

in the last lecture we considered a circuit consisting of an inductance and a capacitance in which the capacitor is initially charged and what we found is that this circuit provides sustained charge and current oscillation with an angular frequency ω equal to $1/\sqrt{LC}$ the last thing that I take up in our discussion on alternating current is an application which is very important for practical use of alternating current and that is a transformer in fact right in the beginning we have pointed out that one of the advantages of using an alternating current circuit is our ability to step up or step down such voltages and

so let's look at how does a transformer work first

so a transformer has two sets of coils

so let me first show it in one type of circuit this is not necessarily the only type of arrangement that is possible

so this is a soft iron core now what I have here is this that on one of the arms I have certain turns of wire which is put in I will assume that to begin with there is no resistance in this part of the circuit but there is an alternating source of voltage with an amplitude V primary

so this part is called the primary part of the circuit we assume r is equal to 0 .

now on this side what I have are windings will see that the number of windings depends upon what you want to do with it

so let me at this moment keep it an open circuit this will of course be connected to load

so this would be connected to load

so let me say

so this side is the secondary part of the circuit this is called secondary once again I assume that at the moment load is not connected and the resistance of the circuit is equal to zero now look at what will happen when an alternating current passes through the turns of the primary it will produce an alternating emf in the secondary because of mutual inductance

so alternating voltage in primary leads to alternating emf in secondary this is actually the basic principle of a transformer is because of mutual inductance effect the role of the soft iron core is actually two folds one is it will increase the strength of the magnetic field produced by the primary current

so soft iron core what it is doing is

so increase magnetic field strength the second thing that it does is to help link the flux in the secondary circuit and make sure that it is linked to each turn help flux linkage now suppose ϕ is the flux through each turn of the primary circuit let me also assume n_p is the number of turns in the primary circuit now notice since I have assumed that the resistance in the primary circuit is equal to 0 which is an unphysical assumption but let's stick with that right now my voltage in the primary circuit must be exactly balanced by the back emf because otherwise the current will become unphysically large

so V_p is $n_p d\phi/dt$ now if I assume that the flux linkage is tight that there is no leakage of flux which is again a slightly unphysical arrangement or assumption then the same flux is linked with each turn of the secondary circuit

so therefore my V_s is if n_s is the number of turns in the secondary it will be minus $n_s d\phi/dt$ because it is the same flux which is being linked

so assume n_s equal to number of turns in secondary and the no flux leakage

so if you compare these two expressions you immediately get the ratio V_s/V_p is equal to n_s/n_p now this is the primary equation of a transformer because it tells me that if I want the secondary voltage to be stepped up meaning thereby secondary voltage to be larger than the primary voltage

so n_s should be then greater than n_p for step up transformer the reverse is

true if you want a step down transformer then of course n_s must be less than n_p for step down now in an ideal transformer what happens is the entire power is transferred to the secondary

so let me write down an ideal transformer all power is transferred that implies that i_p which is the current in the primary times the voltage in the primary must be equal to current in secondary times voltage in secondary in symmetry that's my second equation now if i compare these two x expressions this is this was my equation one then what you find is this that my i_p is i_s times v_s by v_p which is equal to i_s times n_s by n_p

so this tells me that i have an inverse relationship with respect to the turns in the current part of the circuit

so what we notice is that we have n_p by n_s is equal to v_p by v_s and is also equal to i_s by i_p

so what it implies is that supposing we are using a step up transformer then increase in voltage would automatically come with a corresponding decrease in current and conversely if you are using a step down transformer then the voltage will decrease but the secondary current will increase

so there is a trade-off between the current and the voltage and depending upon what our usage is we have to worry about that

so let me illustrate this with some typical examples suppose uh i wish to melt a piece of nail which has a fairly small resistance let's take it typically resistance is let's say 0.004 ohms

now i used to melt it by connecting it to an electric circuit by providing a heating effect of electric current now obviously i cannot directly connect such a small resistance to the mains suppose my mains is 240 volts then the current that you generate would be 240 divided by 0.004

which is equal to 60 000 amperes no household supply can take this larger current and your the fuse if it is there or an mcb will trip

