

a very good morning to all of you we have been discussing electromagnetic induction and today what i want to discuss is a very very important application of electromagnetic induction in the generation of electricity

so we will be discussing an alternating current generator or an ac generator ok so let us recall that according to faraday's law of induction whenever there is a changing magnetic flux through a closed loop then there is an induced emf generated in the closed tube and that induced emf is given

by the rate of change of flux rate of change of magnetic flux through the to that loop and the direction of the induced emf is determined by the lense's law so let us recall

the induced emf is equal to minus $d\phi_b$ by dt where ϕ_b is the magnetic flux and ϕ_b

we are defined as $\int \mathbf{b} \cdot d\mathbf{a}$

so that's the magnetic flux and whenever this flux changes with time then there is an induced emf in the circuit now if i take a region suppose

i take a small region of space where \mathbf{b} is uniform then ϕ_b becomes actually $\mathbf{b} \cdot \mathbf{a}$ and this is equal to b times a times $\cos \theta$ where suppose i could have a circuit like this the magnetic field is pointing like this and i define the area vector like this and this is θ so

remember i must consistently use the calculation of emf on the left hand side

with the magnetic flux which i am defining on the right hand side because the area

i am defining like this if induced emf calculation then the loop calculation must

be like this $\mathcal{E} = \int \mathbf{e} \cdot d\mathbf{l}$ must be integrated in this direction so

that i am in the right handed screw notation

so the magnetic flux passing through this loop is proportional to the magnetic field depends on the area of the loop and the angle

between the area vector and the magnetic field if any of these changes if any of these quantities

change then there is a change in the magnetic flux and that change in magnetic flux will

induce any emf

so we could have for example the magnetic field itself changing with time and

that is what happens when you have a solenoid and you change the current in the solenoid then

you are changing the magnetic field within the solenoid and

so that induces an emf you

could change the area keeping other two terms constant you can change area for example when we had calculated the motional emf where we had a conductor moving

on a on another conductor we showed that there is an area which is changing with time and that area

change with time creates a changing flux with time and that creates an induced

emf it is

also possible that both magnetic field and area remain constant the magnitude of area

remains constant but this angle θ changes

so if you have a coil which is rotating then

because the area vector is rotating with time the $\cos \theta$ term will change with time and that

will induce a change of magnetic flux with time and that magnetic flux change will generate any

emf

so this is the principle that is used in the ac generator

so let me draw the generator which

looks something like this here

so i have a magnet a permanent magnet one pole here

there is another pole on this side

so let me assume this is the

north and this is the south

so the magnetic field lines are

are pointing from left to right now in this i what i have is a coil let me draw the coil like this

so let me draw

the position which is one particular orientation

so there is a coil there is a coil which is

placed inside the magnetic field and what i do is i connect these two ends of the coil into

what are called as rings

so here i have a ring and here it just connected to one ring here and

this is connected to another ring on this side and what i do is i make an arrangement in which i

can rotate the coil with respect to the magnetic field as a function of time

so this is the

construction

so i have a pair of pole pieces here this is a strong magnetic field between the two

pole pieces a uniform horizontal the directed magnetic field i have a coil which is connected

to two rings here

so and this coil can rotate with respect to the magnetic field and these two

contact points are such that they are always in contact with these two rings and what i

do is i take out the output from these two points and look at the potential difference

between these two points as a function of time

so when the coil rotates the area vector rotates

the rotation of the area vector implies that the change there is a change in θ $\cos \theta$

which i wrote before as θ changes with time the magnetic flux passing through this loop

changes with time and the changing magnetic flux will induce an emf which will develop a potential

