

welcome back let me begin by giving a summary of what we did last time we were discussing lcr circuit and we provided an analytical solution for the differential equation this was done by converting the loop law of kirchoff which for an lcr circuit would be $l \frac{di}{dt}$ which is the back emf due to the inductance plus i times r which is the current drop on the resistance and the capacitance voltage drop that's q by c that's equal to the applied voltage $v_m \sin \omega t$ now this equation can be converted to a second order equation either in the current or in charge equation and the current is obtained by differentiating this equation once more which is what we did and if you wanted instead you could have realized that the current i is dq by dt

so therefore this equation is essentially equivalent to a second order differential equation in charge also but what we did is to differentiate this equation once more and obtained $l \frac{d^2 i}{dt^2} + r \frac{di}{dt} + \frac{1}{c} i = v_m \cos \omega t$ we assumed that the solution for the current i as a function of time is given by $i_m \sin(\omega t + \phi)$ where ϕ is the amount of phase by which current leads the voltage and we obtained that i_m is given by maximum v_m divided by z where z is the impedance which is given by square root of $r^2 + (x_c - x_l)^2$ and tangent of ϕ is $x_c - x_l$ divided by r

so you realize that the angle ϕ would be positive meaning thereby the current will be leading the voltage if x_c is greater than x_l that is the circuit is dominantly capacitive alternatively if x_l is greater than x_c ϕ will be negative and that is the case when the circuit is dominantly inductive and x_c is of course $1/\omega c$ and x_l is ωl now notice this that looking at the expression for this current with z being here the expression for z is this we notice that the maximum current in the circuit depends upon the frequency of the applied voltage

so maximum current i_m has a frequency now this maximum for a given circuit if i_m is allowed to vary the ω the frequency will have a peak

so i_m will be saying i_m becomes has a maximum value as a function of ω there are two types of maxima i_m am talking about

so there should be no confusion firstly in a given circuit meaning thereby resistance capacitance and inductance fixed and frequency also fixed i_m have a maximum value of the current now what i_m am asking is this for the same circuit if i_m is allowed to vary the frequency impressed that is the frequency with which the voltage is changing then this maximum will vary also and i_m will have a maximum value of this maximum

so and that happens

so let me write down again the function expression for i_m

so i_m was v_m divided by z which is square root of $r^2 + (x_c - x_l)^2$ whole square

so i_m has a maximum as a function of ω when x_c becomes equal to x_l now this corresponds to ωl becoming equal to $1/\omega c$ that is ω is equal to let's call it ω_0 equal to $1/\sqrt{lc}$ and this is the condition of resonance

so when the frequency of the voltage is equal to the resonant frequency of the circuit the value of i_m itself has a maximum as a function of ω and that is simply given by v_m divided by r it's a good idea for us to look at how does impedance vary with frequency for the various types of circuit we have discussed so far

so let us try to plot z versus ω the simplest of course is the resistive circuit and we know that the resistive impedance which z is equal to r it doesn't have any ω dependence

so therefore for a resistive circuit i_m simply get this

so this is simply equal to r for an inductive circuit the impedance which is inductive reactance which is equal to l times ω

so it increases linearly with increasing frequency

so therefore this is for inductors $l\omega$ is the z the capacitive circuit gives a different type of variation because the capacitive reactance is $1/\omega c$

so therefore what you get is something like this this is capacitive which is $1/\omega$ if you look at a lcr circuit in general then the behavior that you get will have a minimum at a particular value of ω which is the resonant frequency

so this is what you get for the lcr circuit in general and this is ω equal to ω_0 .

now what happens is this that when ω is equal to ω_0 the circuit absorbs maximum power

so let me write down at ω equal to ω_0 that is the resonant frequency this circuit absorbs maximum power now you can see why we have seen that i is proportional to v/z or rather i_m is proportional to $1/z$ for a given v v_m now the power then which is $i_m^2 z$ is proportional to $1/z$ also because i_m^2 is proportional to $1/z^2$ and

so i have got this

so maximum power implies

so maximum power occurs when z is minimum and minimum z happens when ω equal to ω_0 equal to $1/\sqrt{lc}$ that is the reactive components cancel having done this we defined what are known as half power points

so we said that if you are plotting this i_m that i have talked about as a function of ω then the curve that you get has a peak at ω equal to ω_0 as you have pointed out

so this is basically the type of curve that you get with the peak being at ω_0 now the power absorbed is half of possible maximum when the pair of values on either side of ω_0 happens to be

