time at different points

so and ah i must mention here now that all electromagnetic waves travel at the same speed in the in free space and that value is two point nine nine seven nine two four five eight into 10 to the power 8 meters per second this is an exact value now this is defined as an exact value of the velocity of speed of light in free space and the unit of meter length unit which is the meter is defined through this velocity of a light or the speed of light in free space and

so all electromagnetic waves travel at the same speed given by c in free space and in the last lecture i had shown a spectrum which shows different electromagnetic waves radio waves microwaves light waves in between you have infrared waves ultraviolet then you have x-rays gamma rays all these represent

electromagnetic waves all these are characterized characterized by electric and magnetic fields and they all travel in free space at the same speed which is given by this number c and this we normally approximate by three into ten to the power eight meters per second

so that's an approximate value three ten to the power eight meter per second is an approximate value the exact value as defined now is two point nine nine seven nine two four five eight into ten per eight meter per second

so now what i want to do is the following i want to consider a sinusoidal electromagnetic wave and i will show you that this solution which i am going to write is consistent with maxwell's equations

so sinusoidal electromagnetic wave

so in this figure as i have shown here the electric field is pointing along the x direction and the magnetic field is pointing along the y direction and propagation is along z

so this kind of wave will be represented by equations of this form e is equal to $i \hat{c} e_0 \sin(kz - \omega t)$ this $i \hat{c}$ represents the fact that the electric field is pointed along the x direction e_0 is the magnitude of the electric field the maximum value of electric field and the sine function is actually this and let me remind you that this is a plot of electric field and magnetic field as a function of position at a given instant of time please remember this is a snapshot of the magnitudes and directions of electric and magnetic fields as a function of position at a given time t is equal to zero arbitrary time which i call t is equal to zero i could have like last time i also plotted a figure showing electric and magnetic fields as a function of time at a given position that is another figure and when you look at a figure be very careful in noticing what the figure represents precisely

so this is electric field and the corresponding magnetic field as i have drawn here is along the y direction

so b is equal to $j \hat{c} b_0 \sin(kz - \omega t)$

so as you can see here i am using the same sine function as the electric field and that is because electric and magnetic fields are in phase both of them are represented by the same sine function $\sin(kz - \omega t)$ and their magnitudes are e_0 and b_0 and the directions are $i \hat{c}$ and $j \hat{c}$ i also mentioned last time that $e \times b$ vector $e \times$ vector b will be along the propagation direction

so if you look at $e \times b$ is $i \hat{c} \times j \hat{c}$ is equal to $k \hat{c}$ which means propagation along z direction

so this is what this figure represents $e \times b$ must be along the propagation direction

so this is the wave propagating along this z direction having magnitude e_0 of electric field a magnitude b_0 magnetic field both of them are in phase now what i am going to do is the following i am going to show you that these two equations are consistent with maxwell's equations and for this i am going to look at free space

so i am looking at propagation of electromagnetic wave means free space that means there is no medium and there is no charge no charges no currents

so the electric and magnetic fields satisfy the following equations $\nabla \cdot \mathbf{d} = 0$ $\nabla \cdot \mathbf{b} = 0$ $\nabla \times \mathbf{e} = -\dot{\mathbf{b}}$ $\nabla \times \mathbf{b} = \dot{\mathbf{d}}$ that is the gauss's law of electric fields the gauss's law for magnetic fields faraday's law of induction and the generalized amperes law and in all this i have removed the charge from here on the right hand side

so there is zero here there is no current on the right hand side

so there is no current

so there is no term corresponding to current here and these equations contain only electric and magnetic fields and what i am going to show is that the solution which i have written namely these two solutions are consistent with the maxwell's equations now let me recall again that these two equations tell me that a changing magnetic field would generate an electric field and a changing electric field will give me a magnetic field a time varying in a magnetic field will lead to an electric field and time varying electric field will lead to a magnetic field this is the way the electric and magnetic fields couple to each other and this is the way the electromagnetic fields are propagating and generated through the space and these waves do not require any medium to propagate unless unlike other waves like sound waves or water waves or waves on a string in which there is a medium required these waves do not require any medium to propagate and

so these waves can propagate through free space right from stars which are light years away this the light coming from there is essentially electromagnetic character now what i am going to show you is the solutions which i have written namely these two solutions are consistent with these two equations that means that the magnetic field as given by this equation and the electric field given by these equations and the corresponding these two solutions these two equations this equation here the solutions are consistent with this

