

in the last few lectures we have been talking about lcr circuit and through several examples we have tried to explain various concepts connected with that circuit as well as we talked about a very interesting phenomena of resonance that occurs when the impressed frequency equals the what is known as the natural frequency of the system that is  $1/\sqrt{LC}$  in the last lecture we discussed the role that power factor plays in ac circuit let me briefly summarize what we did last time

so first thing is that the power delivered to a load in a dc circuit is simply given by the product of current with the voltage this is a simple multiplication now in ac circuits other than resistances there are elements such as inductors and capacitors and the problem becomes more complicated because the currents that they deliver are not in phase with the voltage

so therefore when we add currents or when we find out currents for circuits which contain capacitors inductors and resistance there is a little more complicated way of adding them taking care of their phases

so so this is not true of ac circuits in general the power that is delivered to resistors this is what we have been calling as the active power this is either active or occasionally you have also called it true power the reason why you call them active power is because this power may be used for doing useful work such as heating lighting etc

so active power can do useful work and as i have pointed out this is usually measured in watts or kilowatts for a capacitive or an inductive load the current is out of phase with the voltage by  $\pi/2$  in case of a capacitor it leads and in case of an inductor it lags

so capacitors and inductors phase difference between let me just call it  $\Delta\phi$  it leads in case of capacitor lags in case of inductor and that's equal to  $\pi/2$ .

so here it leads and here it lags well i meant current leads and current lags now the for a general ac circuit the current may either lead or lag depending upon which reactance is more

so for lcr circuit current may lead or lag depending on reactance this cosine of this phase  $\cos\phi$  this is called the power factor now let's look at what is known as a power triangle remember the three basic elements  $L$   $C$  and  $R$  uh which contribute to electrical power in an ac circuit they are represented by three sides of a right angle in a impedance triangle

so let me first draw an impedance triangle

so an impedance triangle looks like this this is your resistance  $R$  which is equal to  $Z \cos\phi$  and this is the reactance  $X$  which is the net reactance that comes out of the capacitive reactance and the inductive reactance and that is equal to  $Z \sin\phi$  and the impedance itself is represented by the hypotenuse  $Z$

so this is my impedance track now suppose i multiply the three sides of this impedance triangle by  $i$  square let's see what type of things do i get

so firstly i have

so let me say that multiply by  $i$  square

so what i have then is that when i multiply the resistance arm by  $i$  square i get  $i$  square  $R$  which we have called as the active power

so active power i will represent by  $P$  and that's equal to  $i$  square  $R$  which is measured in watts the other side which is the reactive power is obtained by multiplying the side reactance  $X$  by  $i$  square

so that is  $i$  square times  $X$  and this is measured in volt ampere reactive the hypotenuse when multiplied by  $i$  square gives you the apparent power let's represent it by  $S$

so that it doesn't confuse with  $R$

so that's equal to  $i^2 Z$  which is measured by volt ampere

so let us draw this triangle here

so what i have here is this this is the side  $p$  which is my active power this is the side  $q$  which is the reactive power and this is the side  $s$  which is the apparent power and this angle here is  $\phi$  cosine of which is the power vector

so my  $s$  is  $v$  times  $i$   $i$  will not repeat it but it is measured in volt ampere the side  $p$  is  $v$  times  $i$  times cosine of  $\phi$  that is measured in watts and the reactive power is  $v$  times  $i$  times sine  $\phi$  that is volt ampere reactive now suppose i took an inductive case

so for an inductive circuit my  $X$  is  $X_L$  in that case the power triangle will look like this this is my power  $p$  the active power remember the active power is always along the current direction and this is my reactive power  $q$  and this is the apparent power  $s$  and you can see that the current lags the voltage because  $s$  is nothing but  $v_i$

so so this is current lags the voltage and in the capacitive case this triangle will simply become slightly different and that is the way i would do it with this becoming equal to  $p$  this becoming equal to  $q$  and this becoming equal to  $s$  and this angle is  $\phi$  and you can see that the current which is along the direction of  $p$  actually leads the voltage then what we did is to point out that this power factor plays an important role in transmission lines and that is because the reactive power which is actually a wasteful power that tells us that the amount of power that is actually being produced is not all going to service the load and one of the functions or one of the responsibilities that we have in trans in constructing transmission lines is to reduce the effect of such lagging and this is usually done by compensating factors by substituting capacitive elements in the circuit

