hello in this and the next few lectures i will be discussing what is known as alternating current some time back we had talked about direct current circuits and i would like to point out that in our day-to-day use it is the alternating current which is uh more prevalent than direct currents and we will be discussing uh the various properties that go with it but before we do that let me remind you of what you have done earlier on faraday's law in our discussion of faraday's law of electromagnetic induction we had seen that if we have a situation where the magnetic flux changes with time then an emf is produced the emf relationship is given by the mathematical statement of faraday's law which says emf is equal to minus m d phi by dt where phi is the flux through each turn and n is the number of turns if you recall our definition of flux was integral of b dot d s over a surface the formula is written usually with a minus sign and that is because this is a reminder of what is known as lens law the lens law talks about the direction of current induced in the circuit
 so according to lenz's law now the direction of such induced current is always such that the magnetic field that such a current produces opposes the change that produced it in other words it tends to negate whatever change was being shot by the agency producing this current let's push this idea a little further supposing we have a rotating coil in a uniform magnetic field
 so i will just give you a schematic diagram
 so i have a coil which is rotating in a constant magnetic field
 so this is the rotating coil and let's suppose this is the direction of the magnetic field which is uniform and the coil of course ah moves around this axle and suppose the direction of b makes an angle theta with the perpendicular to the plane of the coil
 so let me represent it like this supposing this is the direction of the magnetic field and this is a section view
 so this angle is theta
 so this is the way it's rotating
 so therefore what happens is this that the flux produced is remember this is b dot a that is b dot d s
 so it is b times a times cosine of theta which is of course a function of t and that is equal to b a cos omega t because the coil is rotating with a uniform angular speed omega
 so therefore the emf generated by it according to faraday's law is n d phi by d t which is equal to n b a omega sine n is the number of turns in that question now this i can write as e0 sin omega
 so since you notice that this emf is changing sinusoidally
 so therefore this will produce a potential difference which also changes sinusoidally
 so let me say that the potential the voltage v is given by an expression like vm sine omega t now suppose i am to plot this voltage against time
 so this is a representative diagram in the sense that when i say t equal to 0 it doesn't mean that is the instant when the voltage was switched on but any particular time you can take it as time t equal to 0 and proceed for one cycle
 so supposing at my time t equal to zero this is axis time my voltage happens to be zero and then i go through one cycle
 so this magnitude the maximum magnitude of it
 so this is my voltage v as a function of time and this is vn that's the maximum voltage
 so in this picture what i have done is to say v equal to 0 at time t equal to 0 now the voltage returns to the same value
 so this is time t equal to 0 after a time capital t passing through a full cycle and this t after which the voltage at any point in the circuit returns to

the value that it had time capital t before is known as the period
 so t by 4 after the initial time the voltage becomes maximum and this is t by 2 this point where it becomes maximum but in the negative direction is 3t by 4 and this is full cycle t now if you want to compare this graph with that of what happens in a dc circuit the dc means there is no time variation
 so this is the dc voltage
 so this definition is v of time t plus t is equal to v of t now this omega is related to the linear frequency remember the linear frequency f is equal to 1 over t and omega then is 2 pi times f which is equal to 2 pi over t now let me then write down the simplest possible ac circuit
 so the symbol for an ac remember in dc circuit i had a battery type of symbol but here this is given like this and this is v equal to vm sine omega t and all that i have is a resistance in this element
 so i am looking at what happens when a alternating voltage is applied in a circuit which consists of a resistance only
 so let me assume ohm's law
 so my current i then is given by this quantity vm divided by r times sine omega t which we will write as i m maximum time sine where i m is the maximum value of the current in the circuit now i had already shown you the way the voltage varies with time you notice that the variation of the voltage just for comparison sake was given by vm sine omega t
 so what you realize is that the time variation is identical there the current's maximum is vm divided by r
 so therefore the magnitude of the maximum current would depend upon what the resistance r is
 so if i am in the same diagram plotting the difference plotting both the current and the voltage in the same diagram let us do that
 so i have a time diagram x axis is time and on y axis i will plot both the voltage and the current obviously because their scales are different their units are different
 so i will have two different scales on that
 so let me first plot the voltage for instance
 so this is one cycle but let me just write down another cycle also half cycle
 so this is your vm supposing in the same diagram
 so this is real supposing in the same diagram i also plotted the current in other words the current will be plotted but with this scale being different
 so voltage is in volts and i will be plotting here the current will be in amperes and depending upon the value of r i will get a different value of i maximum but the important point to notice when the voltage becomes maximum the current becomes maximum and vice versa
 so therefore my plot of the current would be something like this the the each part is identical it doesn't look identical because of free and drawing
 so this is your i
 so the point that i am trying to make is this time here or let's say omega t doesn't matter point that i am trying to make is the current i becomes maximum or minimum at the same time as the voltage does now one of the things which is frequently done is to plot what is known as a phasor diagram now a phasor diagram is basically a polar curve with its x x axis being a reference line at time t equal to zero
 so let me just plot this and i will go on explaining what is the phasor diagram
 so this is some reference line i have taken with respect to this time that what i call it the initial time i am plotting everything
 so i assume that at time t equal to 0 i have a vector of length vm which is aligned with this