so let us start from faraday's law first

so i want to show that the solution which i am writing i want to find out under what condition the solution which i am writing is consistent with this equation minus d by d t of integral v dot d a now for this let me again draw a figure x by z

so i draw the electric field variation like this and the magnetic field variation is like this i am just looking at a part of the wave and let me draw again here the magnetic field lines are pointing like this at this instant of time and the electric field lines are pointing upwards at this instant of time ok now what i want to do is the following i want to consider a loop here a loop here which i call p q r s and i want to do an integration in this direction look here the left hand side has a integral over a closed path and the right hand side has the magnetic flux

so i take a loop p q r s in the x z plane and i want to find out that under what conditions am i satisfying this equation

so there is an electric field here and there is a magnetic flux passing through this

so the area which i am looking essentially is this area

so i have magnetic field lines crossing this area which is in the y direction and there is an electric field which is which is present in the space here

so let me write down

so i want to find i want to calculate the left hand side for this path and the right hand side for this path and equate them and get an equation now let me assume that the height of this loop is h and this is the plane z is a z and this is a plane i call it z plus delta z and delta z is infinitesimal very small value of delta z

so its a very very small number and i want to calculate the left hand side and right hand side

so first let me start with the left hand side

so i want to calculate integral e dot t l

so look at the path here the path i am taking is p q r s

so this will be equal to integral p to q e dot d l plus integral q to r e dot d l plus integral r to s e dot d l plus integral s to p e dot d l and i know the

electric field electric field was given by $i \epsilon_0 \sin(kz - \omega t)$
now

so the electric field is along x axis

so if you see already here along the path q r electric field is perpendicular to the d l d l vector is along q r electric field is perpendicular

so $\mathbf{E} \cdot d\mathbf{l}$ in this path will be zero similarly for the path s p e is perpendicular to this line and

so $\mathbf{E} \cdot d\mathbf{l}$ is zero

so the integrals from this this integral q to r q to r and s to p are zero the remaining two integrals e is the same for all values of x because e depends only on z and the value of electric field at z and z plus delta z will be different because as you can see here the electric field is changing with z

so i will get this is eq

so integral $\mathbf{E} \cdot d\mathbf{l}$ will be equal to essentially you will have two components p to q $\mathbf{E} \cdot d\mathbf{l}$ plus r two s $\mathbf{E} \cdot d\mathbf{l}$ the remaining two integrals have become zero and

so p to q $\mathbf{E} \cdot d\mathbf{l}$

so this is nothing but electric field at z plus delta z into the total length because electric field is constant along this length and d l is parallel to electric field

so $\mathbf{E} \cdot d\mathbf{l}$ is $E dl$ and E is constant along this path

so this will into this length h h is this length of integration and then

so in the second part r to s you see here electric field is pointing upwards the path of integration is from r to s

so i will get a negative sign $\mathbf{E} \cdot d\mathbf{l}$ will be negative and i will get minus e of z into h

so this is nothing but e of z plus delta z minus e of z into x and delta z is a infinite decimal quantity now let me recall to you ah you have defined the differential of a function d

so let me take a function $f(x)$ by dx is equal to limit Δx tending to zero $f(x + \Delta x) - f(x)$ by Δx there is a definition of a differential of a function limit Δx going to zero $f(x + \Delta x) - f(x)$ by Δx

so if Δx is very very small extremely small then i can write this as dh $f(x)$ by dx if i choose a very small value of Δx this is approximately equal to $f(x + \Delta x) - f(x)$ by Δx

so this i can write as $f(x + \Delta x)$ is equal to if i simplify this i will get $f(x + \Delta x) \times \frac{df}{dx}$

so the value of a function at $x + \Delta x$ is equal to value of the function at $x + \Delta x$ into the derivative of the function with respect to x at this point

so this is a nice expression in the limit of Δx becoming very small i can use this kind of an expansion of $f(x + \Delta x)$ is equal to $f(x) + \Delta x \frac{df}{dx}$ now if you look at this equation and this term here it is $f(x + \Delta x) - f(x)$ this is $f(x + \Delta x) - f(x)$

so i can write immediately $f(x + \Delta x) - f(x)$ by Δx is equal to $\frac{df}{dz}$ which means that $f(x + \Delta x) - f(x)$ is nothing but $\Delta z \times \frac{df}{dz}$