so let me illustrate this with another example

so let me say that this is my voltage ac voltage  $v$  and this is my load which typically consists of  $r$  as well as  $l$

so i'm not really writing down what is it and we have seen that in order to compensate it i need to put capacitance here in this specific example suppose my input voltage is 220 volts rms and let us suppose that the current  $i$  is 0.5 ampere of course again rms and the current lags the voltage by some angle

so let's say 75 degrees now what we need to do is to calculate the active power reactive power and the apparent power okay

so notice that the apparent power is very easy to calculate because this is the product that we have

so that's equal to  $220 \times 0.5$ .

110 that is the current which is equal to 110 this time volt ampere now what is my true power my true power is 110 apparent power multiplied by cosine of  $\phi$

so cosine of 75 degrees if you calculate this this turns out to some 0.28.

47 works since it's true it's in watts the corresponding reactive power obviously is going to be large because as you notice that the apparent power is 110 whereas the true power is only 28.

47 indicating a small power factor

so this would be given by  $110 \sin 75$  and that is works out to 106.

25 volt ampere reactor

so this is obviously not a great situation to happen because a lot of power that is delivered to the circuit are going waste and that is the reason for which we attempt compensation which you have discussed in detail now what i am going to do today is to take up one of the very special cases of lcr circuit that's a circuit for which the resistance is taken to be 0 which is known as lc circuit and we will see if the lc circuit is provided with an initial source of energy by initially charging the capacitance then that circuit can provide

sustained oscillation but before i do that let me first recall the dynamics of what is known as a one dimensional harmonic oscillator and that picture is something like this you have essentially a mass which is connected by a spring the other end of the spring being fixed to a vertical wall and this mass is initially pulled from its the natural unstretched position and

so let me take this unstretched position to be  $x$  equal to zero that is the origin from which i will measure everything

so what i do is that i stretch this mass such that this mass is at a distance let's say  $x_0$  from this initial point and release it

so at that position what happens is the velocity of the mass is equal to 0

so this is mass  $m$  this is spring constant  $k$  and

so this let me let me say this is at time  $t$  equal to zero my velocity is zero but since this spring has been uh stretched by an amount  $x_0$  uh there is a spring energy there and

so the spring energy which is the potential energy

so let me represent it by  $u$

so  $u$  is equal to  $u_{max}$  is equal to half  $k x_0^2$  and the corresponding kinetic energy is equal to zero because the particle has zero velocity now when we leave this mass when you release this mass this will start moving to the left and for suppose the i still have  $x$  greater than 0

so from  $t$  equal to 0 to  $t$  equal to some time period  $t$  divided by 4 my  $x$  is still greater than 0 velocity is still towards the left

so what happens in that situation is that i have a spring energy  $u$  which is equal to half  $k x^2$  but this time the kinetic energy is not equal to 0 but it is half  $mv^2$  where  $v$  is the instantaneous velocity between these two limits

so now what happens is this that this mass ultimately reaches the equilibrium position now what happens at the equilibrium position is the following

so um as it is moving towards the left when it reaches  $x$  equal to 0 the time  $t$  is equal to  $t$  by 4

so let me redraw that picture

so this is this has just reached the point  $x$  equal to zero now at that stage there is no spring energy

so  $u$  is equal to 0 but the velocity is maximum here

so let me call it  $v_{max}$

so the kinetic energy is half  $m v_{max}^2$  obviously this kinetic energy must be equal to the maximum spring energy which was there at the other end

so kinetic energy is at its maximum  $k x_0^2$  now this mass obviously starts moving still to the left compressing the spring and now  $x$  becomes negative but

so from  $t$  equal to  $t$  by 4 till  $t$  equal to  $t$  by 2  $x$  is less than 0 the velocity is in principle  $v$  which is less than  $v_{max}$

so kinetic energy is there which is half  $mv^2$  where  $v$  is the instantaneous speed and the potential energy of the spring is half  $k x^2$  where  $x$  is the compression that is there now at  $t$  by 2 the compression is maximum and by conservation of energy we know that the amount of compression must also be equal to  $x_0$

