welcome back let me begin as usual with a review of what we did in the last lecture we continued our discussion on lcr circuit recalling that when we have a voltage variation given by v of t equal to vn sine omega t the corresponding current will be given by i m times sine of omega t plus phi where the current amplitude im is given by the voltage amplitude divided by the impedance the impedance z we had seen is defined as r square plus xl minus xc whole square and the phase difference between the current and the voltage actually in this case the amount by which the current leads the voltage in phase is given by tan inverse of xc minus xl divided by r we have pointed out that for a capacitive circuit the current leads the voltage while for an inductive circuit the current lags the voltage then what we did is this to say that supposing i am allowed to value this frequency then the amplitude im itself will attain a maximum as a function of frequency when the frequency is given by omega equal to omega 0 equal to 1 over square root of lc this is when i m as a function of omega reaches a maximum and this frequency omega 0 is known as the resonant frequency of course this is the angular frequency and the corresponding linear frequency is related to this by division by two pi having done that we discussed what is known as the sharpness of resonance

so this was resonant frequency sharpness of resonance

so this is done by looking at the current versus the angular frequency curve

so i am being plotted against the angular frequency and we have seen that the type of curve that you can get is something like this which has a maximum at omega equal to omega 0

so what we look for is the points where the power delivered to the circuit becomes half and the full width at half that maximum this is what is our definition of two times delta omega and the we had shown that this is known as bandwidth

so bandwidth is 2 times delta omega which is equal to r over l another measure of the sharpness of resonance is given by what is known as a quality factor simply written as q that's equal to omega 0 divided by 2 delta omega

so it is omega 0 l divided by r we then showed that the average power absorbed by the circuit which is denoted by p of t it is given by i n square z over 2 times cosine of phi which can be alternately written as vm square over 2z cosine of phi this factor the multiplying factor cosine of i which we had seen is given by r over the impedance z this is defined as the power factor power factor of the circuit now what we found is that for a purely resistive circuit where z obviously is equal to r my cosine of 5 is equal to 1 namely the corresponding phase angle is zero and in this case the circuit absorbs maximum power maximum power also is absorbed when we have a resonance and the reason is that if you look at the expression for z then you realize that when x l equal to x c my impedance is also equal to r

so maximum power is also absorbed at resonant frequency which is of course when xl is equal to xc we had also seen that the value of phi for a purely capacitive or inductive circuit for pure capacitive or inductive circuit the value of phi is equal to pi by 2 in the former case that is in the capacitive case the current leads the voltage by pi by 2 while in the inductive case the current lags the voltage now in which case either case cosine of phi becomes equal to 0.

in other words for either of these cases namely purely capacitive or inductive circuits the no power is dissipated and such circuits are known as wattless circuits for a general lcr circuit we have in general phi is not equal to zero and of course once again the power dissipation only takes place through the resistance what we did is to also discuss a few applications of lcr circuit in particular the use of such circuits particularly the rl circuits as high pass or low pass filters in the present lecture we continue with our discussion of lcr

circuit and elaborate many of these concepts in particular the question of power factor by giving several examples

so notice this that the expression for power average power was shown to be given by vm square over 2z times cosine of phi and as i had seen that cosine of phi is given by r over z

so it is vm square over 2z into r over z

so equal to since v m square by 2 is v v rms square

so we write it as v rms square times the resistance divided by z square now this expression gives the expected answers for a purely resistive circuit for which z is equal to r

so you can immediately see this is v rns square divided by r which is the power that i expect for a purely capacitive or inductive circuit r is equal to 0

so that also gives me power p equal to 0 and also at resonance since at resonance z becomes equal to r i get back again the maximum power dissipation condition let us return back to a bit of a discussion on this power factor and what is its effect uh with respect to power absorption and power dissipation in an lcr circuit

so what we said is that power the the power factor is the phase angle between the current and the voltage now if the load is purely resistant if we have a purely resistive load we know that the voltage and current are in phase now if voltage and current are in phase then the product v i which gives me the instantaneous power it is always greater than which means the power always flows into the load

so this implies power flows to the load now notice that since both voltage and current vary sinusoidal and if they are not exactly in phase then in some part of the ac cycle v times i will become less than 0

