

welcome back in the last lecture we introduced you to what are called as alternating currents or alternating voltages and we defined different terms associated with these

so alternating current and voltages we considered first a simple circuit consisting of a resistance and a source of emf which is time varying

so this time variation was sinusoidal v equal to $v_{\max} \sin \omega t$ and this is the resistance r and the current i then is given by the standard expression for ohm's law which is v by r

so in this case i is given by $v_{\max} / r \sin \omega t$ which we write as i_{\max} times

so you notice that the time variation of both the voltage and the current are the same both of them vary with time as $\sin \omega t$ and this means the current is in phase with the voltage we had introduced a phasor diagram corresponding to a purely resistive circuit and what we showed there was the following we said that my x axis is the time t equal to zero reference line and my both current phasor and the voltage phasor they rotate counterclockwise with an angular velocity ω

so at time t equal to t let's suppose this is my direction in which the voltage phasor is located

so this is a phasor of magnitude let us call it o_a

so o_a magnitude was v_{\max} and therefore if you take the projection along the y -axis of this quantity this gives you the instantaneous value of the voltage now since the current is in phase with the voltage and i know that the current and voltages are measured in different units supposing the in whatever units we measure i that's of course ampere but in whatever scale we measure i supposing this is o_c times d is the magnitude of the current amplitude then if you take the projection along the y axis this gives you i of t you notice here that the voltage and the current are in phase and

so therefore the direction of the phasors are identical

so since the time variation of both the voltages and the current was $\sin \omega t$ we proved that the average of the current also that of the voltage over a cycle is equal to 0 and that's because $\sin \omega t$ $\sin 2 \omega t$ etcetera their average turns out to be for a purely resistive circuit what we found is p average is given by $i_{\max}^2 r$ divided by 2 which led us to define something known as rms current root mean square current which is defined as the maximum current divided by square root of 2.

if you do that the expression for the p average works out to be identical to the expression that we had in case of dc circuits the concept of rms is also applicable to for example the voltage and you can talk about v_{rms} is v_{\max} by square root of two and i also told you that whenever we mention that the household voltage is let us say 240 volts we don't always say it but what is implied is that these are the root mean square values

so the peak voltage will be obtained by multiplying that with a factor of square root of 2.

having done that we considered a circuit consisting of an inductor and

so this was my circuit once again i have an alternating source

so my $v_{\max} \sin \omega t$ is the v of t and this is l and what we found in this case is that the current expression for the current is given by v_{\max} divided by ωl times \sin of ωt minus π by 2

so this of course told us firstly that this is the magnitude of the maximum current and the time variation is not in phase with the time variation of voltage but the current lags behind the voltage by $\pi/2$ returning back to a phasor diagram for this situation

so what we do is this supposing this is my voltage and

so that the projection on the y axis is my v of t

so this is v max the magnitude then my current would be represented in it in this direction

so that this angle is 90 degrees this is what is showing you that it is lagging behind the voltage by 90 degrees

so this is your length of this vector is i max and this is your instantaneous i of t if you notice that the expression for the current was given by i equal to i max times sine omega t minus pi by 2 and in this case i m is given by v m divided by omega l

so in other words the role of resistance in the purely resistive circuit is assumed by the this combination here omega times l and this omega times l is called inductive reactance and it is denoted by x l

so let us look at some example let me consider a resistive circuit in which the rms value of current is 5 ampere we are asking what is the value of the current 1 over 400 seconds after it becomes 0

so since i am given the rms value is 5 ampere i can get the maximum value of current which i call as i m by simply multiplying it with square root of 2

so 5 square root of 2 ampere is the maximum value of the current therefore my i at time t is given by i m sine omega t and that's equal to 5 root 2 i also need what is the frequency of the circuit

so let me say that frequency of circuit f is equal to 50 hertz

so this is sine omega is 2 pi into f which is 100 pi times t

so notice that at t equal to 0 the current is 0.

and at t equal to 100 1 over 400 seconds i get i equal to 5 root 2 into sine 100 pi multiplied by 1 over 400 that gives me sine of pi by 4 which is 1 over square root of 2

so this is simply equal to 5 let me continue with an example this time with an inductive circuit i have a voltage source v equal to 25 sine omega t meaning thereby that the maximum voltage is 25 volts in a circuit which corresponds to omega equal to 400 radians per second this is approximately the type of radiant frequency that you would get for a supply which is slightly more than 60 hertz because corresponding to 60 hertz omega turns out to be about 377 radians per second but this we are taking simply just because it helps easy calculation

so this in a 10 henry with a rather large value i have taken but again for simplicity 10 henry inducted

so let's find various things connected with it that at the instant when the voltage is minus 12.

