

a very good morning to all of you we were discussing electromagnetic induction and we continue with our discussions on the faraday's laws of induction remember we have seen that whenever there is a change of magnetic flux through any closed path there is an induced emf in the closed path and if there is a conductor in that path then then the emf generates a current in that path

so we had seen how the change of flux generates induced currents

so we we are looking at one example ah just as we finished the last class and the example was the motional emf

so we had seen that there was if i assume a magnetic field pointing downward and if i consider a conductor fixed conductor place like this and there is another conductor on this which is moving to the right with a velocity  $b$  then we found that as the conductor moves then there is an induced emf generated and we had interpreted this as if because when the conductor moves the free electrons in the conductor are moving the motion of the frequent electrons in the magnetic field generates a force on the free electrons

so because the velocity is in this direction magnetic field is pointing downward the  $v$  cross  $b$  is upward because the electrons have a negative charge they are pushed down and the electrons then follow this path and

so there is a current current generated the electrons are flowing in this clockwise direction

so that leads to a current in the anti-clockwise direction

so there is a induced emf leading to a current in the in this direction as shown here

so we had interpreted this from this emf from the lawrence force law because the conductor motion creates lorentz force on the electrons within the conductor and that lawrence force leads to a flow of electrons in the circuit and that flow of electrons constitutes a current now i had mentioned in the last lecture that the same emf could be derived from the basis of faraday's law of induction

so lets see how that can be done

so you see here the magnetic field is  $b$  we assume a uniform magnetic field

so if i assume this length is this distance is  $x$  the magnetic flux  $\phi$  is equal to  $b$  into this area which is  $x$  times  $l$  this this distance is assumed to be  $l$

so  $x$  times  $l$

so at any given instant of time the magnetic flux through this closed path is  $b$  times  $x$  times  $l$

so this conductor moves with the velocity  $v$  the rate of change of flux is equal to  $d\phi$  by  $dt$  which is equal to  $b$  times  $l$  times  $dx$  by  $dt$  and  $dx$  by  $dt$  is nothing but the velocity of the of the rod with which it is moving

so we get a rate of change of flux to be given by this

so according to faraday's law of induction the induce the  $\mathcal{E}$  minus  $d\phi$  by  $dt$  which is equal to minus  $b$  times  $l$  times  $v$

so because the flux i am calculating again the magnetic field is pointing downward the flux i am writing is positive

so the induced emf calculation must be in this direction and i get a negative value for induced emf which constitutes a current in the anti-clockwise direction and if you recall back in the earlier lecture we had obtained the same induced emf using lorentz force law

so these are two representations of the same induced emf in the case of conductor motion in the case of motional emf i can interpret the induced emf as if coming from lawrence force law but please remember if i keep the same situation here do not move the rod but if i change the magnetic field with time then there is again an induced emf generated in this circuit in the conducting path and that induced emf is not cannot be interpreted as lorentz force because

there is no motion of electrons and immediately only the magnetic field is changing and it induces an emf

so that induced emf is generated essentially by the change of flux in this so that is a more general law that whenever there is a change of magnetic flux through a closed path there is an induced emf generated in the closed path if the closed path contains a conductor along the path then there is a current generated in that closed path if there is no path if there is no conducting path in that then in free space a changing magnetic field will immediately generate an electric field like we had seen last time

so this is a very general and important law in electrodynamics essentially ah faraday's law of induction now look at the direction of the induced emf and the current the current is flowing like this

so the induced emf is like this and as we saw last time this current that means in this conductor there is a current flowing in the upward direction magnetic field is downward

so there must be a magnetic force on the current kind of conductor which is  $i \mathbf{l} \times \mathbf{b}$  we have seen before

so the magnetic force  $\mathbf{f}$  is equal to  $i \mathbf{l} \times \mathbf{b}$  now in this case  $\mathbf{l}$  vector is perpendicular to  $\mathbf{b}$  vector

so this is nothing but  $i \mathbf{l} \times \mathbf{b}$  the magnetic force magnitude is  $ilv$  and what is the direction  $\mathbf{l} \times \mathbf{b}$

so  $\mathbf{l}$  is up because the current is flowing in this direction

so  $\mathbf{l}$  vector is upward like this  $\mathbf{b}$  is downward and

so  $\mathbf{l} \times \mathbf{b}$

so the magnetic force is in this direction

so the wire is being pulled back towards the this side

so i am trying to move it away from the in the right to the right direction there is a magnetic force to the left and this magnetic force also comes from the fact that the induced emf has minus  $blv$  which means the current is flowing like this