so what does it do the current that i draw from the primary cannot exceed the typical household current which should be within about 8 to 10 amperes now here i am help if i use a step down transformer and let's consider what happens if i use a step down transformer supposing this is 240 volt mains i will work out how what type of transformer do you actually need

so this is the secondary circuit which is connected to the resistance that we talked about now the way to do it is the following i calculate using a property of this piece of nail like its mass its specific heat etcetera and find out how much of heat do i need to melt this over certain period of time now suppose i did that calculation then my Q that is the amount of heat that is required is given by $i^2 r$ times the time which let me take it to be a couple of minutes for convenience and i can easily calculate how much current do i require because everything else is known to me suppose the current that i calculate is let me call it i to distinguish speed from the previous eye supposing the i that i calculate is let's say about 500 amperes now this is also a huge current but remember i am not drawing it from the primary circuit from the mains i'm drawing it from a secondary circuit and we'll see what effect does that happen

so then what i have is the following that the current is times r_s that's equal to that's the secondary voltage v_s that is 500 multiplied by 0.004 that is just equal to two volts

so i need a huge step down and in fact you can see it since this is 240 you want this circuit to provide you two volts

so therefore this is a step down with a ratio of hundred is to 1

so that 240 is reduced to that now

so n_p by n_s which is v_p by v_s is 120 squared

so let's calculate now how much current was actually drawn from the primary in

that case now i_p which is the current run from the primary times n_p that's equal to n_s times i_s and

so therefore i_p is m_s by n_p which is 1 by 120 times i_s which we have taken to be 500 and that you can see is a very reasonable value of 4.

16 amperes how much of power was actually delivered by the secondary circuit

so the power delivered by the secondary circuit is $i_s^2 r$ and that's simply equal to 500 square multiplied by point zero zero four and that's about a thousand watts this is usually a good enough power to melt the piece of nail in let's say couple of minutes now in practice the power transferred this is what i have assumed that the entire power provided in the primary circuit is transferred to the secondary circuit now in practice this is never

so so let's look at what are the now what is the reason for that

so number one is that all the flux will not link to both the primary and secondary this is this was one of our assumptions

so the flux linkage is not total

so what happens in general is some flux might link to one but not to the other

so some of those could be linked to the primary circuit and similarly there would be a leakage from the secondary circuit

so therefore we have to worry about that and this flux linkage in both parts of the circuit will lead to what is known as a self reactance i can reduce the effect by tight coupling

so what is done in tight coupling is

so you can reduce it by tight coupling

so what is done is to wind the turns of the secondary on the same core on which the primary windings are put

so that would make the coupling somewhat tight the second assumption which is wrong is we have assumed that the resistance of the windings is 0 but that is really not true

so resistance of windings they are not zero in other words the transformer is never ideal let us suppose r_p and x_p are the resistances and the reactants of the primary circuit and likewise r_s and x_s are for the secondary circuit then i know that my impedance in the primary circuit is root of r_p^2 plus x_p^2 and like this the impedance of the secondary circuit is r_s^2 plus x_s^2 now this will in turn mean that the induced emf across the primary is not v_p as we have assumed but is reduced by i_p times z_p

so induced emf across primary windings is not v_p but reduced by i_p times z_t likewise the voltage induced across the secondary windings and secondary is not v_s but reduced by i_s times z_s there are other effects which affect the transfer of power power transfer is never complete there are many reasons for this the first one is because of eddy losses in the iron core this can be actually minimized by taking laminated elements but this will lead to heating which will reduce the amount of power transfer and likewise when you repeatedly magnetize the iron core that is because you are passing an alternating circ emf

so repeated magnetization and demagnetization this leads to what is known as the hysteresis loss which also result in heating and of course thereby reducing the power available one of the losses is because of what are known as eddy current laws

so the point is this that the core on which the transformer windings are put is made of metals now as this core is conducting now as your current changes in the windings there would be localized currents induced in this metal because we would require them to oppose the changing magnetic flux because of changing currents in the windings now these are known as the eddy currents you have learnt about it earlier and and this eddy currents will cause heating of the core which of course naturally results in power loss now what do i do to reduce