so at that stage once again the potential energy of the spring is maximum which is equal to half  $k x_0^2$  and your kinetic energy becomes equal to zero now the velocity being 0 and since at this stage the spring is compressed there is a force in the opposite direction that is towards right and now the attempt will be made by the spring to restore to its normal position and

so therefore from  $t$  equal to capital  $t$  by two till  $t$  equal to  $3t$  by 4 once again  $x$  remains negative but velocity is towards right but not equal to zero and so therefore once again what i have is the kinetic energy is half  $mv^2$  the

potential energy is half  $kx^2$  and this continues till three  $t$  by four  
so at three  $t$  by four the velocity is maximum

so kinetic energy is half  $mv_{\max}^2$  and potential energy because this  
spring is neither compressed nor extended is equal to zero and finally it starts  
moving to the right and once again at  $t$  equal to capital  $t$  it completes the  
cycle with all the energy again becoming potential energy now look at what  
happens that at an arbitrary point when the extension or the compression was by  
 $x$

so there is a spring force which acts on the mass and  
so the only force that is there is minus  $kx$  but that must be equal to  $m \frac{d^2x}{dt^2}$  that is the mass times the acceleration

so that is equal to minus  $kx$  now which simply is nothing but the equation to  
simple harmonic motion and the solution being  $x$  equal to  $x_0 \cos(\omega t)$  where  $\omega$  is square root of  $k$  over  $m$  there is no phase which is uh taken in  
the solution for the simple reason my initial condition was at  $t$  equal to 0  $x$   
equal to  $x_0$  okay

so since at  $t$  equal to 0  $x$  equal to  $x_0$

so the spring mass system executes simple harmonic motion and if you plot the  
displacement of the particle as a function of time

so what you find is that since at  $t$  equal to 0  $x$  was maximum

so let me do this

so this is the way motion continues

so this amount this amount is  $x_0$  now

so this is the simplest example of a system which executes harmonic  
oscillations without any damping there is no damping because we have assumed  
that the mass is moving on a frictionless surface now it turns out that there is  
an electrical analog to it which is known as the LC oscillation let me try to  
give you a circuit

so i have a circuit with a dc source a battery let us take a resistance to  
limit the amount of current that is being passed and i take a circuit which is  
like this i will come back to what these things are i have a capacitance here  
and i also have an inductance in the circuit

so notice what i have done here is the following that there are these three  
points here let me mark it one two and three

so also label these appropriately this is  $L$  this is  $C$  and this is of course a  
source of battery which is not particularly important for us right now but let's  
put it this way now notice what happens when i connect one to two

so let me show it by a dotted line because that is not going to be my main part  
of the circuit

so if i connect one to two what happens is that this capacitance comes to the  
circuit but the inductance being disconnected

so this plays no role in that circuit

so one two two connected this as we know will charge the capacitor

so the capacitor will be fully charged and since i have taken this side of the  
battery to be the positive side

so what will happen is that this end of the capacitor will become positively  
charged and the right hand will become negatively charged

so let me count the time from the instant when the charge is maximum

so at  $t$  equal to zero  $q$  is equal to  $q_{\max}$  and this charge will continue to be  
there in the circuit till the battery remains connected now once the charge has  
become maximum

so at this stage of course the transients have died down

so let me also say that there is no current in the circuit because the  
capacitor does not allow dc to flow through now at that stage what i do is i

disconnect 1 and 2

so let me make this point little emphasized

so disconnect one two but connect one three the result of which is simply  
so let me now show it by a solid line i have an lc circuit the battery is out  
of the circuit and it is fully charged capacitor which is there now

so obviously since this end is charged positive this end is charge negative  
when you connect one to three a current will flow in this direction to begin  
with reducing the charge on positive plate and also reducing the charge on the  
negative plate till the reverse situation happens that this side has become  
positive and that side of a big become negative and then the whole cycle will  
continue and this system will show charge oscillation

so let's look at that

so we'll say that current flows

so which means  $\frac{d i}{d t}$  is greater than 0 but notice when my current is there  
the charge is reducing

