

welcome back in the last lecture we considered an ac circuit with purely capacitive load in the lecture before that we had talked about a circuit with a purely inductive load

so let us quickly summarize what we have learned

so far

so what we found is this if you take a purely resistive load then you find that the current is in phase with the applied voltage

so the direction of the current in more complicated circuit becomes the reference direction with respect to which we talk about whether something leads or something lags we'll see that when we take towards the end of today's lecture more complicated combinations of resistors inductors and capacitors what we found is that for an inductive load

so that is a circuit consisting of an inductance and a voltage we found that the current lags behind the applied voltage what is meant by current lagging behind is that if you look at a trigonometric variation let's suppose a sine function or a cosine function for the voltage then the corresponding expression for the current will be the same trigonometric function but with a phase which will lag mean be negative

so something like if voltage is varying as cosine or sine of ωt then the current will go as cosine or sine of $\omega t - \phi$ the reverse situation happens for a capacitive load and here the current leads the voltage in other words the phase of the current is ahead of the phase of the voltage and

so therefore if you are looking at a purely capacitive circuit the current would become maximum before the voltage does

so this thing about what leads what lags it confuses people quite a bit

so the electrical engineers have a mnemonics for that and that is written as ELI the iceman now this tells you that for an inductive circuit which is what this ELI stands for the emf that is the voltage that leads the current

so E is for voltage and I is for current and for the capacitive circuit the current that is given by I lags the emf or the voltage given by E

so what comes first is what is seen in this mnemonics in both these or all the three circuits the ratio of the voltage max to current max is given as follows

so if I have a purely resistive circuit my current maximum which I represent by I_m is given by V_m divided by R R is of course the resistance for an inductive load we define an inductive reactance represented by X_L which is equal to ω times L and this in terms of this my current is given by voltage maximum divided by X_L and for a capacitive load we define a capacitive reactance by X_C which is equal to $1/\omega C$ and once again I have my I_m as equal to V_m divided by X_C

so this is here it is equal to V_m divided by ωL and here it is V_m times ωC you realize that there is a difference in the way the frequency or the angular frequency ω comes into these expressions in all these cases the trigonometric variations that we talk about are the following let v of t be $V_m \sin \omega t$ and let me take the current i of t as $I_m \sin \omega t + \phi$ here ϕ is the amount by which the current leads the voltage ah in this with this notation if I have a purely resistive circuit then of course ϕ is equal to 0 which implies that current and voltages are in phase for an inductive circuit as we have seen current lags the voltage by $\pi/2$ in which case ϕ must be equal to minus $\pi/2$ because I have taken the current expression to be $\omega t + \phi$ and for capacitive circuits the current leads the voltage by $\pi/2$

so therefore ϕ will be equal to $\pi/2$.

I have mentioned earlier though we will not really be using it when it comes to complicated circuits it becomes clumsy to deal with different types of trigonometric variations for voltage current and things like that

so in electrical engineering what is done for making the algebra simple is to use the exponential form this is just a short introduction i am giving in next five minutes if you find it a little difficult you just ignore it because i will not be really using it as i go ahead

so what is done is supposing we take v of t instead of $v_m \sin \omega t$ i take $v_m e^{i \omega t}$ to the power i now remember that the exponential $e^{i \omega t}$ is $\cos \omega t$ plus $i \sin \omega t$ as you notice that this really doesn't represent a physical situation but mathematically the function that you want which is v of $t \sin \omega t$ is nothing but the imaginary part of this function similarly if you wanted v of t to be $v_m \cos \omega t$ then it would be represented by the real part of this function the reason for taking $v_m e^{i \omega t}$ instead of the cosine of the sign mathematically exponential functions are much easier to deal with than trigonometric function and in that case what we'll do is we'll carry on with the calculations assuming the exponential functions and then of course we'll at the end we'll say we have need to take the real part or the imaginary part as the case may be now if you do that then the corresponding i of t would be given by i n $e^{i \omega t}$ plus 5

so once again if we had taken v of t equal to $v_m \sin \omega t$ the corresponding current would be given by the imaginary part of it now notice the simplification in the algebra the complex impedance which should now be defined as v of t by i of t is given by v_m by $i m$ into $e^{i \omega t}$ because of the way we have taken this and we had seen that for resistive circuits ϕ was equal to zero which tells me that z is nothing but v_m by $i m$ for resistive circuit is just equal to i for an inductive circuit ϕ was equal to minus π by 2

so and we have seen that z is then v_m by $i m$ which is ωl times $e^{i \omega t}$ minus $i \pi$ by 2 well minus but ϕ itself is minus