so axis

so let me put it like this imagine one end of that vector is at the point o origin and it has a length equal to vm and end point is a

so this is

so let me say that oa vector has a magnitude equal to vm now what i assume is this that keeping the end o fixed this vector rotates about an axis which is perpendicular to the plane of the paper passing through with an angular velocity omega

so that at time t equal to t the angle that it has swept is omega t and this vector then lines up like this

so the magnitude still remains vm but it goes to the point b where this point this angle is omega times t now suppose i take my variation of voltage with time as i have said a little while back v of t is equal to vm sine omega t this tells me that if you take the projection of this vector v ob at time t equal to t then this gives you the instantaneous value of the voltage now supposing instead i had taken vt equal to vm cos omega t then the projection along the x axis would have given me that now let's look at what happens to the current now in a phasor diagram like this i plot both the current and the voltage in the same diagram but since current and voltage have different units of measurement i can choose my scale appropriately to make these lengths of vector the way i want it

so so what we do is this that suppose i write i of t for a purely resistive circuit to be given by i m sine omega t remember the current and the voltages are in phase then at all time the current phasor is lined up along the direction in which the voltage phasor is lined up and suppose i decide on a scale in which the current magnitude is given by the length of the vector oc

so oc magnitude is im then the projection of this oc at time t equal to t is gives me the instantaneous value of the current now an important point to take home from this is for a purely resistive circuit the current is in phase with the voltage now let us look at what is the average value of subtle current but before i do that let me define what is meant by the average of a quantity over a cycle

so supposing i have a time dependent quantity f of t the average of f of t over a period it is written like this or you could also write it like f bar of t whatever there is no standard way of doing it is 1 over t integral from 0 to t of f t dt

so let us look at a time dependent quantity such as current which is given by i of t equal to i m sine of omega

so average of i of t if you look at this definition would be i m 1 over t integral from 0 to t sine omega t dt you recall that integral of sine omega t is minus cos omega t by omega

so it is i m 1 over omega t and if you take the limit it is cosine of 0 that is 1 minus cosine of omega times capital t

so uh i have to find out what is this recall by definition of time period i have omega times capital t is equal to 2 pi

so therefore what i have here is cosine of 2 pi and but cosine of 2 pi has the same value as cosine of 0

so therefore this quantity is equal to 0 and this would also be true for functions such as also valid for you could have sine 2 omega t 3 omega t etcetera or even cosine omega t cosine 2 omega t etc there is another relationship that we would require as you go along how much is average of sine square omega

so let us plug this into the definition

so average of sine square omega t is 1 over t integral sine square omega t dt from 0 to tin you recall your multiple angle formula which tells me that sine

square omega t is written as 1 minus cosine of 2 omega t divided by 2 and i have just now told you that any multiple of sine or cosine omega t integrates out to averages out to 0

so therefore the only thing that i am left with is this factor half dt which gives me t

so therefore this gives me 1 over 2 and that is because average cosine of 2 omega t will be using this as we go along

so what we have shown in the process is the average current is zero incidentally this doesn't imply that the power dissipated is 0 because power is given by i square r

so average power dissipated in the circuit is average of i square r and which is equal to i m square r average of sine square omega t which i just now proved the average of sin square omega t is half over a cycle

so it is i m square r divided by two

so you notice that this formula has some similarity with the power dissipated in dc circuits but for this factor of 2.