so remember this negative sign we had introduced in the beginning as lense's law that the induced emf is always

so as to oppose any change in the flux in the loop now we can also see this from physical principles that there must be a negative sign here suppose imagine a situation where the induced emf was clockwise and not anti-clockwise

so when i try to move this rod to the right imagine a situation where the magnet negative sign did not exist that means the emf would have been in this direction

so then in that case in this rod the current would have been flowing down and the magnetic force would have been to the right now the problem with this is that suppose i give the rod a small push to the right the magnetic fields magnetic flux starts to change and if the induced current is in the downward direction this will result in a magnetic force to the right side which will increase the speed of the rod which increases velocity

so as the velocity increases the rate of change of flux increases the induced emf even increases further increasing to greater force and this goes on increasing independently obviously this is not a physical situation

so even from this consideration we can see that there must be a negative sign here which is essentially lense's law

so lense's law just comes out of the fact that energy conservation is required and when i try to move this rod to the right there is a magnetic force to the left if i try to move to rod to the left there is a magnetic force to the right

so i have always to do work against the magnetic force to move the rod and this is what is essentially lense's law

so this will essentially ensure that the induced emf is in the direction of minus  $d\phi$  by  $dt$  now let me look at an example and put some numbers into the calculation here

so let me look at an example

so let me assume the uniform magnetic field again pointing down ah these are my conducting rods and this distance is  $l$

so let me as an example take a magnetic field  $B$  of point five tesla ah let me assume a length  $l$  of ten centimeters which is point one meters let me assume a velocity of this to be equal to two meters per second suppose i am trying to pull it at the rate of two meters per second and let me assume the resistance of this loop is point zero five ohms

so i have a situation like this in a uniform magnetic field which is pointing downward with point five tesla this length i am assuming to be about point one meter ten centimeters and i am trying to pull this at a uniform speed of two meters per second to the right and let me assume as i mentioned before the resistance primarily is here and there is no resistance of this of this part of the circuit

so the resistance remains constant at point zero five ohms

so what is the induced emf  $\mathcal{E}$  is equal to  $B \times l \times v$  which we have just derived which is equal to point five tesla into point one meters into two meters per second which is about point one volt

so there is an induced emf of point one volt across the circuit here and this induced emf comes in because i am trying to pull the rod to the right and this is an induced emf and this induced emf will generate a current  $i$  is equal to  $\mathcal{E}$  by  $R$  which is equal to point one by point zero five ohms which is about two amperes

so there will be a current of two amperes the resistance is only point zero five ohms i am assuming

so there is a current of 2 amperes generated in this now this current as we saw before will generate a force magnetic force to the left side and i can also calculate the magnetic force on this rod magnetic force on the rod which is equal to  $i \times l \times B$  which is equal to two amperes times point one meters times point five tesla which is equal to point one newton

so there is a force on this rod of point one newton to the left side

so if i keep the velocity constant at two meter per second i need to apply a force of  $\theta$ .

1 newton to the right to keep the velocity constant as 2 meter per second otherwise the the magnetic field which is inducing this emf is trying to pull that out back in the reverse direction

so that is an example which gave me a some kind of numbers which are involved in this process now till now what we have been doing is we have been ah assuming a certain conducting path and calculating what is the induced emf in that conduct in that path and if there is a conducting path in there there is a current generated and we are calculating the current now many situations what happens is there is no conductor conducting wire there but there is a conducting solid

so if i have a conducting solid in a magnetic changing magnetic field then they can in the conducting solid there will be currents generated just like currents in a conducting path because as we have seen a changing magnetic field will generate an electric field and this electric field will then try to move the charges within the conductor and that will lead to a current

so there are this these are called eddy currents

so whenever you have a conducting material which is subjected to a changing magnetic flux then induced current gets generated in the throughout the volume

of the conductor

so if you had a wire the conducting part is essentially determined by a line and the current flows along that line but if you have a solid conductor then the conducting the induced electric fields will tend to generate the current right through the volume of the conductor and these are called eddy currents because these currents resemble eddies on a on in water and

so they are called ed currents in fact remember the demonstration that we had done ah in the beginning of the course in the beginning of the discussion about magnetostatics we had taken a i had taken a solenoid a bound solenoid here and then we had a a soft iron piece inside through the hole of the solenoid here and then we had essentially an aluminium disc sitting on it and we had shown that if you change the current in the if you had applied an ac current through this then what happens is the current in the coil changes with time that changes the magnetic field generated by the solenoid with time and that solenoid just changing magnetic field will induce a current in this ah in this conducting aluminium sheet here and what we saw is essentially this is a sheet here