the effect of such losses now obviously we need to reduce the eddy currents now what we do in such cases

so to reduce any losses we use what is known as a laminated core i will explain what is the laminated core seen what is done in a laminated core is instead of using a single block i use layers of conductors which are glued to each other

so let me they roughly the picture is as follows

so this is my core let me show you a section of that now see what happens here is this that i take instead of a single block layers

so let me try to show you the various layers by trying to draw these and of course likewise i have this section and here my windings are there

so these layers are laminates they're glued to each other by thin coatings which are insulating and this helps reduce the eddy current because the thickness of individual layer being smaller the available area for these currents becomes smaller and as a result it reduces the ideas

so if there is a single block then the eddy currents which will flow on the surface would be something like this

so this is eddy current now if instead we use lamination because the surface area for each laminate is smaller

so that i have these laminates glued to each other by a non-conducting material

so the type of eddies that i will have would be something like this

so this is effect of lamination a more artistic presentation of the lamination that one uses in transformer can be seen in the following slide the other problem which we have talked about is due to hysteresis now we have seen that hysteresis arises because of the remnant magnetize now in order to reduce it what is done i will not be able to go through the physics behind it but what is done is to use soft magnetic material such as there are many materials with low hysteresis and these are typically silicon steel steel alloys manganese zinc ferrites etcetera and these sort of are materials with properties which have less amount of remnant magnetization and hence the hysteresis loss can be minimized by using proper material

so use of soft magnetic material with low hysteresis typically silicon alloys steel alloys silicon steel steel alloys manganese zinc ferrite that's it now in addition to the above losses that is hysteresis and eddy another loss is what normally goes as copper loss this loss arises due to the resistance in the winding wires

so you notice that if the primary current is i_p and the resistance of the primary windings is r_p then my loss in primary is $i_p^2 r_p$ and likewise the loss in secondary is $i_s^2 r_s$

so notice that both these losses depend upon the current that flows in primary and the secondary circuit and

so therefore such losses depend upon load now obviously we cannot completely eliminate the copper losses but we can reduce copper losses is possible and first one is fairly simple you use very thick wires

so use of thick wires as winding wires there are other engineering solutions is to keep the transformer inside high vacuum and pass high pressure varnish to the container

so that all small holes are plugged suppose i have a transformer which has n_p equal to 200 and n_s equal to 10 and the supply voltage is 240 volts now clearly this is a step down transformer step down by a factor of 20 because n_p by n_s is 20.

so clearly my voltage sorry this is voltage in the primary is 240.

so voltage in the secondary is 240 divided by 20 that's equal to just 12 volts

the current in the secondary can be obtained if i knew what is the secondary load

so load r_s let me take it to be 20 ohms

so that should be simply 12 divided by 20 and that's equal to 0.6

6 amperes how much is the primary current assuming the complete power transfer i know i_s into v_s is equal to i_p into v_p

so that is 0.6

6 into 12 is equal to i_p into 240 which gives me i_p equal to 0.025

0.025 amperes an important application of the step down transformer happens in power distribution or power transmission now because of the fact that the power is produced far away from the cities which actually consume the power there is a significant loss due to resistance of the cables and the power lost is given by the current that is flowing times the resistance of the cables let me write it by P_c

so P_c is cable resistance now clearly my interest is to reduce this lost power as much as possible which would mean that i want i to be

so reduce P_c lost implies that i should be as small as possible now remember that the power that is produced is actually i times v

so if i want small i for a given power i want v should be as large as possible

so v should be large but this is met with certain amount of danger because you are going to transport power at fairly high voltage

so what is done is that power is actually produced at a fairly high voltage

so let me talk about a typical power plant

so this is where

so this is plant

so maybe let me take a small plant which produces let's say 20 kilovolts now what is done is that this is stepped up in order to reduce this loss

so i need a step up transformer suppose i make it some 200 kv or 300 kv and then i transmit it

so this is transmission this is where loss occurs

so cable loss there are two steps in which it is brought down there would be a substation in which it would be reduced to let us say 10 kilowatts

so this is step down once again before it is given to the consumers a further step down to let's say 230 to 240 volts as the case may be now this is a schematic diagram