so this is this this value of the current is $i_{m\max}$ now when i look at a pair of points for which i_m is equal to $i_{m\max}$ by square root of 2

so this is $i_{m\max}$ by root 2 about 70 percent of the maximum

so there the power absorbed is half of the possible maximum

so therefore i equal to i_m by square root of 2

so let us say i_m is equal to $i_{m\max}$ by square root of 2 the power absorbed is half the possible value

so if you look at the supposing this value we call it ω_2 and this i call it ω_1 let us say then ω_1 minus ω_2 that is the width of the curve at half maximum

so full width at half maximum

so this is i represent it as $2\Delta\omega$

so this mark from here to there is 2 times $\Delta\omega$ this is known as the bandwidth

so this is called the bandwidth in fact there is no very specific symbol for that we simply write it as bw remember that the curve is drawn for the current maximum against the frequency ω but when we talk about f_{whm} that is full width at half maximum we refer to the distance between the frequencies where the power becomes half the maximum possible power

so therefore a smaller bandwidth means a sharper result the curve becomes sharper width becomes smaller if the bandwidth is smaller now you can we calculated and found that $\Delta\omega$ is equal to $r/2l$

so that the full width that half maximum is simply r/l then we defined

something called a quality factor let us write down bandwidth $\Delta\omega$ is equal to 2 $\Delta\omega$ which is simply equal to r by l we defined something called a quality factor represented by q which is defined as ω_0 that is the resonant frequency divided by the bandwidth and that obviously is ω_0 divided by r so therefore the quality factor is also another measure of the sharpness of resonance and in fact i pointed out that when you tune in radio station you will find that at the resonant frequency you will find the maximum reception and this for circuit application

so typical circuit application the value of q is between ten to hundred so these were some of the things that we discussed last time i will illustrate these points using some examples

so let me first start with an example of an lcr circuit

so an lcr circuit has the following parameters r is equal to 5 ohms c is equal to 20 micro farad and l is equal to 200 milli hundred

so first let us calculate the resonant frequency that's a very simple job the only thing that you have to remember is that usually the capacitances are given as micro farads that is 10 to the power minus 6 farads whereas the inductances are given as millihenry which is 10 to the power minus 3

so therefore just take care of those factors

so my ω_0 which is equal to 1 over square root of lc

so that's equal to 1 over 200 nearly hundred means 2 into 10 to the power minus 1 and 20 micro farad is 2 into 10 to the power minus 5

so that's equal to simply this is 10 to minus 6

so in the numerator i have 1000 divided by 2 which is equal to 500 radians per second which corresponds to approximately a linear frequency f of about 80 hertz you can calculate it but it's probably slightly less than 80 hertz now next what is the value of ω for which half power maxima occurs value of ω for which half hour maximum occurs we have already seen that that value corresponds to the situation when i is i_{max} by square root of 2 actually i is i_{max} square root of 2 happens now if you look at the expression for the current maximum which is here and i want this i to be equal to i_{max} by 2 .

now i_{max} is V_m by r

so obviously what i require is this part $X_C - X_L$ should be equal to r

so this implies that the reactance part namely $\omega L - 1/\omega C$ whole square is equal to r square let's find out what the solution of this equation is

so taking the square root what we get is $\omega L - 1/\omega C$ is equal to plus or minus r we can easily convert this to a quadratic by multiplying throughout by ωC

so that i get $\omega^2 LC - 1$ equal to rC or plus rC

so that my ω becomes plus or minus rC there should be another plus or minus but let me write it as a plus and we'll say see why square root of $r^2 C^2$ square plus $4LC$ divided by $2LC$ the reason for taking only the positive square root is

so that this quantity remains bigger than this side

so that even with that minus sign i get my number to be positive

so if you solve this by substituting these values you get ω to be either 512 .

5 radian per second or 487 .

5 radian per second which is a spread of plus or minus 12 .