now we could remedy this situation and make the two formula very similar provided we defined a new quantity which is known as the root mean square current usually denoted by i rms i can similarly define root mean square voltage but let's stick with this now as the name suggests the root mean square is take the square take the mean of the square of the thing and then take a square root of it

so the way one defines it is like this it is square root of average of i square t but we have just now seen that i square of t is im square by 2

so therefore i rms happens to be i n divided by square root of 2 and likewise we can define a v rms equal to vm divided by square root of 2.

now if you are plotting for example the current i of t remember that my current has a sinusoidal variation

so this was my maximum i max the root mean square which is i m by square root of 2 is about 70 percent of this value because 1 over square root of 2 is about 0.
707

so therefore my root mean square value is somewhere here 1 by root 2 i n now once you start using root mean square current instead of the maximum current in this circuit you realize immediately that i can write p average as equal to i rms square that is because there is a 1 over square root there 2 there which will take care of the factor of 2 times r and the formula would then be looking very similar to what we had seen in case of a dc circuit now i would like to point out that when we talk about the voltage supplied to our homes for instance in india for instance the supplied voltage is ac and varies generally it is supposed to be 240 volts but it generally varies between 220 to 240.

is between 220 to 40.

supposed to remain around 240 and the frequency linear frequency nu is taken to be 50 years since many of our textbooks are of american origin i would like to point out that in the usa the household supply is about 120 volts and the frequency is 60 and that is the reason why when you go abroad to a different country you need adapters to match the if you your equipments are designed for a particular voltage or a frequency then you need adaptations

so we have discussed already what happens when an ac source is connected to a register but that's not really a very interesting situation the alternating voltage they become more interesting when you put into the circuit other elements in particular the inductance and the capacitance about which you have learnt in your previous lectures

so let me first talk about an alternating source applied to or connected to an

a purely inductive load what it means is that this circuit does not have any resistance no resistance and the only thing that is there in that circuit other than your ac source which will take it to be vm sine omega t is an inductance now once again i'll use the kirchhoff's law here and you remember from your discussion of faraday's law and the properties of inductances that inductance provides what is known as a back-end

so what we get is this and this back emf provided by the inductance is minus l d i by d

so therefore if i use the kirchhoff's law in the circuit i get v of t at any instant of time minus l d i by dt is equal to 0 which tells me that di by dt is vt over l but vt is known to be vm sine omega t

so this is vm over l times sin omega t if i integrate this now

so my i as a function of time will be vm over l sine omega t which is equal to minus vm over l omega cosine of omega now here i have taken the constant of integration to be 0 because we have seen that the voltage has no constant component and it is symmetrically oscillating around zero

so therefore my current should also not have any constant component and should also oscillate symmetrically about 0.

so notice this in case of a purely resistive circuit the trigonometric form of the time variation of both the current and the voltage was identical but now i have a difference and since i can write this cosine function as sine of omega t minus pi by 2 that takes care of the minus sign as well

so what i am getting in this process is that the amplitude current amplitude is given by v m over l omega

so this is current amplitude another interesting thing you notice is this that since the voltage is varying as sine omega t but the current in this circuit is varying as sine omega t minus pi by t

so what i notice is this that the current falls behind the voltage or in our language there is a phase lag of pi by 2 with respect to voltage the purely resistive circuit had no such lag but now we have realized that the current lags behind the voltage this quantity which comes here as in the expression for current is given a name this omega times l now if you compare this with the direct current circuit you realize that this is playing the role of resistance but there is a difference this quantity depends on omega and this is given a name it's called inductive reactance usually denoted by x l