5 volts and its magnitude is increasing what is the value of current this example will sort of help you understand various terminologies that i have been using

so first is the maximum value of the voltage v m is 25 volts from the given data i can also find out what is inductive reactance

so x l which is equal to omega times l omega is 400 and l is given to be 10

so which is 4000 the unit of reactance happens to be just ohms

so maximum current then would be v m divided by omega l which is equal to 25 by 4000 and if you calculate it it is just 6.

25 milliamps obviously the current becomes maximum when the voltage is equal to 0 that's because we said that the current lags behind the voltage voltage by 90 degrees now let's look at what is the instant when voltage value is minus 12.

5 volt and it is increasing in magnitude now look at what it means

so what it says is my v of t is minus 12.

5 and the general expression for v of t is v m which is 25 times sine omega t

so that tells me that sine omega t is minus a half minus 12.

5 divided by 25 is equal to minus r now my angle omega t varies from 0 to 2 pi

so i know that the sine function becomes negative in third and the fourth quadrant

so my solution for ωt then happens to be either $7\pi/6$ or $11\pi/6$.

that only takes care of the variation of v of t but let us look at the other condition that is there that the magnitude is increased let's consider the way the voltage varies with time and what i am going to do is to plot the voltage not as a function of time but in terms of ωt which as you know is measured in radians or degrees whatever

so this is ωt equal to 0 and let me plot the voltage like this this is just one cycle i'm sure now

so this is 0 this is of course the phase remember i am doing it in terms of ωt this is π this is 2π corresponding to time t by 2 and the full scientific

so this maximum here we have said is 25 volts now we are looking for the voltage becoming minus 12.

5

so roughly i'm looking at half this value

so the this will be either a point here or a point there now what we want is v of t equal to minus 12.

5 volts and its magnitude should be increasing but if you look at it minus 12.

5

so this is minus 12.

5 is here as well as there now if you just look up a table and find out where does it actually occur you will find that this is occurring at ωt equal to $7\pi/6$ and $11\pi/6$ and that's purely because uh this magnitude is half of the maximum value and this is just because uh what we are looking for is minus 12.

5 which is just half of the maximum magnitude and

so therefore it is i'm looking essentially for values of angles for which the sine of the angle is minus half and

so these are the two angles um now out of this i pick up $7\pi/6$ because that is the place where the voltage is increasing in magnitude this is also satisfies the same value of minus 12.

5 but there you notice that the magnitude is actually decreasing

so therefore this value $7\pi/6$ is what i have

so the magnitude is increasing at ωt equal to $7\pi/6$

so my current i is then given by maximum amplitude of the current 6.

25 times sine of $7\pi/6$ minus $\pi/2$ that is because of the lag which is equal to 6.

25 sine of $4\pi/6$ sine of $4\pi/6$ is square root of 3 by 2 and if you calculate this it will give you 5.

41 milliamps having explained how a purely inductive circuit works let's

consider a purely capacitive circuit with the same alternating source as before

so this is $v = V_m \sin \omega t$ now this case would be different from the way the capacitors behave under a dc supply because in a dc what happens is a capacitor behaves like an open circuit and doesn't allow the current to pass through

so all that happens is there are transients created and the capacitor plates get charged and once they are fully charged they remain

so as long as the battery remains connected to the capacitors now here what happens is this that assuming that initially my capacitor plates are uncharged they there would be a charging current and this charging current would make this capacitor plate let's say this plate positive display negative and as the voltage keeps on changing the direction and