difference across these two points in the outside circuit

so these two potential this potential

difference i can use to drive current through an external circuit

so let me try to illustrate

what will happen ah through a slide here
 so let me see that let me show that this is
 the slide
 so these are the two points here p is here and q is here
 so i have purposely drawn
 one as a red line and the other has a blue line
 so i i show this two and
 so what
 happens is at some instant of time the coil is like this
 so let me assume the
 magnetic field is horizontal and let me assume the magnetic field is coming
 out of this
 paper
 so when i rotate this with respect to time you see that the area vector is
 changing and after
 some time the coil becomes horizontal when the coil becomes horizontal there
 is no magnetic field
 passing through the coil and the flux becomes zero and then it rotates further
 and becomes like this
 and again the flux becomes maximum because $\cos \theta$ becomes zero θ
 becomes zero and \cos
 θ becomes one and then if i rotate further it again becomes horizontal and
 the flux becomes
 zero and here the flux becomes maximum
 so what is happening is the flux is maximum because the
 coil is perpendicular to the magnetic field
 so $\mathbf{b} \cdot \mathbf{dA} = b \cdot a \cos \theta$ is one then
 after quarter cycle when it becomes horizontal $\cos \theta$ becomes zero
 so there is no flux because
 area vector is above our area vector is below and there is a magnetic field
 which is perpendicular
 to the area vector
 so the dot product is zero after another quarter cycle the coil becomes
 again
 vertical with the maximum magnetic flux and then the coil becomes horizontal
 with zero flux and
 a maximum flux
 so what is going to happen is the flux through this coil is going to change
 with
 time and that will induce a in emf in the coil now there is something to be
 noted here
 so let
 me assume that the magnetic field is coming out of this paper
 so ah towards this side
 so the flux
 is
 so if i consider this loop to be the in this direction because the
 so ok
 so if the flux if
 the flux is like this if the area vector is up please remember the integral
 has to be done like
 this
 so when i rotate this the flux is decreasing with time the flux is positive
 and decreasing

with time

so $d\phi$ by dt is negative and so e is positive and as it rotates

so here for

example let me assume that the e is such that the current is flowing like this from blue side to

the red side after half a cycle you see here the red side becomes below and the blue side becomes

up the current is now flowing from red to blue please note half a cycle before the current was

so

so this is the e is like this for example

so initially this was at a higher potential

compared to this when it was rotating like this q was at higher potential than p then it

rotates to half a cycle now p is come below q

so p is at a higher potential than q

so what you

can see is the potential difference between p and q is will oscillate with time starting from

this position to this position and it will continuously change with time generating

what is called as an alternating current

so one thing to remember here is because i am

rotating the coil the area is changing area vector is rotating and

so $\cos \theta$ is changing and as

$\cos \theta$ changes the magnetic flux changes and because of the reversal of the orientation

of the coil the e will reverse itself so let me draw a figure here just to explain

so in this figure what is going to happen is that initially for some time this is

at a higher potential compared to this and then after half a cycle this will be this

will be at higher potential corresponding to this

so the the e the potential difference

will keep keep on reversing itself

so let me assume that this rotation is at an angular frequency ω

angular frequency of rotation

so let me try to draw what happens as a function of time

so here is the ah here is the diagram

so let me draw this as a function

of time let me draw the flux magnetic flux

so let me start by looking at the coil

so let me start by assuming

that the coil is perpendicular to the magnetic field

so the flux is maximum the flux

will go through one full cycle in a certain time

so this is the time for one full

cycle of revolution of the coil now this at this position the coil was like

this at this position the coil is like this at this position

again the coil is like this at this position the coil is horizontal and at

this position the coil has become again vertical and the coil is rotating like this

so if i were to draw an area vector here
 the area vector was pointing like this here the area vector is pointing
 downward
 here the area vector is pointing left here the area vector is pointing up and
 here the
 area vector is pointing to the right
 so you see this this arrow is rotating as a function of time
 it is pointing like this after sometime it becomes like this then it becomes
 like this then it goes
 up and then it becomes
 so this cycle is repeated again and again and the frequency of oscillation
 the frequency of rotation i have said as ω and
 so the flux is changing with time now if i want
 to draw on the same figure an emf generated please remember ϵ m f is
 proportional to minus $d\phi$ by dt
 so $d\phi$ by dt is the slope of this curve minus
 $d\phi$ by dt is minus the slope of this curve
 so let me see in this region
 so in this region $d\phi$ by dt is less than zero till this
 point and then here $d\phi$ by dt is greater than zero in this half of the cycle
 $d\phi$ by dt is less than zero because the slope you can see of this curve as
 a function
 of time is negative then at this point the slope becomes positive ϕ is
 increasing
 with time here ϕ is decreasing with time
 so $d\phi$ by dt is negative here $d\phi$
 by t is positive here
 so because $d\phi$ by dt is negative here the induced emf is
 positive and induced c m f here is negative
 so the induced e m f between these two terminals
 keeps on changing its time periodically with time and
 so if i were to draw the induced emf here
 what will happen is it looks something like this
 so this is this point this is this point this
 is this point here
 so it will go to maximum
 so this is emf
 so at this point rate of change of flux
 is zero because the curve is horizontal $d\phi$ by dt zero then $d\phi$ by t is
 negative so
 ϵ m is positive it becomes maximum at this point when the rate of change of d
 ϕ by the rate
 of change of ϕ is maximum slope is maximum then as you come to this point d
 ϕ by dt becomes
 again zero and
 so there is no induced emf beyond this point $d\phi$ by dt is positive the flux
 is
 increasing with time which means essentially that the individual cmf is
 negative and induced mf goes
 like this and this repeats itself periodically
 so this is a generator this actually this
 is a device which generates alternating emf between these two terminals
 so half the
 cycle this is positive with respect to this another half of the cycle this is