so if not in phase v times i may become negative for some part of the cycle what does it imply instead of the circuit drawing power from the source the power flows back from the circuit to the source let us recall the time variation of voltage and the current under different conditions

so for example if i have a capacitive circuits i have the following situation this is my voltage

so this is v as before i plot with time both the current and the voltage and the corresponding current because it's a capacitive circuit because a capacitive elements the current leads the voltage i would have something like this

so this is current

so you notice that if you look at the time scales the this is t by 4 this is time 0 and this is t by 2 3t by 4 and t

so notice in this case from 0 to t by 4 the v times i is positive

so the circuit absorbs power from t by 4 to t by 2 because the product of the current and the voltage is negative the amount of power absorbed in the previous quarter cycle is returned and again from t by two to three t by four both current and the voltage are negative

so therefore once again it absorbs power and in the last quarter cycle from 3t by 4 to t it returns the power

so what happens is the result is that the average power over a cycle for a capacitive circuit is equal to zero now we could do the same thing for a inductive circuit and in inductive circuit the situation is essentially an image of this with the current being reflected about the timeline this is t once again let's draw our voltage the way we did but this time because the current lags the voltage the type of curve that i have is something like this

so this is i of t and that was v of t

so here you notice what happens is this that in the first quarter cycle from 0 to t by 4 the product is negative you recall that the power is being returned in

what it had absorbed in the preceding quarter cycle and i have already pointed out that this is not really the point where you actually switch it on but it's a reference point from some time and in the next quarter cycle both of them are positive

so therefore power is absorbed and again return and absorb like this now let us look at a situation where i have a mixture the situation corresponds to the case where i neither have a purely capacitive circuit nor a purely inductive circuit because there are resistive elements in the circuit

so let's look at a situation like this let's as before draw the voltage for one cycle this is time this is v of t and let me take this situation where the current leads the voltage by phi is equal to pi by 4 which corresponds to a time lead by t by 8.

now let's look at how the current behaves now remember because the current leads the voltage and not quite by pi by 2 but by pi by 4

so what will happen here is the following that let me first try to make the time scales here

so this is t by 4 this is t by 2 3t by 4 and of course the last one is t now if in the same diagram i plot the current now remember unlike the pure capacitive case where the current leads the voltage by a full pi by 2 in this this case the leading is by pi by 4 only

so therefore it hasn't quite reached the maximum but is going to take another t by 8 time before it reaches the maximum

so the current is probably something like this and the it will also naturally not quite become zero when the voltage is maximum but you will have to wait for another t by eight time

so this is the way the current would go and likewise it would another t by eight later it will become maximum

so this is good

so if you look at this picture something like this happens from 0 to 3t by 8 and where is 3t by 8 this is this is the point which is 3t by from t equal to 0 to 3t by 8 my voltage and the current both are positive v greater than 0 i greater than 0

so therefore for a time duration of three t by eight it absorbs power from the supply now from three t by eight till t by two that's in this section here the voltage is still positive but the current has now become negative

so therefore v is greater than 0 but i is less than 0

so therefore my power is negative and the power is returned and i can continue like this from t equal to t by 2 till 7 t by 8.

my current and voltage are both negative

so once again power is absorbed and from 70 by 8 this is slightly because of freehand drawing slightly to the left of where it should have been and from 70 by 8 to t

so my voltage still is negative but the current has now become positive

so once again the power is returned now look at what it actually implies

so what we have actually said here is the following that my current expression

so let me write it here my current expression was i am sine omega 2 plus 5 whereas my expression for the voltage was vm sine of omega t and in deciding which is positive and which is negative i realize that the sine function remains positive for the first two quadrants and becomes negative in the next two products and if you look at this summary then you will realize for a larger part of the cycle the circuit absorbs power from the source and for a smaller duration relatively smaller duration it returns energy to the source this indicates that the net power is absorbed from the circuit now which indicates that there are resistive elements in the circuit now this has an important