so does the current
 so i will have alternately charging and discharge
 so this thing is given by dq by dt is equal to $c dv$
 so this is my charging current and this is what is going to flow into the capacitor and
 so therefore what I have is that my voltage difference between the two plates since there are no other elements in this circuit this is the same as v of t
 so what I get is my v of t is given by the instantaneous q of t divided by
 so therefore q of t is c times v of t which is equal to c times v_m times sine of ωt and then my charging current i of t would then be I need to differentiate this because dq by dt
 so it is $c v_m \omega \cos \omega t$ and I will rewrite this as v_m times $c \omega$ now cosine of ωt I will write as sine of ωt plus $\pi/2$
 so if I write this current as equal to i_m sine ωt plus $\pi/2$ my current amplitude is given by v_m times $c \omega$ therefore the ratio v_m by i_m is one over c
 so notice that the role of resistance in a dc circuit is played by this factor 1 over $c \omega$ and that is usually denoted as X_c and this X_c is known as capacitive reactance
 so let me make a note here that X_c is equal to 1 over ωc is equal to capacitive reactance that's one thing just like the case of the inductive reactance capacitive reactance also depends upon the source frequency
 so the variation now if I am plotting capacitive reactance X_c which is equal to 1 over ωc
 so that would be going like this
 so that is your X_c now recall that my current amplitude i of m is v_m divided by X_c which is v_m times ωc
 so if in the same plot if I am plotting the behavior of i am this is just linear the amplitude increases with increasing frequency let us look at the variation of voltage and the current for one period
 so this is the way the voltage varies and let me just show quarter of cycles separately this is t equal to 0 .
 this is t by 4 this is t by 2 this is $3t$ by 4 and this is t
 so let me show the current in the same diagram
 so this the black curve is v of t and the red curve is the current i of t
 so you can see that the current has become maximum a quarter of a cycle before the voltage does
 so our variations would be as follows if I take v of t to be equal to v_m sine ωt then the corresponding current is given by i_m sine of ωt plus $\pi/2$
 so since the current leads the voltage $\pi/2$ it would have become maximum a quarter of a cycle that is t by 4 before the voltage bridge I just want to point out that these are representative curves in the sense it is not as if the voltage is switched on at time t equal to 0 but basically as the circuit is still on we take the time t equal to 0 at some instant where the voltage is 0 and the current is positive maximum now what happens to the phasor diagram in this case
 so you recall that we had said that our x axis here is t equal to 0 that's the reference line that we have been having and at time t we said the phasor is a vector directed at an angle ωt to the t equal to zero axis and this phasor is rotating counter clockwise with an angular velocity ω and since the current leads the voltage by $\pi/2$ the corresponding current phasor would be given like this
 so this angle is 90 degrees and

so the magnitudes of these are this is i_n is the magnitude of the current phasor v_m is the magnitude of the voltage phasor and if you take the projection along the y axis then this gives you the instantaneous voltage at time t and likewise if you take the projection of the current phasor at time t along the y axis that gives you the instantaneous current at time t

so therefore let me make this point here that is current leads the voltage by $\pi/2$ which is another way of saying that i becomes maximum $t/4$ before the voltage

so let us look at the charging and discharging of the circuit for a capacitive circuit let me redraw the current and the voltage with time for a capacitive circuit with the voltage shown in black and the current in red since my current leads the voltage by $\pi/2$ that is by a quarter cycle

so when the voltage is maximum the current becomes zero

so the type of curve that i have is something like this of course it'll again okay

so notice what's happening is this the at supposing this is my time axis and this is time t equal to 0 and this is the $t/4$ $t/2$ $3t/4$ and t

so look at what happens at t equal to 0

so this is my circuit let us suppose at that instant my this side is positive and this side is negative

so my voltage is zero but is increasing and the current has become maximum and current also is positive

so i have i greater than zero

so therefore there will be a charging current which will flow like this and will make this plate positive and naturally this plate will become negative now

so this is what happens at between t equal to 0 to t equal to $t/4$

so charging now t equal to $t/4$ the capacitor plates are fully charged the voltage has become maximum at that time and for a brief time here it is neither increasing nor decreasing

so that there is no current flowing as can be also seen by the current diagram

so let me label it the red one is the current and the black represents the voltage

so what we said is that at t equal to $t/4$ it is fully charged and for a brief moment there is no current now what happens next

so i am now going from $t/4$ to $t/2$ notice from $\pi/4$ to $\pi/2$ the voltage is decreasing though is positive

so therefore the current uh still flows but notice now the current will flow in the opposite direction making the right hand side plate less negative than it was and of course since the positive plate is depleting charge it will also become less positive at the instant $t/2$ for again for a brief instant at this point the plates have been fully discharged

so let me come back

so at $t/2$ the plates are fully discharged

so in this cycle discharging takes

so left plate becomes less positive and naturally the right plate becomes less negative

so the situation at that time is this that i have two plates which have no charge and this is my

so it's there now let us now go from $t/2$ to $3t/4$.