so this is this is the sheet here and we saw that the eddy currents actually lead to a repulsion because induced emf is always

so as to oppose changes

so as the magnetic field is changing with time there is a induced emf in the entire volume of this aluminium disc which essentially pulls pushes the aluminum disc upwards and we saw a levitation

so depending on the magnitude of the rate of change of flux the force on this will be different and you can actually raise it to different heights and that is essentially very interesting concept of magnetic levitation now eddy currents also can happen in other situations for example let me draw another figure which i will show you another kind of eddy current

so you have

so essentially what is happening is suppose i had a suppose i had a conducting ah surface here suppose i had a conducting surface a solid conducting here and if i had a a solenoid carrying current the solenoid will have its own magnetic flux surrounding things

so as i bring the solenoid closer and closer and if i change the magnetic field of the solenoid which means by changing the current passing through the solenoid i will generate eddy current like this in this spot in the volume of the conductor here this is the conductor here and in the volume of the conductor i will generate eddy currents they will circulate like this because the changing magnetic field generates electric fields and those electric fields then drive the electrons within this conductor leading to eddy currents these are the current that will get generated within the solid and this is exactly what is happening in this case of the demonstration that i showed you there are any current generated in this and those eddy currents are responsible for the effective magnetic repulsion of the solid with respect to this now in fact what is happening is you may have noticed that this was not a solid rod it consisted of a large number of small rods here there were large number of rods actually and the reason is the following

so suppose i had suppose i had a solid rod like this in which the magnetic field was pointing like this and changing with time then you will see induced currents like this everywhere and these induced currents as they pass through this eddy currents as they pass through the solid conductor will lead to essentially a heating because the solid conductor has some resistance

so as the currents pass through the solid conductor they will generate heat and the energy that i am feeding actually partly gets converted to heat in this conductor i can actually ah reduce this effect by using whatever called as

laminator

so if i take instead of a single piece i take a larger number of these cylindrical rods smaller diameter rods forming the same diameter overall diameter as before then what will happen is the eddy currents have no path to get completed and

so the amount of eddy current generated in this situation becomes much reduced so this is called lamination and i can do a lamination to reduce effectivity currents by breaking up the entire solid into smaller size smaller pieces and in that situation this can help me to reduce any eddy current losses in the device and this is precisely used in transformers to have instead of a solid core you have laminated core which essentially ensures that the eddy currents do not have a path to complete themselves and

so the amount of eddy current generated gets reduced and that leads to a reduction in overall heating of the of the core there is another very interesting demonstration that could be done and that is the following

so suppose i had again a magnetic field pointing downward a uniform magnetic say pointing downward in this space

so let me assume that i have a a copper plate which is pivoted like this and which is oscillated like this

so i have a copper this is a copper plate here hanging on a on a distinct solid and on a wire here and this is oscillating like this now imagine what happens as this as this copper plate enters the magnetic field the magnetic flux through this changes the magnetic field applied on this conductor changes and that induces an emf and eddy currents are generated

so as this enters inside the magnetic flux is increasing and

so the eddy current will be such as to counter this increase in magnetic flux

so magnetic field generated by this should be opposite to the direction of this magnetic field which must be upwards

so it will generate an induced current in this direction

so please note as this as the solid moves into the magnetic field the magnetic field is increasing with time the magnetic flux is increasing with time

so the induced emf will be such as to reduce this increase which means that the currents generated in the solid must be

so as to oppose this magnetic field which means that they must generate a magnetic field which is pointing upward because this magnetic field is pointing downward and that will be generated if a current is flowing like this

so what happens is this current starts to flow in this direction and as the solid copper piece enters this magnetic field and once it is fully entered there is no change in magnetic flux and there is no effect until the the solid copper p starts to exit exit the magnetic field now what will happen is as the rod as the plate exits the magnetic field the magnetic flux is decreasing with time and

so the induced current in this must counter this effect which means it must generate a magnetic field which is pointing downward and that means the magnetic field must the current must be in this direction

so that this current produces a magnetic field pointing downward and it actually leads to an eddy current in this direction now what happens in this case is there is a a a force which is acted upon because there are currents here that are magnetic fields this current actually this this force magnetic force tries to ah tries to put a friction on the movement of this plate and this is essentially the the plate will not oscillate freely and it will be damped into a very quick stop of the into the magnetic field

so it's a very interesting example again a very nice demonstration that can be shown to show the effects of induced emfs now there are many applications of this induced emf

so let me note down some of them they are used in magnetic braking in trains because they provide a restoring a force in the opposite direction