consequence in power transmission and this results in loss in power transmission
so in order to explain this let me consider an inductive circuit and the main reason is in most circuits the cause of the phase lag is primarily due to the inductive elements they are the only they are the mostly non-resistive elements in circuits in transmission lines etc are inductive in nature
so in such cases as we have pointed out the current lags the voltage now let us look at an inductive circuit
so since we are talking about an inductive circuit we have seen that the current lags the voltage by an angle phi if you recall the impedance was defined through a right angle triangle and the various arms of the impedance triangle was given like this if this is the resistance r and this is xl which is the inductive reactance then this is my impedance and this angle is 5
so therefore what i have here is x square plus r square or x l square plus r square is equal to z square cosine of phi is r by z sine of phi is equal to x y z now what we'll do now is to convert this into a power triangle and this is simply done by observing that if i multiply all sides of this triangle by i square then i square r represents the true power i square r is the power consumed by the resistive load and and this is measured in watts this is called true power now
so correspondingly i have an i square x l and i square z if you look at this i square r this is also nothing but i times v times r by z
so that is equal to i times v times cosine of phi
so basically what i have done is to write one of the i's as v by z and multiplied by r and use r by z is equal to cosine of phi and we have said that this is true and if you look at the corresponding reactive component this is given by i square x or xl and that's equal to i times v divided by z times x l and that's equal to i times v times sine of i
so what we are doing here is to observe that if i resolve the current along a direction which is uh along the voltage then that is the one which provides me with true power but the component i sine phi which is in a perpendicular direction that's what gives me the reactive power which also goes by the name of wattless power now though the dimension of the waterless power is the same as that of true power the electrical engineers prefer to measure it as volt ampere reactive or kilo volt ampere reactive as the case may be now the product i times v itself that is known as the apparent power which is the same as i square z and in this case the unit in which it is measured is volt ampere or kilo volt ampere as might be the case the true power is also called active power and this as you already pointed out is called a reactive a real life example will make you appreciate the difference between apparent power and the true power suppose you go to a fancy coffee restaurant which are very prevalent in our cities nowadays and you order a cup of coffee now this is what you would actually get and uh you notice one thing that though the shop will charge you for a full cup of coffee what you have actually got is a lot of foam on the and coffee only at the bottom of your cup i have marked the actual coffee as the true coffee which stands for my active power or the true power and the form on the top which is marked with red is what corresponds to my reactive component and this is the amount for which you pay at the same rate as that of the coffee but you cannot drink it now what corresponds to the apparent power is the total volume of the coffee that is being shown to you now most of the time you will not be able to see it in a coffee shop for the simple reason that the cup in which they will serve you will not be a transparent cup but a opaque cup and this is the cost that you pay for the hidden uh components in the cappuccino or coffee which you order now
so you notice this apparent power is more because the current and the voltage are out of phase by an angle phi and

so uh reactive components like the true components also draw power but however they cannot use it for doing any useful work what they absorb in one part of the cycle is returned to the source in the other part of the cycle what we do is this that to observe that this power triangle
 so therefore becomes like this
 so this is v i cosine phi this as i said is measured in watts the wattless component is here v i sine phi which is in v a r and the apparent component is along the hypotenuse which is simply equal to v times i and this will be measured in volt ampere itself and the power factor recall that this angle is 5 is simply given by true power divided by the apparent power that would be the cosine of the angle frame now for a given power then the current that is being drawn from the source is given by p divided by v cos pi that's the now notice what happens if the value of cosine phi is small that if the power factor is small this would imply that the current drawn by the circuit for a given true power or the active power is large
 so current drone is large now this implies that there will be higher losses along the transmission line and the reason is we know that the losses occur because of the resistance of the cables and if i represent rc by the resistance of the transmission cables then the power loss is i square rc which is proportional to 1 over cosine square phi
 so therefore the power factor or rather a lower power factor is responsible for a lot of loss along the transmission line now how does one compensate and the reason why we are having this situation is that the apparent power is substantially different from the true power and that arises because there are reactive elements which are in the circuit
 so therefore if we could somehow rather compensate the reactive elements
 so compensation we will illustrate our ideas with a specific example but let's first look at
 so compensation for reducing apparent power
 so this essentially implies neutralize the effect of the wattless component of course one cannot probably do that exactly or completely but one can try to make it as much as possible
 so let us look at the power diagram again now let us look at how the power compensation takes place now what is compensation
 so compensation basically is this remember we said that my ip which is the component along the voltage this is the one which supplies true power whereas let me call this as iq which is the sine phi component and this is the reactive power which is also occasionally called as the wattless power the adjectives true and the waterless here they refer to the powers that these components correspond to and not to the current themselves
 so let's look at that remember that this was my direction of v since it was an inductive circuit i said let the direction of current be here like this this is phi and
 so therefore these are the components by drawing the completing the parallelogram and and this component the component of i along v is what i have called as i p and perpendicular to v is what i called as the iq now ideally what i would like to do is to compensate for this i sine phi component now how this is done is to essentially provide another element which will provide me with a reactive component in the opposite direction now as we recall that the capacitive reactance and the inductive reactances they are in the opposite direction they are aligned but in oppositely
 so therefore the simplest way of canceling this would be to supply a capacitive component which exactly cancels this
 so therefore this is what ideally we would like to do this is i want this to be