so therefore my  $i$  is minus decubed now if  $i$  now look at the kirchhoff's law for  
this circuit now remember there is no battery in this circuit but i had charged  
my capacitor initially

so what will happen is this my circuit equation will become minus  $l \frac{d i}{d t}$   
plus  $q$  by  $c$  which is the voltage across the battery is equal to zero using the  
fact  $i$  is equal to minus  $\frac{d q}{d t}$  i can rewrite this equation as  $d^2 q$  over  
 $d t^2$  plus  $q$  over  $l c$   $i$  divided both sides by  $l$  as well is equal to zero now  
you could compare this equation with the equation that i had given for one  
dimensional harmonic oscillator namely  $d^2 x$  by  $d t^2$  plus  $k$  over  $m$   $x$   
is equal to 0.

both these obviously represent oscillating circuits we had analyzed this one  
but this would mean that the frequency or the angular frequency of oscillation  
is given by  $\omega$  equal to  $1$  over square root of  $l c$  this is for the electrical  
circuit compare this with the mechanical circuit for which  $\omega$  let me call it  
 $\omega$   $m c$  that's equal to square root of  $k$  over  $m$  looking at these two equations  
it tells me that the similarity seems to be that the charge is being similar to  
displacement  $x$  in the mechanical circuit

so let me put in this comparison there the charge  $q$  is analogous to  
displacement  $x$  in the mechanical circuit now

so what happens is this that at  $t$  equal to  $t$  by four the energy has because the  
capacitors are totally discharged

so the energy that was stored in the capacitor that is energy that was stored  
in the electric field has now been transferred into the magnetic field  
associated with the inductance

so the electrical energy has been completely transferred to the magnetic energy  
associated with the inductor

so let me just show you uh the slide which gives me the energy as a function of  
time

so notice that in the beginning my circuit was charged completely

so all my energy was electrical energy now with time the electrical energy gets  
converted to the energy of the magnetic field and at time  $t$  equal to  $t$  by 4 the  
capacitors are fully discharged and all the energy is in the magnetic field

so therefore in this diagram this picture here that's my  $u_e$  and and this other  
one is my  $u_b$  which is the magnetic energy and at any instant of time at any  
instant of time my magnetic energy  $u_b$  is half  $l i^2$  and electrical energy  
 $u_e$  is  $q^2$  by  $2c$  and the total energy which is sum of this and this is  
represented by this horizontal line that is constant

so  $u_{total}$  net is  $u_b$  plus  $u_e$  and that you can write either as  $q m^2$  by  $2c$   
or as half  $l i m^2$

so what happens at time  $t$  equal to  $t$  by 4 now notice at time  $t$  by 4 my capacitor plates have been completely discharged now in such a situation would expect that no current will flow because there is no battery in the circuit there is nothing to provide a current because initially you remember there was a current because my left plate was charged positively right plate was charged negative

so initially i had that but now since both the capacitor plates are discharged i don't expect it but there is a problem the current cannot suddenly switch off to zero because if it did according to faraday's law there would be a very large emf brought into the circuit

so as a result what happens is the current continues in the same direction as before that is the direction in which it was flowing from zero to  $t$  by four and it will now charge the right plate positive and left plate will become negative till the at time  $t$  by two the capacitors are charged fully again though the sense in which the plates are charged have now reversed and the entire energy now is in the capacitor or in the electric field and this oscillation continues like this for again at three  $t$  by four back at  $t$  i will not go through the solution because the equations are very similar

so the solution must be very similar what is happening is that in the mass spring system we had a continuous conversion of kinetic energy into potential energy and vice versa in this case the kinetic energy is analogous to the magnetic energy  $\frac{1}{2}mv^2$  and  $\frac{1}{2}li^2$

so what we find is the kinetic energy for the spring mass system this corresponds to the magnetic energy for the lc circuit that is  $\frac{1}{2}mv^2$  corresponds to  $\frac{1}{2}li^2$  now comparison is now clear remember my quantity which corresponded to the displacement was the charge

so therefore the parallel is between the velocity being similar to the current  $i$  which is as i said should be obvious because  $x$  corresponds to  $q$  but you notice one thing the role of mass in the mass spring system is taken over by the inductance