so x l is equal to omega times l

so basically what is happening is something like this the xl supposing i am plotting this versus the frequency then xl is linear this is your xl variation but as the frequency increases the current decreases

so this is the way your current would let's say current at any instant of time would change provided i am plotting it against the frequency

so this is called inductive reactance

so what actually happens is this that as the frequency increases the inductance value of the reactance increases and as a result the current decreases let me plot the current and voltage as a function of time

so let me draw the voltage waveform because this is completely one cycle but let me also draw a little more than that

so this is my time t and what i have plotted here is the voltage in this blue

so this is v of t this is t by 2 this is t this is of course 3t by 2.

now in the same plot first let me write down what i have plotted here is voltage of course in volts in the in the same figure i'll be plotting the current but since current and voltages are measured in different units i can choose different scales for this and i have seen that for an inductor the

current lags the voltage in phase by pi by 2 it means it lags by quarter of a cycle

so let me make a note here that for the inductive circuit current lags the voltage in phase by pi by 2 which means by one quarter of a cycle

so in this figure when the voltage is 0 my current would be negative and maximum magnitude

so let me plot it here and this is quarter of a cycle

so therefore that when the voltage has reached maximum my current would become zero

so the it's nice to divide this by t by 4

so my current could be something like this well they don't exactly look like a sine curve but since it's a freehand drawing it is sort of an acceptable thing

so what i've got here is this this is the red curve is i of t and you can see that it is lagging in phase by t by four

so that it is maximum negative at time t equal to zero when voltage is zero and when the voltage has reached its reached its maximum the current is zero and then as the voltage remains positive and is decreasing and has just about reached 0 the current has reached its maximum and this maximum is my im value

so let me write down in red i have current in ips of course and this magnitude is what we wrote as vl

so let us look at how does the phasor diagram look this is for an inductive circuit

so let me for reference reproduce the current and the voltage curve remember this was my voltage

so this is my time t and this is time t by 4 t by 2 3 t by 4 and of course time t equal to 0 i repeat that this doesn't mean that i am switching on the emf at time t equal to 0 but it is a representative curve starting from any origin that you have taken for the time and with respect to which i am drawing this curve now the corresponding current curve was something like this remember that this is the way the current behaves

so this is current and this is voltage

so if you look at this diagram and find out what is the phasor diagram looks like for this

so as before i take the initial line to be t equal to 0 which is the reference line voltage which has a magnitude v n is a vector of length ob

so once again ob magnitude is equal to vm and at time t it makes an angle omega times t with the x axis as i have said several times the projection along the y axis gives me the instantaneous value of the voltage now since the current lags behind the voltage by pi by 2 it means that the current becomes maximum a full quarter of a cycle after the voltage does now the thing is this that because the lag the time lag between the voltage and the current is pi by 2

so when this is in the first quadrant the corresponding current will be in the fourth quadrant and this angle will be 90 degrees

so this is my current there this is what i had represented by oc there

so oc is magnitude is i n and of course it automatically means that if the voltage goes to the second quadrant imagine this entire thing being rigidly being rotated by certain angle taking ob to the second quadrant by that time the current would then come to the first quadrant

so current would also become positive

so this is the way it is when voltage is positive the current is negative in the first quarter cycle in the next quarter cycle both of them happen to be positive in the third i have a negative from here and a positive from there and finally to conclude it both negative voltage as well as current are both negative what about the power in an inductive cell

so let's look at instantaneous power

so power is given by i times v this is instantaneous power

so instantaneous current multiplied by instantaneous voltage and this is equal to i m sine omega t minus pi by 2 and v is equal to vm sine omega t

so it is equal to imvm we have seen that this is minus cos omega t and this is of course sine omega t which is nothing but minus i n vm by 2 sine 2 omega which tells me that the power over a cycle is zero the situation is very similar to what happens in a mass spring system supposing i have a mass on a frictionless table connected by a spring no friction now as you set the system into motion the mass gets gains kinetic energy at the expense of the potential energy of the spring and later on returns that amount of energy as the potential energy of the spring and it loses its kinetic energy and it's a conservative system because the only other thing here which could take away or dissipate power that is the friction which we have assumed that doesn't exist here and a identical thing happens in case of your inductors in the circuit