now notice what is happening in this part of the cycle the current is negative and is increasing in magnitude

so what is happening now is this that the direction of the current would be opposite

so the current would now flow in this direction making this plate positive and

that

so this side was minus this side is plus

so the current reverses in direction and plates are charged in the opposite direction once again at three t by four the voltage in the negative direction has become maximum and once again when you go from three t by four to t the discharging takes place

so this is charging circuit again discharging takes place now making the left plate less positive and the right plate less negative till at time t we return back to uncharged plates let me take an example very similar to what we did sometime back for the inductive circuit in fact we take essentially the same example but with a slightly different num set of numbers

so what i said here is this that i have a voltage source which is giving me maximum 25 volts

so 25 sine ωt was my variation this is v ω was 400 radians per second and here i have connected it to 10 micro farad capacitor in the previous example i had an inductor connected to it

so once again my question is the following that first of course we will find out various quantities connected with the circuit and then we say at the instant fire when v of t is minus 12.

5 volts and is increasing in magnitude what is the current well first some numbers my capacity reactance which is given by $1/\omega c$ is given by $1/\omega$ is 400 c is 10 micro farad meaning there by 10 to the power minus 5 farads

so this works out to 0.

25 into 10 to the power 3

so this is 250 ohms the maximum current in that case would be v_m by x_c which is 25 divided by 250 and that's equal to 0.

1 up here now my next question was consider the instant pen the voltage is minus 12.

5 and is increasing in magnitude in a similar situation in the previous example connected with an inductor i had seen that there are two solutions for ωt but out of that the solution for which the value is minus 12.

5 volt and the magnitude is increasing is given by ωt is equal to 7π by 6

so i would point out from previous example then my i becomes i_n sine ωt but this time plus π by 2.

i am i have already found out to be 0.

1 ampere and this is sine of 7π by 6 plus π by 2 which is 0.

1 times sine of 10π by 6 the sine of 10π by 6 takes it to the fourth quadrant

so therefore what happens is this that the i will get a negative value of the current and that would then be equal to minus 0.

1 into square root of 3 by 2 that is equal to minus 0.

0.85 amperes in this problem i am also required to find out what is the timeline between the voltage and the current maximum now we have seen that the current leads the voltage by π by 2 and

so that is a time period of t by 4

so and in my case what is given is ω equal to 400 but by definition of ω it is 2π by t that tells me that p by 4 which is the time lag that we are talking about is given by π by 800 which is 3.

9 milliseconds let me continue with another example we can construct a voltage divider using a capacitor circuit remember that we could do that even with resistance for example if i had a let us say couple of resistances in series then the voltage drop across the first resistor would be i times r_1 and about the second register will be i times r_2

so that enabled us to divide the voltage now this is same thing that happens here but let's see how it works

so this is a voltage divider using capacitors here also i use two capacitors in series this as usual is $v_m \sin \omega t$ let me take v of t to be equal to $25 \sin \omega t$ as before and ω i will take it to be 400 radians per second

so the voltage division works like this there is a capacitor here which is let's call it c_1 equal to let's say 10 micro farad there is another one in series with it which is c_2 let's take it as 20 micro farad

so what happens here is this that between the this point and that point there is a voltage v_1 and between this point and that point there is a voltage v_2 now remember that since the capacitors are in series the charge q at any instant in them is the same but the voltage is different now in order to do that what we need to find out is what is the reactance of the first capacitor this is equal to 1 over ωc_1 ω is 400 c_1 is 10 micro farad

so that we have just now worked out in another problem to give me 250 ohms and x_{c2} since the capacitance is double the reactants would have

so ωc_2 equal to 125 ohms now these of course in a series they just add up so that my net x_c for the circuit is 250 plus 125 equal to 375 ohms

so therefore my maximum current is v_m which is 25 divided by x_c which is at 25 by 375 which if you calculate works out to 67 milliamperes this is the amperes this is 67 milliamps

so v_{c1} is equal to i times x_{c1}

so which is 0 .