π by 2 and that's equal to $i \omega l$ likewise for a capacitive circuit we have seen that v_m by $i m$ is 1 over ωc but this times it is $e^{i \omega t}$ minus $i \pi$ by 2

so therefore this quantity is $e^{i \omega t}$ minus $i \pi$ by 2

so this is just equal to minus i 1 over ωc alternatively it's also written as 1 over $i \omega c$

so if i have r l and c all of them in series in a circuit then i define my complex impedance as z equal to r plus i times ωl minus 1 over ωc and as per our notation for the complex reactance

so this is plus i times x_l minus x_c now you can see that this gives modulus of z equal to square root of r^2 plus x_l^2 minus x_c^2 whole square which is the same as x_c^2 minus x_l^2 whole square and this is what we have been using all throughout the phase of the complex z uh which is ϕ is given by tangent of ϕ equal to x_c minus x_l divided by r now these relations can be shown in an impedance diagram which looks like this that it's a right-handed triangle with one side being modulus of x_c minus x_l this being resistance r and naturally the hypotenuse is modulus of z as we have shown here let me look at what happened to power in such circuits now what we said is that when we had a purely resistive situation then the average rate at which it dissipates power average power

so average for a resistive circuit was $i_{rms}^2 r$ by 2 this came because i^2 square r is the instantaneous dissipation of power and if you take the form of current to be a sine function then the i^2 will have a sine square and over a period thus we have seen that the sine square or the cosine square function gives me the factor of half

so what is done is in order to make this formula look similar to the way it looks for dc circuit we defined what is known as an rms current we could also define rms voltage that way

so rms current was simply i_n divided by square root of 2 which we represent as i_{rms} with this this formula becomes identical to $i_{rms}^2 R$ which resembles the form $i^2 R$ for a dc circuit now this is the only element which actually dissipates power both capacitive and inductive circuits absorb energy in one part of the cycle and return the same to the source in another point

so the average power for both inductors and capacitors is equal to zero so with this let me now go over to a discussion of what happens to an lcr circuit when an alternating voltage is applied

so let us draw this is my voltage which as before i will take it as $v_m \sin \omega t$ i have the resistance R an inductance L and a capacitance C so we will be interested in the next part of the lecture in discussing the properties of an lcr circuit in the presence of an alternating voltage

so look at this i use still the same ketchup's law and i said that whatever voltage is supplied by the source is dropped through R which i know is iR through L which i know is $L \frac{di}{dt}$ and through the capacitor which i know is $\frac{q}{C}$ by C

so therefore my kirchhoff's law tells me that $v_t - v_R - v_L - v_C$ is the drop across the resistor minus v_L the drop across the inductance minus v_C is equal to 0 alternatively my v of t is equal to $iR + L \frac{di}{dt} + \frac{q}{C}$ which is the back emf expression that you remember and plus $\frac{q}{C}$ we will return to a formal solution of this problem little later in this lecture but let us look at what statements i can make about this circuit let's look at this situation and let's see what statements i can make about it now one thing you realize is this that since this element R L and C they are in series this is series lcr circuit we may have different other form of lcr

so let me also write down here series here circuit

so since they're in series there can be a unique current through this entire thing

so the current must be unique through the three elements in other words the current that we are talking about should also have a fixed magnitude and a fixed phase difference with respect to this ωt

so let me take for the current in the circuit i to be equal to $i_n \sin \omega t + \phi$ i have not made any statement on what ϕ is for the simple reason i have three components behaving in three different ways in my circuits when they were acting alone for resistor the ϕ was zero for inductance it was negative for capacitance it was positive

so at this moment all that i have said is the ϕ is unique now source voltage is v equal to $v_m \sin \omega t$ these are the two things that we know now what i will do is this i will first try to solve this problem or try to understand the implications of what we have said in a graphical manner

so let us do that we will do the formal analysis little later but we will see that a lot of it can be done by application of the graphical technique

so as before i take the x axis as my reference line

so this is t equal to 0 reference and what we have said is this that at a time t because this phasor is rotating with an angular velocity ω

so therefore at time t the phasor for the voltage which was initially aligned along the t equal to 0 the x axis points in a direction which makes an angle with the x axis the angle ωt with the x axis

so let us draw this this is ωt now what we have taken is that the current to lead the voltage by an amount ϕ