so let us look at some numbers which will help us in understanding what is happening

so let us suppose i have a small power plant which is producing one megawatt of power

so power output let me call it P_{out} from the plant is one megawatt which is 10^6 watts but that's equal to i times v power lost we have seen is i^2 times cable resistance r_c if you compare these two i get a relationship between lost power to power out that is equal to power out divided by v^2 times r_c that's very simple because this is i^2

so i^2 is P_{out}^2 divided by v^2 and i have a v P_{out} here now let me take some numbers suppose my r_c is taken to be small let me take it as some 10 ohms and i have seen that my P_{out} the power output was 10^6 watts supposing i produce v power is produced at 20 kv then my power lost to power out is power out which is 10^6 divided by v^2 this is 20 kilovolts

so it is 2 into 10^6 divided by 4×10^8 r_c i have taken to be small which is just 10 ohms if you calculate this this works out to 0.025

0.025 which is a loss of 2.5 percent

5 percent now if you increase the voltage 200 kv you can repeat this calculation

and find that this will give me θ .

0.25 percentage of loss

so what we have done today is to consider an important application of alternating current and voltage that is it's used in stepping up or stepping down the voltage using the principle of mutual inductance and we have seen that transformers are of great practical use particularly in case of power transmission or whenever there is a need to either step up the voltage or step down the voltage from the supply that you have with this we come to the end of our set of lectures on alternating current it is an appropriate time to summarize uh the content of this set of lectures on alternating current

so let's do that

so we started with a simple illustration of an ac generator which consists of a rotating coil in a uniform magnetic field as the magnetic flux varies with time so will the emf generated and the

so the rotating coil in B field and the voltage of the emf that is generated will draw it as a function of time and

so it is something like this etcetera etcetera and this is the peak voltage

so and plotting v as a function of time this is time t equal to θ and we also defined what was an rms value of voltage or also current and this was about 70 percent of that

so this is v_{rms} and v was given by $v_m \cos(\omega t)$ the time period which is the this difference distance of time between a time t equal to θ and the time when it again returned back to the same value of the voltage

so time period T which is inverse of the frequency is 2π over the angular frequency ω this oscillating emf also leads to an oscillating current

so i talked about oscillating current then what we found is that both oscillating emf and the oscillating current they may be represented as what we call as a phasor after that we are looking at what happens when we put in various elements in this circuit and we found that for a purely resistive circuit the current is always in phase current is in phase with the voltage what it means is that if my instantaneous voltage through the resistor is given by $v_m \sin(\omega t)$ then the instantaneous current is given by $i_m \sin(\omega t)$ where this i_m is equal to v_m / R is the usual ohm's law expression and we also defined what is meant by an rms current by saying that i_{rms} is $i_{maximum}$ by square root of 2 the power dissipated P of t is i of t times v of t and if you look at the average power because each one of them has a sine variation

so that sine square of ωt whose average value is half will determine the average power and that is equal to i_{rms}^2 times R which is also equal to v_{rms}^2 divided having done that we looked at a purely capacitive circuit in this case if the voltage across the capacitor is given by v of t is given by $v_m \sin(\omega t)$ then the corresponding charge if you like the instantaneous charge is given by C times $v_m \sin(\omega t)$ and you can obtain the current by differentiating this and the current at time t is given by $i_m \cos(\omega t)$ plus $\pi/2$ weight that is the current leads the voltage by $\pi/2$ what it means is that the current peaks a full quarter cycle before the voltages

so i peaks $\pi/4$ before voltage if you look at the phasor then what you find is that the current leads by $\pi/2$

so they would be on two consecutive quadrants of the $x-y$ plane

so for example let us take this as your v then since the current is leading by 90° this should be your current and

so that this angle is 90° the value of i_m is given by v_m divided by what we called as the capacitive reactance

so X_C is capacitive reactance and that is equal to $1 / \omega C$ in fact X_C has a dimension of the resistance that is measured in ohms and if you plot the

xc versus the frequency

so this is the way it behaves because as ω is very large the reactance is ∞ goes to ∞ and for small reactants it starts with infinite value if on the other hand we have an inductor in the circuit