so $f(x + \Delta x) - f(x)$ is nothing but $\frac{df}{dz}$ into Δz

so i get the following equation integral $\mathbf{E} \cdot d\mathbf{l}$ is equal to ah

so $\frac{df}{dz}$ into h Δz now i must ah mention something here that remember this function this electric field e is a function of both position z and time and because my derivative is with respect to one of the variables position this is usually written as a partial derivative

so this is in mathematic this is written as $\frac{\partial E}{\partial z}$ into $h \Delta z$ this only means that i am taking a derivative of the electric field as a function of z keeping time constant the electric field is a variable which depends on position z and time t and here in this derivative all i am saying is i am taking a derivative of the function with respect to z keeping time constant

so i am writing this as $\frac{\partial E}{\partial z}$ it implies it is called the partial derivative it is a derivative of the function with respect to one coordinate keeping the other coordinate constant

so what i have got is $\int \mathbf{E} \cdot d\mathbf{l}$ is $\frac{\partial E}{\partial z}$ into $h \Delta z$ now let me look at the right hand side there is a right hand side of this integral here of this equation which is $\int \mathbf{B} \cdot d\mathbf{a}$ now i must calculate the flux through this area the first thing you notice is because the integration is in this direction the area vector is along this direction because the area because the loop integration which i have done is anti-clockwise area vector is in this direction which is nothing but $y \hat{k}$

so area vector for this loop of integration is along the direction y which is also the direction of the magnetic field

so i will immediately calculate $\int \mathbf{B} \cdot d\mathbf{a}$ is equal to now remember Δz is a very small quantity and

so i can assume that the magnetic field is almost constant between z and $z + \Delta z$ in this area magnetic field is almost constant

so this is approximately equal to magnetic field and said into area area of this entire loop is $h \Delta z$ into $h \Delta z$

so faraday's law for faraday's laws i must calculate $\frac{d}{dt}$ of $\int \mathbf{B} \cdot d\mathbf{a}$ which is nothing but i will again write as before $\frac{\partial B}{\partial t}$ into $h \Delta z$ because magnetic field depends on position and time i am differentiating with respect to time and

so i write as a partial derivative of B with respect to time into $h \Delta z$

so that i have got and i have got the left hand side of the of the parity's law

so left hand side is

so much

so i just substitute into this equation

so i substitute both of these into the following equation $\int \mathbf{E} \cdot d\mathbf{l}$ is equal to $-\frac{d}{dt} \int \mathbf{v} \cdot d\mathbf{a}$

so $\int \mathbf{E} \cdot d\mathbf{l}$ i have calculated as $\frac{\partial E}{\partial z} h \Delta z$ and this is $-\frac{\partial B}{\partial t} h \Delta z$ this implies $\frac{\partial E}{\partial z}$ is equal to $-\frac{\partial B}{\partial t}$

so faradays law implies $\frac{\partial E}{\partial z}$ rate of change of E with z is equal to $-\frac{\partial B}{\partial t}$ now if i substitute the solutions which i have written before for electric and magnetic fields

so let me write the equation again here

so i have shown that $\frac{\partial E}{\partial z}$ is equal to $-\frac{\partial B}{\partial t}$ now E was equal to the the scalar form if E was equal to $E_0 \sin(kz - \omega t)$

so $\frac{\partial E}{\partial z}$ is equal to $k E_0 \cos(kz - \omega t)$ B was equal to $B_0 \sin(kz - \omega t)$

so $\frac{\partial B}{\partial t}$ will be equal to now there is a minus sign here

so i will get $-\omega B_0 \cos(kz - \omega t)$

so this is $\frac{\partial E}{\partial z}$ this is $\frac{\partial B}{\partial t}$

so i substitute here and i get this cosine function cancels off and i get $k E_0$ is equal to ωB_0

so i got an equation that means if the electric and magnetic fields were given by those equations the solutions then i find that the for those solutions to satisfy faraday's law of induction the magnitudes of the electric and magnetic fields must be related to this equation $k E_0$ must be equal to ωB_0

naught that's first equation which I have got now I want to apply the same solutions in amperes generalized amperes law

so now let me look at

so ampere's law was $\int \mathbf{b} \cdot d\mathbf{l} = \mu_0 \epsilon_0 \int \mathbf{e} \cdot d\mathbf{a}$

so I need to calculate both the left hand side and the right hand side

so let me draw the figure again here

so I again have the electric field going like this and I have the magnetic field like this now I take a loop in the other plane