so they can be used to break they are also used in electric motors they are used as induction furnaces remember i mentioned to you that these currents as they plus pass through the material the conductor they will generate heating because of joule heating and that heating can be used for making a furnace they are also used as metal detectors for example airports where you can you have seen that there are detectors which detect presence of metals and this is this can be based on induction ah and i must mention that there are some undesirable effects and the an undesirable effect is essentially that it causes heating in transformer core and as i just now mentioned this can be reduced by laminating the core

so for example in this in this plate in this experiment if i had instead of doing this if i had let me draw another figure here on the side

so if i had a situation in which the the plate was not a solid plate like this but it was laminated

so for example if i had a plate and this plate was like this then what happens is you have sort of destroyed the path for the induction for the emf to generate currents the eddy currents get much reduced and the damping is much reduced here because edd current gets reduced

so instead of having a solid piece like this if you had cut pieces you can reduce you can reduce the effect of eddy currents and that is what essentially is done in lamination of the core of a transformer where you can reduce the effects of eddy currents

so there are many applications ah we will discuss a little later also again but there are many applications of these induced emfs which are very useful in practical situations now this induction leads to electromagnetic induction leads to another very important concept that is concept of inductance

so let me assume that i have ah two coils place close to each other

so let me call this this is this is loop one and this is loop two now i pass a current through this loop suppose i pass a current through this loop this will have a magnetic field associated with this

so the magnetic field may be generated like this

so when i pass a current through this loop one i will generate a magnetic field because of this current there is a loop which is placed close to this loop one which i call loop two

so the part of the magnetic flux will pass through this loop two and

so the loop two will enclose certain kind of flux now notice that the magnetic field produced by loop one is proportional to the current passing through loop one

so suppose i call the current  $i_1$  remember the magnetic field produced by this current is  $\mu_0 i_1$  by four pi integral over loop one  $d\mathbf{l}_1$  cross  $\mathbf{r}$  by  $r^3$  we have seen bio sever law before

so the magnetic field produced by this loop one is given by this and is proportional to  $i_1$  and

so the flux through loop two which is equal to the magnetic field into this area actually integration of  $\mathbf{b} \cdot \mathbf{t} \, a$

so this is  $\mathbf{b} \cdot d\mathbf{a}$  over loop two will be proportional to  $i_1$  note here

so  $b_1 \cdot d\mathbf{a}$   $d\mathbf{a}$  two

so i am integrating over the area of loop two the magnetic field produced by the loop one current  $i_1$  in loop one is  $b_1$  loop two is placed somewhere close to loop one

so some of the magnetic field lines generated by loop one pass through loop two the magnetic field generated by loop one is proportional to current  $i_1$

passing through loop one and

so the magnetic flux passing through loop two is proportional to the current passing through  $i_1$

so  $i_1$  will  $i_1$  can write a relationship flux through two is equal to  $M_{21} i_1$  where  $M_{21}$  is a constant of proportionality and is called mutual inductance its mutual inductance because it is a its how much flux is linked by loop two because of current in loop one

so its a mutual inductance relationship here between these two these two loops and  $M_{21}$  actually is a constant which depends on the orientation of the two loops the position of the loops the geometry the shape etcetera etcetera

so this is a quantity which depends on the position and orientation area etcetera shapes of the loops but this is a constant and the flux passing through the second coil loop two is proportional to current passing through  $i_1$  through the loop one and this point this proportionality constant is called mutual inductance and this is a very very important quantity

so let me look at an example of this mutual inductance to understand mutual inductance let me look at an example of two coaxial long solenoids

so i have one big solenoid like this with a coil and i have another solenoid inside is another coil here

so let me call this solenoid  $S_1$  the solenoid  $S_2$

so i have solenoid  $S_1$  ah

so number of turns per unit length  $n_1$  radius of  $S_1$  is  $r_1$  solenoid  $S_2$  number of turns per unit length sorry this is  $n_2$  and radius of  $S_2$  is  $r_2$

so  $n_1$  turns per unit length for solenoid  $S_1$  of radius  $r_1$  and  $n_2$  turns per unit length of turns per for solenoid  $S_2$  and of radius  $r_2$  now let me assume that i pass a current  $i_1$  pass current  $i_1$  through  $S_1$

so i have ah this  $S_1$  is connected to a current source and first of all i am assuming that these are very long solenoids

so ah the i can assume the magnetic field generated by these solenoids is uniform within the solenoid and this is  $B_1$  as now current carrying a current  $i_1$