so inductor in one cycle part of a cycle would absorb power from the circuit actually retain it and re return it to the circuit in the next quarter cycle the effect of what i said is that for an inductive circuit the power over a cycle is zero

so for a an inductive circuit the average power equal to zero over a cycle which is another way of saying that the purely inductive circuit conserves energy and compare this a purely resistive circuit for which the average power was shown to be equal to i square r well actually i rms square r

so let me elaborate on this a little bit let us plot the current voltage relationship in this case

so this is x axis is time and in the same plot i'll be plotting both the voltage and the current and this blue line on the blue curve is my voltage t v t which is obviously in volts

so v t involves i will also plot now the current in the same uh curve in amperes

so a i t in amperes and since i know that the current for an inductive circuit lags behind the voltage by quarter of a cycle the curve that i get for the current is something like this notice my scales are different because in one case i am plotting current in the other case i am plotting voltage

so this is i of t now let's consider an instant of time when the voltage is zero now since the current at that instant has a maximum

so therefore this is my time t by four and let me also give some labels to these

so this is v max and in the red curve this would be slightly different scale i have taken

so this should be i max

so let us look at how does current and voltage value in this cycle from t by 4 to t by 2 which is here you notice my current is greater than 0 not only that is increasing that is d i by dt is greater than zero which implies that my voltage v of t is also greater

so it can also be seen of course in uh the voltage curve itself

so it tells me that power which is i times v is greater than 0 in this cycle

so since the power is greater than zero it implies that the energy is absorbed from the source

so then various signatures are the following voltage plus current plus now let's go to the next quad cycle from t by 2 to 3t by four the current which had become maximum starts decreasing but it still remains positive

so i is greater than zero but d i by dt is less than 0 which as you can see from the curve and from the signature of di by dt v of t becomes negative

so which means power p iv is less than zero which means the energy that was absorbed in the previous quarter cycle return to the source in the next section
so this was my 3t by 4.
in this section as we have seen my voltage is negative current is positive in the next quarter cycle from three t by four to t my current is less than zero
so is di by dt which means vt is also less than 0 but since both are negative my power is greater than 0 which implies again that the energy is absorbed from the source
so this was from 3t by 4 to t
so here you notice this is negative and
so is the current negative now if i went from p to 5 t by 4 the situation would be simply a replica of what happens from 0 to t by 4 and there you can see that the current is negative the voltage is positive once again it means that whatever energy was absorbed in the previous quarter cycle is returned back
so this is positive and this is negative let me close this discussion with a couple of examples suppose i have a 48 milli henry inductor connected to some of these numbers i have chosen says that arithmetic becomes easy connected to 240 volts 50 hertz supply i need to find out what is rms current i do not have to tell you that whenever we are giving the value of voltage or current their rms values otherwise we would specifically point out that these are peak valves
so in this case my first job is to find out what is my reactance my reactance is omega times l omega is 2 pi nu and nu is 50 earth
so 2 pi into 50.
the inductance is 48 milli henry
so therefore 48 into 10 to the power -3 and that is 4.
8 pi which if you calculate works out to 15.
08 ohms
so my rms value of current is 240 divided by 15.
08 just let's take it as 15.
that's equal to 16 amperes this value of current which we have obtained is much higher than the typical household current which is generally limited to about 8 to 10 amperes but that's primarily because in this artificial circuit that i have considered i have not taken any resistance normally resistances are there in the circuit which would limit the value of the current
so what we have done today is to define an alternating source voltage and seen that when this is connected to a purely resistive circuit the current and the voltages are in phase but when you connect it to an inductive circuit then you find that the current lags behind the voltage the second point is that in a purely resistive circuit there is a power dissipation which is given by the same formula as for dc accepting that i have to change my definition of current to the rms current i square r whereas a purely inductive circuit does not dissipate power whatever power it absorbs in a part of the cycle is removed returned back to the circuit in another part you