so inductive circuits

so this is the representation of an inductive circuit

so once again let us take voltage to be given by $v_m \sin \omega t$ what we found is that the current is given by v_m divided by $l \omega$ times $\sin(\omega t - \frac{\pi}{2})$ meaning thereby the current lags the voltage $\frac{\pi}{2}$ and this maximum current that we have i_m is v_m over $l \omega$ and this quantity $l \omega$ is what is known as inductive reactance which goes up linearly with the frequency and if you are looking for a corresponding phasor diagram for this then if this is where your voltage is then the current would be in the preceding quadrant

so you notice that what we said is for a capacitive circuit the current leads the voltage and for an inductive circuit the current lags the voltage we have given you a mnemonics which says *ellie the iceman* standing for that for an l in the circuit the emf comes before the current does that is emf leads the current which is the same statement as current lags the voltage and for sc in the circuit it is the current which comes before the emf that is current leads the voltage having done this individual elements we discussed a series lcr circuit in this case the relationship between the voltage and the current was v is equal to i times z where z is the impedance of the circuit which has two components a component which is in phase with the voltage that is by resistance and another component which is out of phase with the voltage that is the square of the difference between the capacitive reactance and the inductive reactance and by representing them as just $x_r^2 + x^2$ in this case if i again take v is equal to $v_m \sin \omega t$ the general expression for the current would be $i_m \sin(\omega t + \phi)$ this ϕ that is there may actually be positive or negative depending upon whether the circuit is more capacitive than inductive or vice versa and ϕ is given by $\tan^{-1} \frac{x_c - x_l}{r}$ showing thereby that ϕ is positive if x_c is greater than x_l and is negative if x_c is less than x_l

so that if x_c is greater than x_l the current leads the voltage and vice versa of course the power in the circuit which is given by i times v is equal to $i_m \sin(\omega t + \phi)$ into $v_m \sin \omega t$ and this on expanding $\sin(\omega t + \phi)$ we get two terms the terms are $i_m v_m \sin^2 \omega t \cos \phi + i_m v_m \sin \omega t \cos \omega t \sin \phi$ if you take the average of the power then the second term that vanishes because the sine function this is essentially $\sin^2 \omega t$ it vanishes and we are left with only this term which is $\frac{1}{2} i_m v_m \cos \phi$ because $\sin^2 \omega t$ has a average of $\frac{1}{2}$ times $\cos \phi$ and this $\cos \phi$ we said is the power factor of the circuit which plays a very important role in transmission lines the last thing that we did with respect to lcr circuit is to look at a phenomenon known as resonance

so look at the impedance expression is given by $r^2 + (x_l - x_c)^2$ whole square now for x_l is equal to x_c z has a minimum and that minimum is r actually now if you tune in the frequency of the source then the current would have a maximum when z has a minimum by varying ω we get maximum amplitude at the same place where z has a minimum where x_l is equal to x_c and that frequency turns out to be equal to $\omega_0 = \frac{1}{\sqrt{LC}}$ this is the resonant frequency this occurs when the voltage across the LC part of the circuit is equal to ∞ and what we found is that lower the resistances the sharper is the peak

so the pictures were something like this this is against ω

so for high values of r has another flat scope as you decrease r this is the

type of thing that you get and further decrease of r gives you a still sharper this thing the peak happens at ω equal to ω_0 equal to $1/\sqrt{LC}$

so this is let's call this r_1 this is r_2 this is r_3 and this is this picture is for r_1 greater than r_2 greater than r_3 is just a graphical representation of the resonance phenomena normally this plot would be plotted not directly against ω but against logarithm of ω that's a long skill and in that what you find is corresponding to ω equal to ω_0 you will find there is a peak in the current

so that the current would become like this and

so this is ω_0 and

so this is current or current amplitude actually and this is the variation of Z the impedance the it has a minimum at the same place where the current has maximum and if in the same plot I am also plotting the phase

so this is minus 90° and this is plus 90° .

so the way it does is the following now this is your phase ϕ .

so with this we conclude our series of lectures on the alternating currents you