so therefore in this picture my current would be let me use the slightly different color my current would be along this direction

so this angle is fine and this is of course v_m and this is because we have

taken i to be given by $i_m \sin(\omega t + \phi)$ now what we want to do is this we will try to or we will be drawing the voltage phasors across the three elements namely the resistance the capacitance and the inductance now remember that the v_r that is the resistance record between the the voltage across the resistance is along the current direction because we have seen that a resistive circuit is in phase with current

so therefore $i_m \times r$

so let me generally take it here that will help me in completing this

so this end of this red arrow is my $i_m \times r$ now since i know that the inductive voltage it leads the current remember that in case of an inductor the current lags which is another way of saying the inductive voltage leads the current $\pi/2$

so this would be the direction in which the inductive voltage

so this is v_l and correspondingly the capacitive voltage would be in the reverse direction

so this is v_c since v_l and v_c are oppositely directed

so they will be along v_m where the capacitive reactance is larger than the inductive reactor

so you subtract the two and put it somewhere here now this v_m that we have drawn then if you complete this rectangle here a parallelogram here

so this amount from o to whatever let us say a

so o a magnitude is $x_c - x_l$ times current

so this tells me this graphical construction tells me that v_m^2 is v_r^2 resistance square plus v_c^2 minus v_l^2 square and that is equal to this is $i_m^2 \times r^2$ whole square and this is $i_m^2 \times (x_c - x_l)^2$ whole square and

so therefore my v_m is given by i_m times square root of r^2 plus $x_c - x_l$ whole square which is nothing but i_m times z where z is the quantity which is within the square root

so this is r^2 plus $x_c - x_l$ whole square now remember when i was discussing the complex nature of the impedance i had said z is $r + i(x_c - x_l)$ i am not really going to be doing it

so so far as i am concerned i am only interested in magnitude of that quantity and which obviously is r^2 plus $x_c - x_l$ whole square square root

so this is the impedance that i look at

so repeating once again the impedance consists of

so that is z it has a factor which is r and it has x_c and x_l in a vector diagram the resistance and the these reactances they are perpendicular to each other and x_c and x_l themselves are oppositely aligned in the vector diagram

so this is the reason why i repeat z is given by square root of r^2 plus $x_c - x_l$ whole square where x_c as before is $1/\omega C$ and x_l is ωL let me continue with my graphical analysis for little longer by giving you examples

so let me take consider an example numerical example

so let me consider lcr circuit supposing this is the 80 ohm resistance i have a θ .

1 henry inductance and a 25 microfarad capacitance the source is well i'll just give you the frequency of the source which i will conveniently take it as 400 radians per second ω is let me sort of point out that 400 radians per second i am taking purely for calculation of ease of 60 hertz which is reasonably common in us corresponds to 377 ohms that's 377 radian per second but 400 is close enough

so that we can take it as a reasonably physical number

so let us first make the following thing suppose i say that an rms current of two amperes is passing through the circuit now we need to first find out various

quantities and we would be interested in knowing that if this is the situation what is my voltage like the source voltage but before that let's calculate various things r is of course very simple that's given to me 80 ohms the let's calculate the reactances

so x_c is equal to $1/\omega C$ both x_c and x_l they have the dimensions which are the same as that of ohms resistance

so i have taken ω conveniently to be 400 this is 25 micro farad

so 25 into 10 to the power minus 6 that is 10 to the power minus 4 in the denominator and

so that takes me up there and that is 100 ohms and x_l which is just ωL ω is 400 L is 0.

1

so that's equal to 40 ohms let me find out what is the total impedance like

so impedance i repeat is $r^2 + x_c^2 - x_l^2$ whole square

so this is equal to 80 square plus 100 minus 40 that is 60 square

so that is just equal to 100 ohms

so therefore my rms voltage is given by i rms current multiplied by Z rms current is given to be 2 amperes Z is 100

so it is 200 volts rms peak will of course be square root of 2 times bigger but let us take this opportunity of finding out what are the individual voltage drops

so resistance drop is just i times r which is 2 into 80 that's equal to 160 volts but remember all these are rms voltage if you want peak you have to multiply by square root of 2 the capacitive voltage that is voltage drop across the capacitor is 2 that is the current times x_c we calculated the x_c to be 100

so this is equal to 200 volt rms and v_l that is the voltage drop across the inductor which is 2 times x_l and we calculated x_l to be 40

so therefore 40 into 2

so it is 80 volt now you can check that the source voltage rms in rms source voltage also satisfies the law of addition of vectors