so I take a loop like this

so this is again this is this is point z this is point $z + \Delta z$

so I take p, q, r, s now I take a loop in the y, z plane

so this is z this is x this is y this loop now happens to be in the y that plane and

so we have electric flux crossing this plane and there is a magnetic field

so I want to calculate $\int \mathbf{b} \cdot d\mathbf{l}$ in this loop and the right hand side for this area which is enclosed by this loop and substitute into this equation and find out again a relationship between the electric field \mathbf{e} naught and the magnetic field \mathbf{b} naught

so let me start calculating $\int \mathbf{b} \cdot d\mathbf{l}$ $\int \mathbf{b} \cdot d\mathbf{l}$ again p to q $\mathbf{b} \cdot d\mathbf{l}$ q to r $\mathbf{b} \cdot d\mathbf{l}$ plus r to s $\mathbf{b} \cdot d\mathbf{l}$ plus s to p $\mathbf{b} \cdot d\mathbf{l}$

so look here it contains p to q $\mathbf{b} \cdot d\mathbf{l}$ plus q to r $\mathbf{b} \cdot d\mathbf{l}$ plus r to s $\mathbf{v} \cdot d\mathbf{l}$ plus s to p $\mathbf{v} \cdot d\mathbf{l}$ that's a complete loop now just like before look here the magnetic field is along the y direction and this line is along the direction and similarly this line

so the $\int \mathbf{b} \cdot d\mathbf{l}$ from q to r and s to p are zero

so I will simply get $\int p$ to q $\mathbf{v} \cdot d\mathbf{l}$ plus $\int r$ to s $\mathbf{v} \cdot d\mathbf{l}$ the remaining two integrals are zero now at this plane in this direction in this line the magnetic field is calculated at z plus Δz here the magnetic field is at z the direction of magnetic field is along the path of integration

so $\mathbf{b} \cdot d\mathbf{l}$ is $b \cdot dl$ similarly here $\mathbf{b} \cdot d\mathbf{l}$ I can calculate

so I will get this is equal to nothing but b at $z + \Delta z$ into again I will assume that this distance is h before as before

so b at $z + \Delta z$ into h now look here in the r to s integral magnetic field is pointing along y direction and my integration is along the minus y direction

so I will get a minus sign

so I get minus b of z into h which is nothing but b of $z + \Delta z$ minus b of z into h now as before b of $z + \Delta z$ minus b of z is approximately equal to Δz into $\frac{db}{dz}$ using again the same argument as I had given for calculating $\frac{de}{dz}$ I can write immediately that like here remember I had written here e of $z + \Delta z$ minus e of z was $\Delta z \frac{de}{dz}$ b of $z + \Delta z$ minus b of z is approximately $\Delta z \frac{db}{dz}$

so $\int \mathbf{b} \cdot d\mathbf{l}$ will be equal to $\frac{db}{dz} \Delta z$ again I am writing partial derivative because the magnetic field depends on position and time and

so this is a derivative with respect to z keeping time constant now I must calculate the right hand side which is depends on the electric flux $\int \mathbf{e} \cdot d\mathbf{a}$

so here this is the area enclosed by the loop and remember now the integration is in this direction p, q, r, s

so the area must be pointing down this using right handed rule this is the path of integration is in this direction and because of right hand rule area is pointing downward electric wheel is pointing upward and

so the electric flux is negative please remember in this equation electric

field is pointing upward area is pointing downward and

so $\mathbf{e} \cdot d\mathbf{a}$ will be will have a negative sign now again as before i am going to assume that the electric field is almost constant within the area of this loop

so this integral will simply be electric field at this point multiplied by the area of the loop

so this is equal to minus of e of z into area of the loop which is $h \Delta z$ i am assuming that the electric field is almost constant within the area of the loop

so $\mathbf{e} \cdot d\mathbf{a}$ is minus e $d\mathbf{a}$ because area is pointing downward electric field is pointing upward and electric field at this point is approximately e of z and multiplied by area which is h times Δz

so $\mu_0 \epsilon_0 \oint \mathbf{d} \cdot d\mathbf{t}$ of integral $\mathbf{e} \cdot d\mathbf{a}$ will be equal to minus $\mu_0 \epsilon_0 \oint \mathbf{d} \cdot d\mathbf{t}$ into $h \Delta z$ again i am writing a partial derivative because electric field is a function of both position z and time and this derivative is only with respect to time