5 radians per second on either side of the resonance

so that my bandwidth which is simply equal to 2 times $\Delta\omega$ that's equal to 25 radians per second of course you could simply obtain it directly by application of the formula for the bandwidth

so that's equal to $2 \Delta \omega$ which you have shown to be equal to r by l and that's equal to 5 divided by 2 into 10 to the power minus 1 and that's as expected equal to 25 radians per second the quality factor q for this circuit is $\omega \theta$ divided by $2 \Delta \omega$ and that's equal to 500 divided by 25 which is equal to 20 .

i'll give another couple of examples which are some interesting applications and that is if you take a combination of r and l in an alternating current circuit you can use the combination properly to get what is known as a high pass filter or a low pass filter

so let me explain what it actually mean

so example of a high pass filter this uses rl we will see later that i can also construct a circuit which acts as a low pass filter but let us look at this situation

so basically the way it works is this that if you look at a circuit like this there's a resistance here let's say r and there is an inductance here l this is $v_m \sin \omega t$

so let us measure the voltage which is output between these two points

so this i will call as a v in input now notice one thing if this supply was in dc then we have seen that for dc supplies the inductance simply conducts now in which case the current will pass through the l like a short and my v output will be zero

so for dc supply l conducts and v output is equal to zero let's see what happens now that as you increase $\omega \times l$ will increase as ω increases ωl which is ω times l will increase this would imply there is a voltage drop across l okay

so this means that as frequency increases the voltage drop across these two points they will increase and this will result in what we call as a high pass filter

so let me give you a specific example to illustrate this point

so the circuit that i have is this

so let me call this input as v in that's equal to $10 \sin \omega t$ at this moment i am not giving you what the value of ω is for reasons to follow

so i have a resistance which is 40 ohms then the circuit has a 200 milli henry inductance and another resistance here often now the output voltage which is the voltage which is across these two parts this what let me call this as a v out now notice that in this case the time dependence of v out is obviously the same as that of v in that is $\sin \omega t$ because i am taking the voltage across these two points and suppose we are looking for ω such that v out divided by v in is equal to half now remember that the trigonometric metric terms cancel out

so therefore what we are looking for is v out equal to since this is 10 we are looking for five $\sin \omega t$

so let's look at how to compute this first is what is the input impedance which will determine how much of current is passenger now since the circuit has series resistance which is 50 40 plus 10

so i get 50^2 square plus l which is θ .

2 henry

so i get θ .

2 ω square

so that is θ .

04 ω square now the output impedance is simply given by this part of the circuit

so which is again but now only the 10 ohm resistance comes in

so i get 10^2 square plus the same θ .

04 omegas

so my current maximum in the circuit which is determined by Z_n is I_m which is 10 divided by Z in which is 10 divided by square root of 50 square plus 0 .

04 omegas now V_{out} maximum because we have pointed out all already that the time variation remains the same is I_m times Z_{out}

so if you plug it in there

so I_m is 10 divided by square root of 50 square plus 0 .

04 omega square multiplied by 10 square plus 0 .

04 omega square

so we can write this as equal to V_m in which is the strain multiplied by let's just write this as 10 square plus X_L square divided by square root of 50 square plus X now I am looking looking at V_{out} ratio V_{out} divided by V_{in} to be equal to half

so by substituting this I can immediately find out what the value of X_L X_L s

so this X_L square turns out to be 700 and that's equal to omega square into 0 .

0 if you simply solve this equation you get omega to be given by 132 radians per second which corresponds to a linear frequency of 21 hertz

so what we have said is this that when we have an inductance and a resistance in the circuit for dc the inductance works like a resistanceless wire and it passes the current without dropping a voltage whatever drop is there that occurs only across the resistors as the frequency increases because the reactance increases there is a drop across the inductance which is what we can tap

so this amount of voltage that drops across the inductance increases as we increase the frequency of supply and hence this is an example of what is called a high pass filter the basic idea behind this was the following that if I have a resistance r and an inductor cell the current as we know is given by V_{in} divided by square root of r square plus L square omega square this is just the current amplitude

so this is V_{in} maximum and

so therefore V_{out} is simply given by V_{in} maximum divided by square root of r square plus L square omega square into L omega and if my L omega becomes large this gives me V_{in} maximum L omega divided by let's expand this binomial you get omega L into 1 plus r square by L square omega square to the power half and that's approximately equal to

so L omega and L omega will cancel out you will be left with V_{in} maximum into 1 minus half r square by L square omega square

so you can see immediately as omega increases this term goes on decreasing

so as omega increases this term will become smaller and smaller and V_{out} will approach V_{in} but as omega goes to 0 then of course this expansion is not right but I have directly use it here then V_{out} becomes equal to zero

so this is the principle of a high pass filter we can use the same circuit with a slight modification as a low pass filter and let us see how it works

so let us use the same a supply I have an inductance whose reactance is L omega there is a resistance r and that's all that is there in the circuit but this time I take the volt output voltage across the resistance r now by the same principle the V_{out} is V_{in} now this time r divided by the impedance which is r square plus L square omega square and you see what happens in this case if my L omega is large