so  $m$  is  $l$  and if you look at the potential energy expression which is  $\frac{1}{2}kx^2$  this corresponds to  $\frac{1}{2}q^2/c$  and since we know  $x$  and  $q$  are comparable then my analogy between the two circuits would be the spring constant  $k$  is analogous to the inverse of the capacitance and the total mechanical energy which is the sum of the potential energy plus the kinetic energy now that obviously corresponds to the total magnetic energy plus the electric energy which is potential energy is  $u$  electric kinetic energy is  $eu$  magnetic and that is the total electromagnetic energy let me put these are analogies and that remains constant in both these circuits

so let me give you an example of lc oscillation supposing i have an lc circuit with  $l$  is equal to 50 millihenry and  $c$  is equal to 20 micro farad and it's given that current is initially maximum initially actually simply means at time  $t$  equal to zero my question is how long does it take to fully charge the capacitor now notice that we have already pointed out that there is a phase lag between the magnetic energy and the electric energy which is time  $t$  by 4

so here my  $\omega$  is given by  $1/\sqrt{lc}$  and that is  $1/\sqrt{50 \text{ milli henry} \times 20 \text{ micro farad}}$  is  $2 \times 10^3$  and that is equal to  $10^3$  radian per second

so that tells me that the time period  $T$  which is  $2\pi/\omega$  that if you substitute you will get it to be 6.

3 millisecond now realize what we have been having here is something like this i have given you a current which is like this and

so this is the current  $i$  and correspondingly the voltage is like this and and

this is  $t$  by four that is the time lag between the voltage maximum and the current maximum which is  $t$  by four

so my time lag between these two will be one fourth of the this which is approximately 1.

6 milliseconds later the capacitor will get fully charged let's take another example suppose i have a radio tuner remember i told you that a radio receiver or a tuner it works on the principle of variable capacitors when you are rotating the dial of a radio tuner what you are doing is actually to change the capacitor in the circuit now suppose i have a radio tuner which can work in what is popularly known as the mw band medium wave band and let us say it can tune in in the range 800 kilohertz 1200 kilowatts and what have been given is this that the inductance in the circuit is given to be 200 micro henry which is same as  $2 \times 10^{-4}$  henry now my question is that what is the range in which my capacitance varies

so let's look at that

so remember these are linear frequencies  $f$

so i first need to convert them to the angular frequency

so corresponding to 800 hertz kilohertz this corresponds to multiplied by  $2\pi$

so  $2\pi$  into this kilohertz already  $10^3$  is there

so this is 5.

$0.3 \times 10^6$  radians per second and corresponding to 1200 kilohertz multiplication by  $2\pi$  will tell you this is 7.

$5.4 \times 10^6$  radians per second

so these are omega values but i know that omega is  $1/\sqrt{LC}$  which tells me that the capacitance is given by  $1/\omega^2 L$

so all that we now have to do is to put the value of  $L$  and substitute the values of two ranges of omega

so if i take the first case omega equal to 5.

$0.3 \times 10^6$  i get  $C$  is equal to  $1/2 \times 10^{-4}$  and this is omega square

so it is 5.

$0.3 \times 10^6$  square

so that's about let's just approximately say it's about  $25 \times 10^{-12}$ .

so therefore this quantity as you can see it is  $1/50$  in the denominator and  $10^8$  in the denominator

so put them back this gives you about  $2 \times 10^{-10}$  which is same as about 200 picofarad one pico is  $10^{-12}$  and if you look at the other extreme then your omega is 7.

$5.4 \times 10^6$  the calculation is trivial the corresponding capacitance becomes approximately 90 picofarad

so therefore your radio tuner must have the ability of varying the capacitors from the minimum of 90 picofarad to a maximum of 200 picofarad let's take another example in an LC circuit given  $C$  is equal to 64 micro farad the expression for current is given by  $2 \sin(500t + \theta)$ .

4 this is of course an ampere and the phase constant  $\theta$ .

4 is in radians i have a few questions a at what time  $t$  does the current reach maximum is actually trivial only thing that you have to realize is since this is the type of expression that i have given for the current it tells me the origin of time is different from what we have been assuming in our previous discussion and

so i know that  $i$  is equal to  $i_m$  that's maximum when this argument here when  $500t + \theta = 0$ .