positive with respect to this

so this keeps on changing with time and this is called an ac generator so if

i were to draw again another figure here

so the ah the coil will be looking like this with area

pointing here here the coil is looking like this with area pointing up sorry down here the coil is

like this with area pointing to the left here the coil is like this with a sorry the area pointing

up and here the coil is like this with the pointing here and in between you will see that

this has rotated like this with area going like this here it is rotated in this direction

here it is rotated like this and here here rotated

so it starts from being oriented like

this then after sometime it becomes like this then it becomes like this then it rotates to

this position then it rotates this position then this position then this position then this

position and this position and as you can see here the orientation of this area area vector

direction if it was like this the area vector is to the left then it changes down then it becomes

to the to this direction then it goes like this and then rotates

so that is one complete cycle

and this results in induced emf which is changing as a function of time

so i can actually write an equation

so the magnetic flux $\phi = B A \cos \theta$ ah let me assume that this is my

coil this is the magnetic field direction and this is the area vector and this angle

is θ

so that is a coil that is a side view of the coil and this coil is rotating as a

function of time

so because the coil is rotating as a function of time θ will be varying as a

function of time θ at any time will be ωt its a rotating coil

so as a function of

time if i start from θ is equal to zero at t equals zero θ is equal to zero

as time progresses the θ keeps on changing with time and

so the magnetic flux is

actually given by $B A \cos \omega t$

so induce emf $-\frac{d\phi}{dt}$ which is equal to $-B A \omega \sin \omega t$ and you can

see here that this is

what exactly what i plotted here at t is equal to zero the flux is maximum as time

time increases the flux starts to decrease and the induced emf starts to increase because $\cos \omega t$

t is decreasing and $\sin \omega t$ is increasing and this is the plot of

essentially $\cos \omega t$ and that's the plot the way the emf generated where i is $\sin \omega t$ and this is why you see that the emf is changing sign after every half a cycle and this is precisely indicated in this figure

so half a cycle

so this time this is the 2π by ω this is the time taken for one full cycle which is 2π by ω

so depending on the speed of rotation of this coil the the emf cycles will be determined by that speed of rotation or angular rotation and you get essentially an alternating emf between these two terminals

so in this generator what is going to happen is as you rotate this coil some half the cycle this is the higher potential than this the remaining half cycle this is at a higher potential with respect to this and the potential difference keeps on changing with time and this is the alternating current generator

so this is a very very important application of ac generator where you can use induced emf to generate alternating currents or alternating potential differences and if you connect this to an external circuit you can actually generate a current which is alternating current in the external circuits so the potential difference between this can be a generated potential difference and as you can see here if I use mechanical energy to rotate this I am converting mechanical energy to electrical energy through this generation process

so I can have this rotation generated by different mechanisms this rotation of this coil has to be done by an external agency

so if I rotate this as a function of time I will generate a potential difference here as a function of time and that will be generating a current for myself

so there are different generators so one is for example a hydroelectric generator hydroelectric generator here mechanical energy falling water a mechanical energy from a falling water is the mechanical energy that is converted and this can happen in for example a dam

so water from a height when it falls down it has a kinetic energy from the potential energy and that kinetic energy can be converted to rotation of this coil and that gets converted to electrical energy then you can have thermal generators where water is converted first to steam using coal or other sources then steam at high pressure is used for rotation you can also have nuclear generators where you convert nuclear fuel instead of coal and as I mentioned the frequency of this