so I have V_{in} r divided by L omega into 1 plus r square by L square omega square to the power half for large omega V_{out} decreases

so for large omega V_{out} decreases and if omega is equal to 0 you have to come back to this because no expansion is possible

so if omega equal to 0 then of course I have square root of r square which is r I get V_{out} omega out is equal to V_{in} for omega it going to 0 V_{out} goes to V_{in} when

we have an inductance and the resistance in the circuit we know that the inductance conducts for V_C remember that a capacitor is an open circuit now as frequency increases X_L increases because remember X_L is nothing but ω times it

so therefore the drop across the inductance also increases and since in our circuit this drop precedes the resistance with increasing ω the drop across the resistance will decrease hence this is an example of a low pass filter let's look at this circuit again

so i have V in equal to $10 \sin \omega t$ and i have a 200 milli henry here and a resistance in series which i take to be one kilo and i'm looking at what is the drop across

so what is this is V out we have pointed out that the time dependence remains the same between V and V out

so let's look at what do i get

so my i_m which is V in maximum divided by Z which is 10 divided by square root of since it's 1 kilo ohm i get 10 to the power 6 plus $0.04 \omega^2$ as before and V out then which is maximum of course gives me 10

into 10 to the power 3 divided by square root of 10 to the power 6 plus $0.04 \omega^2$ and if i just want V out then it's obtained by multiplying with

$\sin \omega$ now let me take ω equal to 500 radians per second which corresponds to a linear frequency of about 80.

now we compute using this formula by simply substituting for this ω you find V out maximum works out to 9.

95 volts now let me make ω 10 times larger

so 5000 radians you can substitute the same values there and you find that the V out naturally your ω is increasing

so therefore and ω being in the denominator the value of V out is decreasing and in this case it's once works out to 7.

07 volts make it 10 times bigger

so if you take ω equal to 50 000 radians this is all radians per second

so if you compute Z V out you get this to be 0.

995 volts

so notice that as frequency increases the output voltage becomes smaller and smaller alternatively as frequency decreases the output voltage becomes bigger and bigger therefore the circuit that i have shown here is what is known as a low pass filter having discussed some application of lcr circuit let me now switch over to a different topic that is how much of power is absorbed in an ac circuit now before i work this out i would like you to understand one thing that in an lcr circuit the only circuit element that dissipates power is the resistance both the capacitance and the inductance they do not dissipate powers even though you may have the their reactance is written as in units of ohms

so let us look at what happens and try to get some ideas about this firstly we know that V of t is given by $V_m \sin \omega t$ this is my starting voltage the corresponding i of t we have seen is given by $i_m \sin \omega t + \phi$ where i_m is simply V_m divided by Z which is the impedance and ϕ is a phase by which the current leads the voltage is given by $\tan^{-1} \frac{X_C - X_L}{R}$

so my instantaneous power P of t is given by i of t into V of t simply multiply these two terms i get $V_m i_m \sin \omega t \sin \omega t + \phi$ now expand $\sin \omega t \sin \omega t + \phi$ and multiply through

so $\sin \omega t \cos \phi + \cos \omega t \sin \phi$

so i get two terms $\sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi$ now if i take the average power P of t i notice that the first term has a $\sin^2 \omega t$ and we have seen that the time average of $\sin^2 \omega t$ is

half the second term is $\sin \omega t$ into $\cos \omega t$ which is also equal to half $\sin 2\omega t$ and we have pointed out that quantities like $\sin 2\omega t$, $\sin 3\omega t$ etcetera they all become 0

so in other words when I do the average the only term that contributes to this average is this term and

so therefore this is $V_m I_m$ divided by 2 into $\cos \phi$ there are many alternate ways in which you can write it

so for example writing that V_m is equal to $I_m Z$ I can write it as $I_m^2 Z$ divided by 2 $\cos \phi$ also V_m^2 divided by $2Z$ into $\cos \phi$ if you express I in terms of V_m now you notice that in the average power there is a product factor coming in which is $\cos \phi$ and this $\cos \phi$ factor is known as a power factor

so my power factor is related to this by the average power is given by $I_m^2 Z$ by 2 into $\cos \phi$ we had seen that $\tan \phi$ was X_C minus X_L divided by R which gives me $\cos \phi$ is equal to R divided by square root of R^2 plus X_C minus X_L whole square which is nothing but R divided by that

so therefore the expression for average power is $I_m^2 Z$ by 2 into R by Z which as you can see is I_m^2 into R divided by 2 since I know that I_m by square root of 2 is the rms current I can rewrite this as I_{rms}^2 times R let us look at what is its variation like