so i substitute both of them into this integral of ampere's law $\oint \mathbf{b} \cdot d\mathbf{l}$ is equal to $\mu_0 \epsilon_0 \oint \mathbf{d} \cdot d\mathbf{t}$ of integral $\mathbf{e} \cdot d\mathbf{a}$

so i just calculated $ah \oint \mathbf{d} \cdot d\mathbf{l}$ integral

so $\oint \mathbf{b} \cdot d\mathbf{l}$ integral is $\mathbf{d} \cdot \mathbf{b}$ by $\mathbf{d} \cdot \mathbf{z}$ $h \Delta z$ is equal to minus $\mu_0 \epsilon_0 \oint \mathbf{d} \cdot d\mathbf{t}$ $h \Delta z$ which implies $\mathbf{d} \cdot \mathbf{b}$ by $\mathbf{d} \cdot \mathbf{z}$ is equal to minus $\mu_0 \epsilon_0 \oint \mathbf{d} \cdot d\mathbf{t}$

so just like i had an equation relating $\mathbf{d} \cdot \mathbf{e}$ by $\mathbf{d} \cdot \mathbf{z}$ $\mathbf{d} \cdot \mathbf{b}$ by $\mathbf{d} \cdot \mathbf{t}$ i have another relationship between $\mathbf{d} \cdot \mathbf{b}$ by $\mathbf{d} \cdot \mathbf{z}$ and $\mathbf{d} \cdot \mathbf{e}$ by $\mathbf{d} \cdot \mathbf{t}$

so if if the solutions which i have written have to satisfy ampere's generalized ampere's law then the electric and magnetic fields must satisfy this equation $\mathbf{d} \cdot \mathbf{b}$ by $\mathbf{d} \cdot \mathbf{z}$ is minus $\mu_0 \epsilon_0 \mathbf{d} \cdot \mathbf{e}$ by $\mathbf{d} \cdot \mathbf{t}$

so let me substitute the solution

so let me rewrite this again

so $\mathbf{d} \cdot \mathbf{v}$ by $\mathbf{d} \cdot \mathbf{z}$ is equal to minus $\mu_0 \epsilon_0 \mathbf{d} \cdot \mathbf{e}$ by $\mathbf{d} \cdot \mathbf{t}$ now \mathbf{b} was equal to $b \sin k z - \omega t$

so $\mathbf{d} \cdot \mathbf{b}$ by $\mathbf{d} \cdot \mathbf{z}$ is equal to $k b \sin k z - \omega t$ \mathbf{e} was given by $e \sin k z - \omega t$

so $\mathbf{d} \cdot \mathbf{e}$ by $\mathbf{d} \cdot \mathbf{t}$ is equal to minus $\omega e \cos k z - \omega$ differential \sin is \cos and because there is a minus sign here i get a minus here and ω

so if i substitute this into this equation i get k times $b \sin$ is equal to $\mu_0 \epsilon_0 \omega e \cos$

so that is another equation if the solutions which i have written must satisfy the generalized ampere's law then \mathbf{e} \sin and \mathbf{b} \sin must satisfy this equation

so now let me recall the other equation which i got which is the condition to satisfy ah faraday's law

so i got two equations if the solutions which i have written must satisfy ampere's law of induction \mathbf{e} \sin and \mathbf{b} \sin are related by this if the solutions must satisfy generalized ampere's law \mathbf{e} \sin and \mathbf{b} \sin are related by this

so let me rewrite this equations and simplify

so i have two equations now

so k times $e \sin$ is equal to ω times $b \sin$ and k times $b \sin$ is equal to $\mu_0 \epsilon_0 \omega e \cos$

so let me multiply both these equations i get $k^2 e \sin b \sin$ is equal to $\mu_0 \epsilon_0 \omega^2 e \sin b \sin$

so if i cancel of $e \sin b \sin$ i get k^2 is equal to $\mu_0 \epsilon_0 \omega^2$