so you can see that 200 square that's v_{rm} square that's given by v_r square which is 160 square plus v_c minus v_l

so 200 minus 80 square you can check this is 160 square this is 120 square and that exactly works out to 200 square let's look at the phase

so return back to the diagram the in doing these five types of problem you have to realize if i am plotting a voltage the statement that we have made that for an inductive circuit the current lags the voltage implies that for an inductive circuit voltage leads the current

so therefore when you draw you have to draw it keeping this in mind

so let's look at that

so let's suppose first i am trying to draw a vector diagram

so let me draw the current direction along the x-axis that's also the direction in which the resistance drop takes place

so this is let's take this is v_r and we just now calculated that my v_r was 160 volt rms

so this is 160.

now

so let me let me write it down here v_r is 160 these are all rms values v_l was 80.

and v_c was 200 volts

so let us let us drop it right here

so the v_l because this inductor it leads the corresponding voltage for the resistors i repeat again the voltage leads the current for an inductive circuit the current lags but voltage leads

so let us take the same scale and put some 80 here
so so this is my v_l and since v_c is 200 in terms of length it will be slightly bigger

so let us do that

so this is v_c which is 200 volts

so what we now do is this that we find out what is v_c minus v_l

so all that we need is to chop off here by an amount 80

so this is v_c minus v_l and if i am drawing the parallelogram here then
so this quantity is 200 minus 80 which is 160.

so 120

so this is 120 this is 160 and the result is obviously this

so this is my v_{max} for the source let me just put s there to indicate source
and we have seen that 120 square plus 160 square is 200 square

so therefore the length of this is 200 it is just accidental that this 200 and
that 200 happens to be the same number but look at this phase here this is the
amplifier

so notice the resultant voltage is lagging behind the current okay

so resulting voltage that is the supply voltage lags by 5 and how much is 5 you
can immediately calculate ϕ is $\tan \phi$ is 120 by 160 which is equal to 3 by 4
and if you look up your trigonometric tables you will find this is 37 degrees or
 θ .

64 radians

so this is the angle by which that this is the phase lag of the total voltage
with respect to the current or with respect to the resistive load now
incidentally this situation that the net supply voltage lags behind the current
happened because the capacitive reactance is larger than the inductive reactance

so as a result the this circuit is primarily or let's say predominantly a
capacitive circuit this circuit is predominantly capacitive in nature hence
voltage lags current and the reverse would be true which will also show you by
taking some other example if you had taken a situation where the inductive
reactance is bigger than the capacitive reactors now look at this what does it
actually mean what does this phase signify now this is telling you that the
there is a time lag between the time the current maxima occurs or the voltage
maximum occurs now look at what we have said we have said that my current is i_m
 $\omega t + 5$ and the voltage maximum occurs at ωt equal to π by 2
whereas current maximum occurs when $\omega t + \phi$ is equal to π by 2

so therefore there is a time lag

so time lag between current maximum and voltage max this is done by observing
that i_{max} becomes occurs when $\omega t + \phi$ is equal to π by 2 the voltage
max occurs when ωt is equal to π by 2 because it's just a sine ωt

so therefore the time lag is given by ϕ by ω is equal to now we have said
now you have to be careful this ϕ must be in radian uh

so this was θ .

64 radian divided by ω which is 400 radians per second which is equal to 1.

6 milliseconds now one of the things that i would like to point out is to for
you to observe what happens when ω increases that is what's the situation
with higher frequencies you see what happens if your ω increases then this 5
which we had worked out is 10 10 of ϕ is given by x_c minus x_l divided by r and

so if ω increases let's suppose i am talking about a capacitor now in that
case my ϕ would go to θ for a capacitor and the reason is very simple we had
said that my $\tan \phi$ is x_c minus x_l divided by r and if i have a dominantly
capacitive circuit or just a capacitor let's say then x_c is 1 over ωc

so when ω becomes large my ϕ becomes θ

so what does it actually mean it means that a capacitor will essentially behave like a conductor

so high frequency current will simply pass through it the reverse situation happens when ω approaches 0 that is the circuit is resembling now a dc circuit in which case the capacitor becomes like an open circuit that is no current passes now that is of course obviously something which we already know now remember that for an inductive circuit the current magnitude

so here what we said is capacitor behaves like a conductor for an inductive circuit on the other hand the current magnitude is proportional to $1/\omega$ exactly the reverse occurs because as ω increases the circuit essentially behaves like an open circuit

so for high frequency and of course reverse is true if you have essentially a dc passing through it now in this lcr circuit that we have talked about which we calculated various things what is the average power delivered now remember that the only element of an lcr circuit which dissipates power is the resistive element because on an average the capacitor and the inductor do not dissipate power they absorb and release

so average power is simply $i_{rms}^2 R$ square times that we have actually calculated already i_{rms} or rather i_{rms} has been given to be equal to 2

so this is 4r

so 4 into 80 equal to 320 watts as another example let me take an rc circuit this is an rc circuit with an alternating voltage let me take numbers r is equal to 3 ohms c is equal to 2.