so this is nothing but V_{rms}^2 divided by R^2 plus X_L minus X_C whole square multiplied by

so if you plot this average power as a function of the frequency where does the frequency dependence come in it comes in in X_L and X_C what you get is this that the power absorbed will be maximum when X_L is equal to X_C but you also remember that X_L equal to X_C is also the condition of resonance

so therefore what we get is this that average power is maximum at X_L equal to X_C and if you are plotting this as a function of ω you get the formula a typical curve would look like this this is for let us say some R_1 and if you were to increase the value of R it will become more flat and it will become like this the peak is still here at ω equal to ω_0

so we have seen that P_{avg} is maximum at X_L equal to X_C in which case it has a value V_{rms}^2 divided by R

so let us look at certain properties here

so firstly suppose I had a purely resistive circuit now remember I said that $\cos \phi$ is equal to R by Z resistive circuit simply means Z is equal to R so which is equal to one

so in which case ϕ is equal to zero and the power dissipation is at its maximum and as R increases the peak power decreases

so we have seen that for purely capacitive or inductive circuits the phase is plus or minus $\pi/2$ plus for capacitive circuit minus for inductive circuits that gives me $\cos \phi$ in either case to be equal to zero zero power means that no power is dissipated such circuits are also known as wattless circuits if you now have an LCR circuit we have seen that tangent of ϕ is X_C minus X_L by R and ϕ in general is not equal to 0 or $\pi/2$ and this means that the current may lead the voltage or lag as the case may be but even here the dissipation is only through the resistance and finally if I have a circuit at resonance the reactive and the inductive reactances cancel and we again get ϕ is equal to zero very similar to what we got for purely resistive circuit and once again maximum power is dissipated and of course needless to say through resistance only of course we should emphasize that the inductive and the capacitive elements that we have considered here do not dissipate power because of our assumptions that they are resistanceless in practice the inductive elements will always have some amount of resistance and there would be some leakage from the

capacitor plates and

so therefore even in an ideal LC circuit which we will see later that they sustain oscillation the oscillations will gradually die down because of such small currents and charge location now what is the connection remember even when we plotted i_m versus ω we had used what are known as half power maximum point by pointing out that the current maximum was seventy percent actually one over square root of two times the maximum possible value of the current maximum but this time I have directly calculated how much is the power and let us see how the power dissipation curve looks like

so let me let me return back to this power curve again

so what we said is this that this is a different resistance r_2 which is greater than r_1 now let me look at what happens to the curve instead of comparing just look at a fixed resistance and look at this power p

so the power curve was like this and we have seen that the peak here is at ω equal to ω_0 and the expression for average power was v_{rms}^2 square r divided by z actually z^2

so r^2 plus x_L minus x_C whole square now clearly this average power becomes a maximum when the denominator is minimum which of course happens at resonance when x_L is equal to x_C

so let me say p_{max} as a function of ω is simply v_{rms}^2 divided by r when x_L equal to x_C meaning thereby ω equal to ω_0 now let us ask the question where in this curve the power average is half now remember in my current expression it was 70 but I am not looking at that

so this is the power max average of course I am looking at half of it half the side

so this is the point I am looking at p_{max} by 2 .

okay this is the measure of the half power maximum the full width at half power so let's see how much is that

so if you look at this expression the power would be half its maximum value remember how the maximum value occurs when this this part excel minus x is equal to zero

so I want my power to be equal to v_{rms}^2 divided by $2r$ now this implies that this quantity square is r^2

so let's look at that

so we say x_L minus x_C square is equal to r^2 we get the solution of the quadratic equation to be plus or minus r by $2l$ plus or minus 1 over 2 square root of r^2 over l^2 plus $4\omega_0^2$ now I have to choose this properly because if I choose the negative sign this term will dominate because its magnitude is greater than r by $2l$

so I have to choose the positive sign only and with that I get ω_1 equal to r over $2l$ plus 1 by 2 root of r^2 over l^2 plus $4\omega_0^2$ and the lower frequency is minus r by $2l$ plus again 1 over r^2 over l^2 plus $4\omega_0^2$

so therefore $2\Delta\omega$ is ω_1 minus ω_2 which is simply equal to r over l and the corresponding quality factor is ω_0 divided by $2\Delta\omega$ and that's equal to $\omega_0 l$ over r

so therefore we have now given an interpretation of the quality factor in terms of the width of the power curve at half maximum you