iq let's call it iq prime now in practice what however happens is that a complete cancellation is never possible

so therefore what we do is to provide a capacitive element which will not completely compensate it but maybe compensate up to this distance now if you look at what is happened as a result is that because this much of capacitive component has cancelled here

so this triangle that i have got has really come up to this point now which is your iq minus iq prime whatever this i can talk about now if you now complete this rectangle here then you notice that my direction of apparent power is going to be like this and the angle that it will make will be theta such that theta is less than phi which of course means that cosine of theta is greater than cosine of phi this is as we have talked about is done by introducing a capacitive element

so this is the way it works

so supposing this is the voltage and i had a situation of this type that i had resistance r and an inductance here and

so this is my i let me call it irl because it's passing through both r and l

so what is done is to introduce a capacitive element in parallel with it and

so therefore what basically happens is it divides the current this was resistance r and this is f let us illustrate this with some numerical examples

so that it becomes slightly clearer suppose i have a 250 volts 60 hertz source many of these numbers i take because the calculations become easier as you know that 250 volts is not a household supply or things like that but it doesn't matter and suppose this source supplies 1.
5 kilowatt power now notice one thing when we say that the source supplies 1.
5 kilowatt power what we are talking about is the rate at which the energy is being supplied to the circuit by the source

so this is not dependent upon the properties of the circuit but it is dependent upon the source itself

so this is the rate at which we are supplying uh power to the source now and and this is usually measured by something which is known as a watermeter but we will not get into this but suppose we find we find that the rms current draw remember that we had already pointed out that this voltage is given are usually rms voltages

so rms current drawn by the load this is as observed by the reading of an ammeter is found to be 10 amperes these are data given

so let me calculate a few things

so first thing that i want to calculate is the power factor

so notice one thing that we have been given that the true power which is as i said is the amount of power supplied by the source this is equal to 1.
5 kilowatts now i know how much is the apparent power the apparent power is known to me because i know what is the voltage of the source and i know the current that the load is drawing

so this is 250 multiplied by 10 amperes and that is equal to 2.
5 now it is not written in kilowatt but it will be written in kilo volt amperes now if you now look at the power triangle see what i have is this that you have at 1.
5 kilowatt of true power and you have a 2.
5 kva along the hypotenuse

so this is 2.
5

so this is your reactive power

so this is the reactive power this is kilowatt and that's equal to square root of 2.

5 square minus 1.
5 square and that simply is 2 now 2 kilo volt ampere reactive these are standard notations used by electrical engineers okay

so this seed angle is fine

so therefore my power factor which is cosine of phi works out to 1.
5 divided by 2.
5 and that's simply equal to 0.
6 not a very low power factor but not very large either now suppose i want to compensate for this what would i like to do ideally though it may not always be possible ideally i would like to make this cosine 5 as close to 1 as possible now what it implies is this that you must compensate the reactive power component and we have already seen that you can do that by introducing a capacitor capacitive element is

so the reactive power is 2 into 10 to the power 3 and that must be equal to the voltage square divided by xc and that is 250 square divided by xc now you can immediately check how much is xc

so xc is 250 square divided by 2000 there and that if you work out it works out to 31.
25 what is the corresponding capacitor the capacitance that i need c that's equal to 1 over x c omega

so which is equal to 1 over 31.
25 multiplied by remember i told you that it's a 60 hertz supply