$\epsilon_0 \omega^2$

so now i got a relationship between ω and k which appear in the solutions in the solutions there is $k = \omega / v$ and remember when i was discussing waves on a string i had defined the speed of wave as $v = \omega / k$

so ω and k are related by this

so speed of the wave is equal to $v = \omega / k$ which is equal to $1 / \sqrt{\epsilon_0 \mu_0}$

so this is the speed of the electromagnetic wave what i have shown you is that i started with solutions of electric and magnetic fields in the form of a wave ah let me let me show you the slide again here the electric and magnetic fields we are written in the form of a wave here electric field written as $E = E_0 \sin(kz - \omega t)$ which is a wave propagating along the z direction magnetic field $B = B_0 \cos(kz - \omega t)$ a magnetic field propagating is a wave in the z direction these two are the electric and magnetic fields associated with the electromagnetic field and if these two have to be solutions have to satisfy the maxwell's equations we find that these are waves these are because i have written solutions like this these are waves and these are the electromagnetic waves and the speed of electromagnetic wave in free space is given by $v = 1 / \sqrt{\epsilon_0 \mu_0}$ which is actually called c the speed of light in free space and this is the speed which i wrote earlier

so you see here that in free space independent of the frequency of the electromagnetic wave all electromagnetic waves no matter what frequency you take whether you take radio waves at megahertz frequencies or microwaves at gigahertz frequencies or light waves or x rays or gamma rays all these waves which consist of electric and magnetic fields propagate at the same speed c which is $1 / \sqrt{\epsilon_0 \mu_0}$

so this is a ah this is a very important relationship that we have got i have what i have shown you today is that electric and magnetic field can propagate as waves in the in free space and the speed of these waves is nothing but given by $v = 1 / \sqrt{\epsilon_0 \mu_0}$ in free space

so essentially what i have not actually solved the maxwell's equation then got solutions but what i have essentially shown you is that if i write a wave solution of electric and magnetic fields then i can satisfy maxwell's equations these solutions satisfy max those equations provided these waves travel at the speed which is given by $v = 1 / \sqrt{\epsilon_0 \mu_0}$ and this was the prediction of james clark maxwell and when he found that the speed of light is the speed of the electromagnetic wave is related to the ϵ_0 constant and μ_0 constant and the value of c which he obtained from this equation was

so close to the velocity of light in free space the measured speed of light in free space that he predicted that light must be electromagnetic and as i mentioned before it was hertz in 1888 who conducted experiments and showed the generation detection of these electromagnetic waves and then we have now we know that electromagnetic waves exist at all kinds of frequencies and they have we have given them different names for different frequencies and just

so just like before the speed of electromagnetic waves in free space c which is approximately three ten per eight meter per second and remember the wavelength of the electromagnetic waves is equal to $\lambda = c / \nu$

so different frequencies are characterized by different wavelengths these are all wavelengths in free space

so i will urge you to substitute the values of frequencies for radio waves microwaves light waves x rays and gamma rays and calculate the wavelengths and you will see typically radio waves are in the range of a few hundred meters of wavelength microwaves are in centimeters light waves are in nanometers x rays are even much smaller than that fraction of a nanometer and then you have gamma

rays in the range of picometers

so the wavelengths range over an entire range of wavelengths and supplement similarly frequencies

so these are all electromagnetic waves now once i have got these electromagnetic waves remember i have shown you long time back to when we were discussing electrostatics and magnetostatics that electric and magnetic fields can have energy stored in them and the energy stored is given by

so energy stored energy in electromagnetic waves

so i had shown ah before that the electrostatic energy density is half epsilon zero e square that is electric energy density energy per unit volume similarly the magnetic energy density is one by two mu zero b square now we have got these two solutions here and i have shown you the speed s one by epsilon zero mu zero square root

so also note this this equation this equation let me see what that implies k e naught let me read out that equation

so k times e naught is equal to omega times v naught this implies b naught is equal to k by omega into e naught and k by omega is one by c

so this is equal to e naught by c

so the if e zero represents the maximum value of electric field in the electromagnetic wave the maximum value of magnetic field in the electromagnetic wave is e naught by c where c is the speed of light in free space

so this is a very important relationship again i must remember that in free space the electric and electric and magnetic fields of the electromagnetic wave are related to b naught is equal to e naught by c

so let me substitute this here for example and what i get is u b becomes equal to u b was one by two mu zero ah b square which is equal to one by two mu zero b naught square sine square k z minus omega t which is equal to one by two mu zero now p naught was e naught by c

so e naught square by c square sine square k z minus omega t and this is one by two mu naught now one by c square is epsilon zero mu zero e naught square sine square k z minus omega t which is nothing but one by two epsilon zero e naught square sine square k z minus omega t that is the magnetic energy density the electric energy density is one by two epsilon zero e square which is nothing but one by two epsilon zero e naught square sine square k z minus omega t