5 into 10 to the power minus 4 farad which is 250 micro farad let us take ω somewhat high frequency 1000 radians per second and let us also take the supply voltage V_{max} is equal to 5 volts now since it's an rc circuit the current would lead the voltage only thing the difference that occurs in all these is that if i had a purely resistive circuit the voltage and the current would be in phase if you had a purely capacitive circuit the current would lead by 90 degrees if you have a combination the current would still lead but not by $\pi/2$.

let's see how it works

so so we said let v be 5 sine ωt this is what is given this for the source the current i'll take the general expression as $i_m \sin(\omega t + \phi)$ i expect ϕ to be positive for the simple reason that current leads the voltage how much i don't know had it been a purely capacitive circuit it would have been private

so let's look at that

so the first thing that i do here is find out what is the capacitive reactive X_C that's equal to $1/\omega C$ ω is 1000 and this is 2.

5 into 10 to the power minus 4

so you calculate this there is already a 10 to the power 3 here and it works out to 4 resistance has been given to be 3 ohms

so my impedance which is equal to $R^2 + X_C^2$ i have got

so X_C^2 square

so it is 3 square plus 4 square square root which is equal to 5 ohms now that immediately tells me that my maximum current would be maximum voltage divided by Z which is just equal to 1 ampere what about V_R max which is simply equal to $i R$ i is 1 ampere R is 3

so it is 3 volts what is the V_C max now this is where you must remember i am not adding in series resistance circuits the drops simply added but i'm not adding here and that would be given by i divided by ωC $i X_C$

so this is $1/\omega C$ was 4

so 4 into 1

so that's equal to 4 once again you realize that i have 3 volts drop across

resistance four volts drop across the capacitor but the total drop is square root of three square plus four square which is equal to five and let us show it in a diagram

so this is my current direction that's my v_r the now remember again i am drawing the voltages

so though current leads the voltage voltage lags

so hence the negative y axis

so this is my v_c which is equal to $i x c$ and if you complete this you find that that is your supply voltage and you can easily calculate how much is this angle 5 this was 3 this is 4

so $\tan 5$ is equal to 4 by 3 let me give an illustration for a circuit which is predominantly inductive

so let's do this i have a resistance which i take it to be 1 kilo ohms i have an inductor which i take it before henry i have a capacitor which i take it to be 4 micro farad and my source voltage is 140 sine 500 that is ω is still 500 i will not repeat the calculation but you can immediately find out what is x_l x_l is ωl

so ω is 500 l is for henry

so it is 2000 ohms x_c is 1 over ωc just do the same calculation this will be 500 and z which is equal to once again r^2 plus x_c minus x_l whole square is the simple calculation will give you 1800 ohms

so maximum current is 140 divided by 1800 which is equal to 0.

078 amperes rms is obtained by dividing by square root of 2 which will work out to be 55 milliamps now repeat the same term how much is v_r max you have got i r already i is this r you know and you will get if you do it correctly r is one kilo ohm 78 volts simple mathematics arithmetic i'm not doing it i'll simply illustrate the last thing v_c max is i max times x_c this will turn out to be 39 volts v_l max will work out to 156 volts

so if you calculate the \tan final which is equal to x_c minus x_l by r you will get it as minus 56 degrees the corresponding vector diagram is this this is your v_r in this case my v_l is bigger

so this is a much bigger one v_c is smaller

so therefore this is the way i would draw the diagram and this would be fine

so in other words you can see the current lags the voltage by 56 degrees

so in this lecture what we have done is to look at a combination of lcr circuit and we have defined what meant by reactances for the inductive and the capacitor developments we defined impedance and then we talked about a graphical analysis of the lcr circuit to determine current voltage and things like that in the next lecture we will be doing a formal analysis which will require the solution of a second order differential equation but we'll sort of take it up next time you