induced emf that means this time period or the frequency at which the current is changing with time depends on the frequency of rotation of this coil and typically in india this frequency is about 50 hertz and in some other countries it is 60 hertz and so on so depending on the frequency of rotation of the coil you will generate current frequencies now by simple modification of this of this design i can convert into a situation where instead of an alternating emf i can generate emf in the same direction by the following arrangement so what i do is the following that i have again the same two magnets and what i do now is the coil which i am using here coil is like this and what i do is the following so i connect this to a what is called as a split ring so i have so this is connected here and this is connected here so i must draw it ah to the full here so this ring here there is another ring here splitting and the two contacts are taken from here and as before there is a magnetic field in the direction this is north this is south and this entire thing is now rotating around this axis now unlike the other earlier situation what you can see here is the this particular part of the ring is always in touch with the coil on the left hand side here what happened was the the coil on the left hand side was partly partly connected to this point and half a cycle is connected to the other point so because of this arrangement you will find that the e m f here is not changing its sign but it will be something like this let me draw a figure again here so if i were to draw ϕ_b as a function of time suppose it is like this just like before one cycle ah let me draw the induced emf here so this is one quarter of a cycle half a cycle another quarter of a cycle full cycle so here so at just like before the india cmf will first do this and in the second part instead of going down it again does this because the two terminals have interchanged themselves at the out for the outside circuit the emf is always positive and so here again i will have a so here the coils will be looking something like this here the coil was oriented like this with arrow here then the coil became slightly rotated then became rotated further then it became rotated

you have a charge potential difference and between these two plates there will be an electric field pointing in this direction between these two plates now my objective is to find out what is the magnetic field at this point so what i will normally do as before is take a a loop like this and so this is my loop and here i define my because my current is flowing like this let me define my area a loop of integration like this okay so thats a loop which i take at a distance say ah r from the from the axis and i use this formula for calculating magnetic field now if i am quite far away from this capacitor plates for example if i am deep inside here then i will find that the magnetic field because of symmetry again has to be azimuthal parallel to this circle here at every point and we have seen already this fact and using this i can do an integration of the left hand side immediately and get integral $\oint \mathbf{v} \cdot d\mathbf{l}$ now what is the current enclosed the current enclosed is determined by i must draw a surface for which this particular loop is a boundary and the current enclosed is the current crossing this surface so given a loop of integration i must draw a surface with this loop of integration as the boundary and that surface as i mentioned before has to have this as the boundary now i can have any i can choose any surface as long as i am consistently defining the enclosed current direction and the direction of integration of the loop so if i integrate like this the current must be positive current is towards me if i am integrating like this positive current is away from me so depending on the direction of loop integration the enclosed current has positive or negative sign now so obviously the first effect as you will see is why not take the surface the flat surface which is on which the loop is lying and in that case the current enclosed is simply the current passing through this wire so if i am given this loop of integration then i will choose i can choose a surface one surface which i can choose is the surface which is a flat surface and the current passing through the surface is simply i now there is no requirement that i choose only this surface for example i can choose another surface so let me draw another figure here and this figure so let me again draw the capacitor

so here is the capacitor plate here is another capacitor plate the wire is coming from here this wire is going away from here and so this current is flowing in like this and again the loop looks something like this that's my loop now there is no necessity that i must choose the flat surface i can choose a surface which looks like this that loop that surface as you can see here still that surface has this loop as the boundary but that surface does not intersect the wire anywhere it passes between the capacitor plates please remember in this equation in this equation i am free to choose any surface of integration to calculate the current enclosed the current loss in the surface for a given loop integral $\oint \mathbf{B} \cdot d\mathbf{l}$ is known and so if i take this loop and if i take this surface of integration which is surface cutting through the wire i enclosed is simply the current passing through the wire on the other hand if i happen to choose a surface which is passing between the capacitor plates and as the current changes i can see that on the right hand side there is no current enclosed because the surface is not crossing the wire at all and this wire is beyond that point so this current is passing away from here so it looks to me that the right hand side is \emptyset and i get a different result if i use the surface i get a finite value for the right hand side if i use the surface i get a \emptyset value for the right hand side so there is something wrong there is something incomplete in this equation and this was actually discovered by maxwell james clark maxwell and he modified this equation by adding a very very important term which is what i will call as the displacement current so there is some incomplete this equation seems to be incomplete because depending on the surface which i take i get a different value of the right hand side and there must be a problem with this equation so to to analyze this problem let me take a surface which is looking like this ok so just to be a little more specific let me take a surface in my current carrying wire here current is flowing like this flowing out that's my loop of integration and i take a surface which looks something like this okay so that's the surface which is a cylindrical surface which is lying like this for example ok so between the between the two plates the surface the surface is the flat flat surface between the two plates and ah is crossing this so if tossing the area between the two plates but it does not touch the wire now let me try to calculate so please remember here there is a electric field within this capacitor plates so let me calculate what