so you see that because of this relationship b naught is equal to e naught by c what you find is the energy density of the electric field in the electromagnetic wave and the energy density of the magnetic field in the electromagnetic wave are exactly equal both are equal to half epsilon zero e naught square sine square k z minus omega t

so the electromagnetic wave as it propagates is carrying this energy as it propagates

so i can write the total energy density u is equal to u e plus u b which becomes equal to epsilon zero e naught square sine square k z minus omega t i have added these two this is the electric field energy density magnetic field energy density they are equal and

so i get epsilon zero e naught square sine square k z minus omega t now this is a time varying and a position varying function as you can see it varies as sine square k n minus omega t now at optical frequencies the frequency is very very large

so so it is very difficult to follow this

so what we normally do is to calculate a time average time average of this energy density and i can calculate the time average as one by

so let me call this u dash one by t integral zero to t u d t i integrate over one period of the wave with

so t is equal to 2π by ω

so i integrate over one period of the wave divided by the time of integration and i get an average value

so to take an to calculate an average i integrate over a certain region and divide by that width of that region and i get an average

so this is equal to $\epsilon_0 E_0^2$ by $\int_0^t \sin^2(kz - \omega t) dt$ with t given by $2\pi/\omega$ and ah i believe it is a problem to you to show that this is equal to half of $\epsilon_0 E_0^2$

so you show that $\frac{1}{t} \int_0^t \sin^2(kz - \omega t) dt$ is actually half you must know that the average of a sin square function is half the average of a cosine square function is half

so that average is half and that is the time average total energy density associated with the electromagnetic wave and this energy is actually propagating like here

so i can actually look at the following situation

so i have let me take a unit area

so this is a unit area and length c i take a cube with length a cuboid with length c and unit area

so as you can see here the waves are propagating in the direction this is the direction

so in a unit time all the energy contained within this volume will cross this area all the energy that is contained within this volume will cross this area

so i can calculate average energy crossing unit area per unit time as equal to energy density into the volume which is c into one which is equal to one by two $c \epsilon_0 E_0^2$

so this energy which is contained in electromagnetic waves is propagating and in a unit time the energy contained in this volume of length c and unit area will cross the surface and that energy happens to be this and this is also called as intensity and usually referred to as I

so intensity of the electromagnetic wave is given by $\frac{1}{2} c \epsilon_0 E_0^2$

so intensity and the electric field are related through this relationship very important relationship if you know the intensity if you know the power crossing per unit area per unit time then you can calculate the corresponding electric field here and

so let me write this equation again here

so I is equal to $\frac{1}{2} c \epsilon_0 E_0^2$ and E_0 is also equal to square root of $2I$ by $c \epsilon_0$

so if you know the intensity you can calculate the electric field if you know the electric field you can calculate the intensity of those waves now ah

so what we have done now is we have actually written down electric and magnetic fields as waves and i have shown you that the solution which i wrote down is consistent with maxwell's equations satisfying maxwell's equations provided i take the speed as c and i get a relationship between the magnitude of the electric and magnetic fields and this is what the prediction was now i want to take a few examples and calculate what kind of electric fields are generated in typical situations

so first example which i want to look at is sunlight light from sun light is an electromagnetic wave the sunlight is falling on the earth ah as it comes outside the earth it is much higher but as it propagates through the earth atmosphere it gets scattered

so finally the average intensity on the earth as it falls on the ground is approximately 1000 watts per square meter that is unit of intensity

so much power per unit area thats intensity which is watts per square meter

so i can use this i am assuming that it is a single frequency wave and it is just on an average i can use this equation to calculate the electric field
so E_{naught} the electric field of the sunlight is equal to $\frac{2I}{c \epsilon_0}$ which is equal to $\frac{1000}{3 \times 10^8 \times 8.85 \times 10^{-12}}$ which is equal to 3.77×10^6 volts per meter

so the sunlight is producing a potential voltage electric field over about eight hundred and seventy volts per meter and i can also calculate the corresponding magnetic field B_{naught} is equal to $\frac{E_{\text{naught}}}{c}$ which is equal to $\frac{3.77 \times 10^6}{3 \times 10^8}$ which is approximately three into ten to the minus six tesla an electric field of the order of eight hundred and seventy volts per meter and a magnetic field of the order of three micro tesla