4 that is equal to  $\pi$

so that sine function reaches its maximum you compute  $t$  from this and that works out to 2.

34 into  $10$  to the power minus 3 seconds what is the value of the inductance in order to do that you have to recognize that this must have been my  $\omega = 500$

so  $\omega$  is equal to 500 and that is equal to  $1/\sqrt{L C}$

so compute that will give you  $L$  is equal to  $1/16$  henry rather a big value of inductance but these are all illustrative problems

so it doesn't really matter what is the total energy the total energy  $i$  can simply compute the magnetic energy when it is maximum because  $i$  know at that time the capacitor energy is zero

so it is simply half  $L I_{\text{max}}^2$  now that  $i$  have computed  $L$

so it is half into  $1/16$  and  $I_{\text{max}}$  was 2 ampere that's the maximum

so therefore this into 4 and that's equal to  $1/8$  joules

so let me give a an illustration consider an LC circuit with  $L$  is equal to 2 milli henry  $C$  is equal to 80 micro farad and supposing at  $t = 0$  initial charge in the capacitor plate is 4 micro coulomb and the circuit is set to oscillation

so let's look at various things associated with it first is the frequency of oscillation which is  $1/\sqrt{LC}$  and that is  $1/\sqrt{2 \text{ milli henry}}$

so  $2 \text{ into } 10 \text{ to the power minus } 3$  and this is  $80 \text{ into } 10 \text{ to the power minus } 6$  because it's micro farad

so this is clearly  $1/4 \text{ into } 10 \text{ to the power } 4$  and that is 2500 radian per second now look at the electrical energy  $U_e$  that's  $q^2/2C$   $q$  is 4 micro coulomb

so it was  $16 \text{ into } 10 \text{ to the power } -12$  divided by  $2 \text{ into } 8 \text{ into } 10 \text{ to the power } -5$  that's 80 micro farad

so that is  $10 \text{ to the power } -7$  joules

so we have calculated what is the maximum energy stored in the electric field now with time what happens is the electrical energy gets converted into magnetic energy and vice versa means that the electrical energy from being maximum goes to zero when the magnetic energy goes from 0 to being maximum and how much is the magnetic energy maximum remember  $i_{\text{max}}$  is given by  $q_{\text{max}} \omega$  and we have given  $q_{\text{max}}$  to be 4 micro coulomb

so this is  $4 \text{ into } 10 \text{ to the power } -6$  times 2500 which is  $10 \text{ to the power } -5$

so the maximum magnetic energy let us write it as  $U_{\text{mag max}}$  which is the energy associated with the magnetic field is half  $L I_{\text{max}}^2$  and since  $i$  am looking for the maximum it is half  $L I_{\text{max}}^2$  and substituting this  $i$  got half two milli hundred two into minus three into ten to the power minus four because it's  $i_{\text{max}}^2$  and that as expected works out to ten to the power minus  $i$  said as expected because obviously at this time all the energy stored in the capacitor has got converted into the magnetic energy associated with the inductors and this push and pull mechanism where electrical energy gets converted to the magnetic energy and vice versa continues as long as the oscillation persists

so this is about an oscillating circuit and the only point to realize is that what we have assumed what we have assumed is slightly unphysical because we have said there is no resistance in the circuit but then if there were resistance in the circuit this will lead to damping of these oscillations just the way the friction would lead to damping in the mechanical circuit

so what we have done in this lecture is to consider LC circuit we have assumed that the capacitor was initially charged and the only thing that is there now in the circuit is an inductor and a capacitor and what we found is that there is an

oscillation of both charge and current and the oscillation frequency

so charge and current oscillation takes place the oscillation frequency or the angular frequency  $\omega$  is given by  $1/\sqrt{LC}$  what we did is to also establish a parallel between this oscillating circuit and a mechanical circuit consisting of a mass and the spring system without having any friction the comparisons were as follows the charge  $q$  corresponded to the displacement of the mass the current was parallel to the velocity inductance  $L$  was similar to mass the capacitance was  $1/k$  the correspondingly the electrical energy of my LC circuit was like the spring energy of the mechanical system and the magnetic energy was similar to the kinetic energy of the system and both these systems are conservative systems in that the total energy remains the same you