is the electric flux through this area electric flux to this area ϕ
 electric is equal to $\int \mathbf{E} \cdot d\mathbf{a}$
 so let me calculate the electric flux through
 this entire surface which I have now drawn and that is given by $\mathbf{E} \cdot d\mathbf{a}$
 so if I neglect
 fringing fields in the capacitor the field is uniform within the area between
 the two capacitor
 plates and this simply becomes equal to E times A area is the area A enclosed
 by this surface
 and if this surface depending on the the area now what is the current
 that is passing through the wire i is equal to dq by dt now which is equal to
 $\epsilon_0 \int \mathbf{E} \cdot d\mathbf{a}$
 so the flux is given by E times A
 so let me rewrite this equation
 again here A flux is given by E times A A is σ by ϵ_0 into
 A which is equal to q by ϵ_0
 so the electric flux is given by ϕ
 E is equal to $\int \mathbf{E} \cdot d\mathbf{a}$ and the flux is simply given by because
 electric field
 is uniform A is the area of the plates
 so E times A and the electric field is given by σ of
 ϵ_0 σ is the surface charge density and σ times A is q
 so $d\phi$ by dt is equal to $\frac{1}{\epsilon_0} dq$ by
 dt and the current is nothing but current flowing through the wire is dq by
 dt
 so let me put a subscript
 here called conduction current just to differentiate between another current
 remember we have already introduced a conduction
 current before a bound current and
 so this is the conduction current this is actually the
 current flowing through the wire because electrons are moving
 so this is i_c which is the
 conduction current
 so I get the fact that $A \frac{d\phi}{dt}$ is equal to $\frac{1}{\epsilon_0}$ by
 ϵ_0 into conduction current
 so conduction current is equal
 to actually $\epsilon_0 \frac{d\phi}{dt}$ ok
 so if I for example modify if I modify the
 ampere's law
 so this is the ampere's law
 so this is usually whenever I have been discussing
 ampere's law this is current enclosed is nothing but conduction current and
 closed
 so let me I can write this as μ_0 times conduction current enclosed ok $\oint \mathbf{B} \cdot d\mathbf{l}$
 stands for
 conduction current and on the right hand side it is always the current which
 is enclosed and
 in this case its conduction current enclosed now suppose I modify this law to
 the following integral $\oint \mathbf{B} \cdot d\mathbf{l}$ is equal to $\mu_0 i_c$ plus μ_0
 $\epsilon_0 \frac{d\phi}{dt}$
 so let me modify the ampere's law to this now you see what happens if I if I
 take a surface
 which is which is like this which I wrote before if this is my loop of
 integration and if this

is the surface then the right hand the second term is zero there is no electric flux and the first term gives me $\mu_0 n i c$ so if i use this on the right hand side if i use the surface to calculate the right hand side as the flat surface containing this loop then in this right hand side of this equation the second term is 0 because there is no electric flux and only the first term contributes which is $\mu_0 n i c$ on the other hand if i take a surface which is like this then in this case there is no conduction current i only have the second term and remember this term $\epsilon_0 \frac{d\phi_e}{dt}$ is exactly equal $i c$ so this term also becomes $\mu_0 n i c$ exactly equal to the right hand side when i took the flat surface

so let me repeat again this is the loop on which i am trying to calculate the magnetic field this is the position and the magnetic field i know i can integrate the left hand side and i get a value for the left hand side the question arises as to what i choose as a surface of integration to calculate the current enclosed so i can choose any surface which i want if i happen to choose the flat surface in which the current is crossing then the right hand side is simply $\mu_0 n i_{enclosed}$ and i conduction which is the first term the second term is absent because there is no electric field flux if i choose a surface which is like this which does not cut the current but it encloses the space between the two capacitor plates then the first term in this equation is zero and i am left only with the second term and the second term as you can see from here $\epsilon_0 \frac{d\phi_e}{dt}$ is exactly equal to the conduction current that is passing through the wire so this equation i becomes valid whether i take a surface which is and with inverse like wireless cutting or i take a surface into the wire is not cutting but i am passing through between the capacitor plates so this equation is more general and this is the generalized form of ampere's law this term was introduced by james clark maxwell in the year sixty five eighteen thirty one to eighteen seventy nine introduced modification to ampere's law in 1865 and this term is referred to as the displacement current this term which is coming here is called the displacement current and that happens we call a conduction current in this case so this is called the displacement current is $i_{d} = \epsilon_0 \frac{d\phi_e}{dt}$ so this modified form of ampere's law or generalized form becomes equal to $\mu_0 n i c$ plus $\mu_0 n i_{d}$ so there is a conduction current term on the right