so it is 16 to 2 pi if you work it out this works out to 84 micro farad remember micro is 10 to minus 6

so 84 micro farad okay now let us suppose let us suppose

so this is my second thing for full compensation what do we need for full compensation

so i have seen that i need something like 84 micro farad and let us suppose i have with me a 80 micro farad

so let me see how much of correction i have made

so my circuit now becomes 250 volts 60 hertz i had a load here which i need not worry about right now and i decided to put in a capacitor which is 80 microfarad now the capacitive reactance corresponding to 80 micro farad you can easily calculate its 1 over omega c c is 18 to 10 to the power minus 6

so put those numbers omega is 2 pi into 60 you will get 33.
15 ohms remember it's slightly more than this

so the current that is being drawn is 250 divided by 33.
15 this is the capacitive current that is being drawn which is 7.
54 if you multiply this with v i times v

so this is 250 multiplied by 7.
54 and if you work it out it works out to 1.
885 kilo volt ampere react

so the reactive power is reduced by 1.
885 kilo volt ampere reactor now that is it becomes actually the new reactive power 2 minus 1.
885 that's equal to 0.
115 kilovolt ampere reactive and that will make the apparent power as equal to square root of 0.
115 square plus 1.
5 square and that if you calculate works out to 1.
5044 but since this is an apparent power the unit is kilo volt ampere and this is very close to 1.
5 kva as was the true power

so let me give another example i have a 230 volts 50 hertz supply this is what

is supplied to a load and this results in 280 kilo volt ampere reactive problem is the following that if the power factor is given to be 0.

86 we need to find the value of capacitance which will fully compensate this lagging phase now in order to look at the solution of this problem let us look at the power triangle that i have my true power here and this is my reactive power let us call this according to our previous notation vp and vq and this vq is given to be 280 kv ar and this is my completion of my power rectangle or triangle and this angle is 5 which is given to me since power factor is cosine of phi

so cos phi is approximately equal to root 3 by 2

so my phi is of course 30 degrees and this is my apparent power let's call it vr

so my apparent power vr is vq divided by sine phi and since cos phi is root 3 by 2 sine phi is half

so this is equal to your 280 which is what is given for v vq divided by half which is equal to 560 remember the units its kilo volt amperes this is the apparent power the principle of compensation is to make this apparent power as close to the real power as possible but how much is real power the real power vp real or active power vp is equal to vr cosine phi and cosine phi is of course root 3 by 2.

so if you compute this this is 484 0.

4 and this of course is proper since it's a true power it's in kilowatt now what does power factor of 1 imply power factor of 1 implies that the apparent power should also be equal to 484.

4 kilovolt ampere let's call it vr prime equal to 484 0.

4 kva now clearly this implies that my compensation should just be such that the capacity of reactance gives me a reactive power which is obviously in the reverse direction and that v q prime must be equal to 280 kilovolt ampere reactor and let us look at what this implies we are saying then v q prime for compensation for full compensation that must be equal to 280 kilo volt ampere let me put that units properly now 10 to the power of 3 and that must be equal to omega c times v rms square

so this is equal to omega corresponding to 50 hertz and that is equal to 314.

16

so this is 314.

16 c and this is vrms square which is 230 square you compute the product on the right it is 1.

66 into 10 to the power 7 times the capacitance

so capacitance is immediately found as 280 into 10 to the power 3 divided by 1.

66 into 10 to the power 7 farad of course and this is equal to 16.

86 millifarad

so what we have done in this lecture is to have a close look at the lcr circuit in particular the power factor the which decides how much of loss is going to be there because of reactive elements and this is very important because as i pointed out right in the beginning that if you look at power transmission the losses which are popularly known as copper losses in electrical engineering is given by i square times the resistance of the cables

so if this i is more then of course the losses will be long and a low power factor implies that the current drawn by the load is more and

so what we find is that there is a true power which is being supplied by the source which is nothing but the rate at which the energy is being supplied to the circuit and there is an apparent power and the two things are out of phase and the if you complete the power triangle which i explained graphically then

there are reactive powers which come into the picture what we need to do in
order that the amount of loss is minimized in transmission is to somehow rather
compensate for the losses that are due to the reactive you