so the electromagnetic waves from sun is creating these kinds of electric and magnetic fields let me take another example there was a satellite launched in nineteen seventy seven which is called voyager in september 1977 it has traveled over all the last 30 years or

so and it has left the solar system and is out into the space and
so the current distance is about 2×10^{13} meters and the transmitter power is about 20 watts now this transmitter is not transmitting in all directions but it is a form of an antenna you must have seen the antenna that you are using for cable television antennas

so there is an antenna which actually directs the waves into a particular direction and

so instead of the electromagnetic waves going in all directions you can actually reduce the spread of these electromagnetic waves and increase the intensity of the electromagnetic wave in the direction that you want and

so we define what is called as the antenna gain which is approximately six point five into ten per four which is essentially it gives me how much increased intensity that i get because i am now directing the electromagnetic waves in one particular direction and not radiating in all directions and i can then calculate the received intensity which is equal to $\frac{P_{\text{emitted}}}{4\pi r^2}$ there is a gain of six point five times the power four divided by 4π into distance square which is 2×10^{13} square and if you calculate this it comes out to be about 2.6×10^{-22} watts per square meter an extremely small value of intensity that is coming here from that voyager spacecraft and we can actually calculate immediately the corresponding electric field electric field is $\frac{2I}{c \epsilon_0}$ raise per half and that comes out to be 4.4×10^{-10} volts per meter and the magnetic field is equal to $\frac{E_{\text{naught}}}{c}$ which is approximately 1.5×10^{-18} tesla this one its a very small electric and magnetic field that is coming from the spacecraft and our detectors here are able to detect these these signals and one final example which i want to give you is the is a laser suppose i take a laser all of you must have seen laser pointers

so they have typically if i take a laser power off ten milli watts which is equal to 10^{-2} watts and if i assume the radius of the laser beam is about one millimeter then the intensity is equal to $\frac{P}{A}$ which is equal to $\frac{10^{-2}}{\pi \times (0.0005)^2}$ which is 10^6 watts per square meter

so that is equal to 10^6 watts per square meter and i can immediately calculate E_{naught} which is $\frac{2I}{c \epsilon_0}$ raise per

half that comes out to be one point five kilo volts per meter and the corresponding magnetic field is $\epsilon \text{ naught by } c$ that comes out to be five ten to the minus six tesla

so you can see that the power levels are quite strong here that's about a thousand watts per 3000 watts per square meter and the corresponding electric field is the magnetic fields

so these were two or three examples which i thought may be of interest you that you can actually calculate from the intensity of the electromagnetic waves you can actually calculate the corresponding electric and magnetic fields and so what we have done

so we have now come to the end of this course on electromagnetism

so let us recall we have through the lectures the past lectures we have obtained laws which describe electric and magnetic fields we try to understand what are these electric fields and what are the magnetic fields we wrote down equations and finally we combine all the equations into a set of equations called maxwell's equations maxwell added a very very important contribution to ampere's law in that equations and which we called as displacement current and we got four very very important equations which describe all of electromagnetism

so those equations along with lorentz force law which we have already used give us complete electromagnetic behavior of all systems that that you can imagine and

so these equations form a very very important part of physics engineering we are using electromagnetic waves in

so many applications our communications today our mobile communications depends on radio waves or microwaves we have light waves which are being used for various applications we have communication satellites rate transmitting radio waves from far off distances we have we are using electromagnetic waves over all possible applications and these form a very very important component of our society and i think we have been able to use this using these equations try to understand some of the physics very interesting physics behind electric and magnetic fields and how we can generate and propagate electromagnetic waves in free space its a very very important aspect that these waves do not require any medium to propagate

so you have electromagnetic waves propagating in free space and this velocity the speed of light c which is here also forms also the basis for the special relativity which einstein had postulated and

so i i hope i have been able to convey you some of the excitement and interest and wonderful physics behind the maxwell's equations and the kind of predictions that they give there is a lot more to understand ah for all of you we have not discussed electromagnetism in materials electromagnetic waves in materials

so we can actually further understand many of these and as i mentioned in the first lecture very interesting concept of meta materials and negative refractive index and

so on

so all these come under purview of electromagnetism we can actually form structures we can we can design structures in which we can as we can get very interesting properties of electric permittivity ϵ and magnetic permeability μ and

so these are a form of a very very important aspect of physics today and i hope you have enjoyed the course of lectures on electromagnetism and wish you all the best thank you very much you