hand side and there is a displacement current term on the right hand side both together have to be taken into account in this and this was a very major modification of the ampere's law introduced by Maxwell and it not only corrects the ampere's law as we will see that this introduces a completely different picture to electromagnetic equations because it predicts as I will show you later on the existence of electromagnetic waves the existence of waves which are just electric and magnetic fields and light is a form of electromagnetic wave radio waves are electromagnetic waves gamma rays electromagnetic waves x-rays and electromagnetic waves so there is the electromagnetic waves have a very broad spectrum of wavelengths and frequencies and the existence of electromagnetic waves came up through mathematical formulation in which Maxwell introduced this term and this is called the displacement current and so because the flux is determined by area and the electric field I can also define a displacement current density $\epsilon_0 \frac{d\phi_e}{dt}$ this is in free space I can define a displacement current density which is called which is $\epsilon_0 \frac{d\phi_e}{dt}$ let me put a vector here that's vector current current density and that is so this right hand side of the generalized ampere's law contains conduction current and displacement current so depending on the situation you may find a contribution to the right hand side because of conduction current only or displacement current only or both conduction and displacement currents so it is possible in situations where there is a conduction current and there is also displacement current both of them contribute to the generation of magnetic field now what is very significant is the fact that this term is a very very important term in the following sense suppose I have a situation where there is no conduction current a situation where there is no conduction current then according to Maxwell's equation $\int \mathbf{b} \cdot d\mathbf{l} = \mu_0 \left(\int \mathbf{j} \cdot d\mathbf{l} + \epsilon_0 \frac{d\phi_e}{dt} \right)$ because of the modification I have $\mu_0 \epsilon_0 \frac{d\phi_e}{dt}$ that is the displacement current and I am assuming that I am taking a region where there is no conduction current look at the other equation $\int \mathbf{e} \cdot d\mathbf{l} = -\frac{d\phi_m}{dt}$ this is the Faraday's law a changing magnetic flux induces an electric field a changing electric flux induces a magnetic field Faraday's law of induction tells me a changing magnetic flux induces an electric field in space a changing electric flux induces the magnetic field in space so this term actually makes the electric and magnetic fields coupled

to each other and symmetrizes the equations symmetry is very beautiful but here what is happening is essentially that changing magnetic flux generates electric field a changing electric flux generates magnetic field

so this at this term actually symmetrizes the equations electromagnetic equations and we will see later on when we start to discuss electromagnetic waves that this term actually predicts the existence of waves now let me take an example an example i want to consider is a parallel plate capacitor with circular plates of radii r and a capacitor is getting charged

so let me draw the two capacitor plates so that's one plate here another plate and a current is flowing like this and this is accumulating positive charge here and this is accumulating negative charge here and there is an electric field between these two now i want to calculate so this tells me this equation tells me that the changing electric flux generates a magnetic field

so i want to calculate so according to this equation because when i am charging the capacitor if i varies with time i am charging the capacitor the charge on the capacitor varies with time σ varies with time if σ varies with time electric field varies with time and if electric field varies with time the electric flux through any close surface to any surface not closer to any surface will vary with time if i take a surface like this the electric flux through this surface will vary with time and that should induce a magnetic field according to this equation because if the flux changes if the electric flux changes i should have a magnetic field

so let me try to calculate the magnetic field between the plates of a capacitor generated by the changing electric field now so in this case if i take this loop here and if i apply this equation $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 i_c + \mu_0 \epsilon_0 \frac{d\phi_e}{dt}$ now i had this general equation which was $\mu_0 i_c + \mu_0 \epsilon_0 \frac{d\phi_e}{dt}$ now for this surface there is no conduction current so this becomes equal to $\mu_0 \epsilon_0 \frac{d\phi_e}{dt}$ now because the plates are circular there is circular symmetry there is no variation with this direction with this distance here between the two plates the electric field is uniform and so the magnetic field will have only an azimuthal component it cannot have a radial component because of the magnetic flux total flux through any close surface being zero there cannot be radial magnetic field there must be