0.67 i am writing it in ampere still into 250 and that works out to 16 .

6 volts v_{c2} will be i times x_{c2} and this is simply the same same thing multiplied with 125 and that happens to be 8 .

33 volts you can see that basically this voltage division has been in a 2 is to 1 ratio these are of course the maximum values of voltage across the capacitors and the variation with time which we have not shown here is the same as that of the source namely it varies as $\sin \omega t$ let us continue with some more examples that will help us in getting some of the terms that we have been using clearer a five microfarad capacitor is connected to an ac generator with v_{rms} equal to 60 volts we observe the peak current peak current is the same as the maximum current to be 0 .

42 amperes we are required to find the frequency of the source

so you remember that the relationship between v and i and that is applicable to whether we are talking about rms or the instantaneous values

so v_{rms} is i_{rms} multiplied by x_c i_{rms} equal to 0 .

42 divided by square root of 2 because i have been given that the peak current is 0 .

42

so rms value is obtained by dividing it by square root of 2 and that is approximately 0 .

3 amperes and my x_c then is rms by i_{rms} that's 60 divided by 0 .

3 which is 200 ohms but x_c is also 1 over ωc that's equal to 200 i have given c to be 5 micro farad from this it's trivial to see that ω works out to a thousand radian per second which corresponds to a frequency f because ω is $2 \pi f$

so f is 1000 divided by 2π which is equal to 160 hertz before we conclude this topic let me talk about power in a capacitive circuit remember that capacitors can store energy and they don't dissipate energy but can return that energy that they have stored to other parts of the circuit depending upon what is the condition that's there

so my instantaneous power that's equal to $i t$ into $v t$ and that's equal to $i m$

v and i had a sine ωt multiplied by sine of ωt plus π by 2 which is the same as $\cos \omega t$

so that is equal to $i_n v_m$ by 2 times $\sin 2\omega t$

so my average power is 0 because we had said in the first lecture that averages of things like $\sin \omega t$, $\sin 2\omega t$ etcetera is 0

so my average power is 0.

the energy that is stored is given by half cv^2 as you can see it here that if you want to rewrite what is it $v dt$ you can write this as c times dv by dt that is your dq by dt which is same as i times v these are all the voltages across the capacitor plates

so this can be rewritten as d by dt of half cv^2 and this is the amount of energy which is of course positive and that is let's call it w that w is given by half $c d m^2 \sin^2 \omega t$ now let us look at the way the power changes

so let us look at what happens to the power curve in this case in this graph I have plotted the voltage v of t in black and the current in blue and the power p of t instantaneous power is given by i of t into v of t and since i varies as cosine and v varies a sign what I get here is $i_n v_m$ by 2 into $\sin 2\omega t$

so notice that the period of the power is half the period of either the current or the voltage and in the first quarter cycle that is from here to t by four that is t equal to zero to t equal to t by four to notice both current and voltages are positive and

so this implies that we are talking about ωt equal to 0 to π by 2

so $\sin 2\omega t$ remains positive now this is shown by the red curve here

so i is greater than 0 v is greater than 0 and naturally the power p which is product of i and v is also greater than here the energy is absorbed from the source power is positive means energy is absorbed in the next quarter cycle that is from t equal to t by 4 to t by 2.

you notice that power becomes negative because though the voltage remains positive the current has now become negative

so what it implies is the energy that was absorbed in the first quarter cycle is now returned to the source and

so here p is less than zero the energy absorbed earlier is returned and the same thing is repeated in the third and the fourth quarter cycle namely in the third quarter cycle the energy is absorbed and in the form it is returned back again now if you look at this power curve namely the way power is absorbed from the source and returned to the source you will realize that in the quarter cycle in which charging is taking place the power is always absorbed and in the next quarter cycle when discharging is taking place the power is returned to the source

so what we have done in this lecture is to talk about a circuit which consists of a source of an alternating voltage with a capacitive element put in there we found out that in a purely capacitive circuit the current leads the voltage by an amount of π by 2 and we discussed how a purely capacitive circuit can be used for purposes such as being applied as a voltage divider and things like that you