an azimuthal magnetic field

so the magnetic field must be pointing azimuthally like this
so let me calculate me use

this argument to calculate the magnetic field

so first thing is what is the electric flux ϕ_e is equal to the electric field into this area

so let me take an area radius r
so an electric field

into area which is equal to σ by ϵ_0 into πr^2 and what is σ if the area of the plates is A

so the area of the plates have a radius r
so area is equal to πr^2

so this is equal to

so this is equal to q by $\pi r^2 \epsilon_0$ into πr^2 which is equal to $q r^2$ by $\epsilon_0 r^2$

so that is the electrical flux passing through this

so rate of change of flux $d\phi_e$ by dt is equal to r^2 by $\epsilon_0 r^2$

dq by dt and dq by dt is nothing but the current that is charging the capacitor

so this

is r^2 by $\epsilon_0 r^2$ into i

so the rate of change of flux through this loop

small r i am assuming to be less than capital r that means i am taking a a loop within the

capacitor plates and smaller radius than the radius of the capacitor plates itself

so i get

$d\phi_e$ by dt this thing and $\int \mathbf{b} \cdot d\mathbf{l}$ now as i mentioned because of symmetry the

magnetic field will have to be azimuthal and if i calculate the $\mathbf{v} \cdot d\mathbf{l}$

i will get $2\pi r$ times b please note that i must take the right correct direction is the electric field is pointing to the right and i am integrating the flux as a positive

quantity which means the area vector of the area is pointing to the right here which means

that the the direction of the magnetic field must be like this and

so i get an equation

so i use if

i use this equation $\int \mathbf{b} \cdot d\mathbf{l}$ is equal to $\mu_0 \epsilon_0 d\phi_e$ by dt this gives me $2\pi r$

times b is equal to $\mu_0 \epsilon_0$ into $d\phi_e$ by dt i just now calculated

r^2 by $\epsilon_0 r^2$ into i which is

so b is equal to

so $\epsilon_0 \theta$

goes off and

so i get $\mu_0 i$ by $2\pi r^2$ into r

so one of the r cancels off

and i get $\mu_0 i r$ by $2\pi r^2$

so magnetic field within the area of the plates increases with small r that means at the axis the magnetic field is zero and as you increase small r up to capital r this will be the magnetic field so this is r lying between zero and r similarly i can calculate the magnetic field outside the plates of the capacitor so if i draw the figure again i have the capacitor plate like this now my loop is outside the place of the capacitor but the electric field is only in this region the electric field only exist in this region so $\phi = E \cdot A$ the electric the electrical flux becomes $\phi = E \cdot \pi r^2$ although this radius is this radius is small r there is only flux up to capital r so this is equal to $\sigma \cdot \pi r^2$ because πr^2 is the area of the plates σ is the charge density and so $d\phi/dt = \sigma \cdot 2\pi r \cdot dr/dt$ which is one by $\epsilon_0 \cdot i$ so if i again as use the fact that the magnetic field is azimuthal then i will get $2\pi r \cdot B = \mu_0 \cdot i$ $B = \mu_0 \cdot i / (2\pi r)$ so B is equal to $\mu_0 \cdot i / (2\pi r)$ this is for r greater than capital r so if i were to draw the magnetic field as a function of distance and this is capital r the magnitude increases and then decreases and at this point the magnetic field is $\mu_0 \cdot i / (2\pi r)$ so what we have seen is there is a magnetic field generated between the capacitor plates when the current is when the capacitor is charging once the charging is over once the current has become constant the rate of change of flux on the right hand side is zero there is no conduction current through this area there is no $d\phi/dt$ so there is a magnetic field magnetic field becomes zero so magnetic field is generated as long as the current is flowing in this in this region here or it is actually the flux is changing with time so there is no change of flux and so the magnetic field there is no there is no magnetic field generated by changing flux so i will discuss further examples in the next class and we will then move on to a very very important aspect of electromagnetic waves that is electromagnetic waves and how these equations predict the existence of waves which are the electromagnetic waves you