so magnetic field produced produced by  $S_1$  is equal to  $\mu_0 n_1 i_1$  and this is within the solenoid of course i must remember that this magnetic field is generated within the solenoid  $S_1$  and there is no magnetic field outside the solenoid of  $S_1$  when the current is passing through  $S_1$

so now please remember the solenoid  $S_2$  is surrounding solenoid  $S_1$

so the magnetic field generated by  $S_1$  forms a part of magnetic flux that is crossing  $S_2$

so magnetic flux is now because of the magnetic field generated by  $S_1$  there is a magnetic field flux through  $S_2$

so what is the magnetic flux magnetic flux through each loop of  $S_2$  which is equal to the magnetic field  $\mu_0 n_1 i_1$  into this area only because although  $S_2$  is bigger there is no magnetic field outside between  $S_1$  and  $S_2$  the magnetic field only exists within  $S_1$

so the flux is calculated by magnetic field times area but area in which magnetic field exists only in this region

so this must be equal to  $\mu_0 n_1 i_1$  into  $\pi r_1^2$  please note although the area of  $S_2$  is  $\pi r_2^2$  the magnetic field occupies only an area  $\pi r_1^2$  square within the solenoid  $S_2$  and that magnetic field is uniform

so every loop of  $S_2$  has a flux of  $\mu_0 n_1 i_1$  into  $\pi r_1^2$  square through the ah through the solenoid  $S_2$

so the flux through length  $l$  of solenoid of which is equal to

so this is magnetic flux through every turn  $n_2 i_1$  into  $\pi r_1^2$  square into

the number of turns in a length  $l$  which is  $n$  two times  $l$

so this is equal to  $\mu_0 n_1 n_2 \pi r_1^2 l$  into  $i_1$

so this is  $i_1$  will write this as  $M_{21} i_1$  and in this example  $M_{21}$  happens to be  $\mu_0 n_1 n_2 \pi r_1^2 l$

so that's the mutual inductance between these two solenoids

so all it means is that when I have a current  $i_1$  passing through  $S_1$  then over a length  $l$  the flux passing through solenoid  $S_2$  is simply  $M_{21} i_1$  and  $M_{21}$  for this example happens to be  $\mu_0 n_1 n_2 \pi r_1^2 l$  and that's the mutual inductance between these two coils and it depends on as you have seen here it depends on the number of turns per unit length of solenoid  $S_1$  the number of turns per unit length of solenoid  $S_2$  the area of the solenoid is  $\pi r_1^2$  and the length of the entire divide that I am looking at

so I can define here mutual inductance per unit length as  $\mu_0 n_1 n_2 \pi r_1^2$  now

so what happens suppose I pass a current  $i_1$  through the outer solenoid instead of the inner solenoid

so I can also again associate a flux enclosed by  $S_1$  because of a current in  $S_2$  then I will get another mutual inductance

so for example if I pass a current  $i_2$  through  $S_2$

so current  $i_2$  through  $S_2$  then what I will get is the magnetic field generated is equal to  $\mu_0 n_2 i_2$   $\mu_0 n_2 i_2$  is the magnetic field generated by  $S_2$  now that magnetic field is part of the flux is passing through  $S_1$

so  $S_1$  now encloses a certain flux and there is a mutual inductance between  $S_1$  and  $S_2$  when I pass a current through  $S_2$

so what is that let me calculate

so flux through each loop of  $S_1$  which is equal to  $\mu_0 n_2 i_2$  into look at look here the flux to  $S_1$  depends only on area of  $S_1$  which is  $\pi r_1^2$

so the magnetic field lies inside and outside  $S_1$  when I pass a current  $i_2$  through  $S_2$  there is a magnetic field generated and within the entire solenoid  $S_2$  but the flux that is linked by  $S_1$  is only in this area

so that is equal to the magnetic field into area

so flux through a length  $l$  of  $S_1$  is equal to the flux through each loop into  $n_1$  into  $l$   $n_1$  is the number of turns per unit length multiplied by the length and this is  $\Phi_{21}$

so this is equal to  $\mu_0 n_1 n_2 \pi r_1^2 l i_2$  which I write as  $M_{12} i_2$  where  $M_{12}$  is the mutual inductance which is  $\mu_0 n_1 n_2 \pi r_1^2 l$

so that's the mutual inductance between two and one this is the mutual inductance I get when I pass the current  $i_2$  through  $S_2$  and the mutual inductance which I got when I pass a current  $i_1$  through  $S_1$  is essentially this quantity and as you can see here they are exactly the same  $M_{21}$  is equal to  $M_{12}$

so the mutual inductance between these two coils

so if I pass a current  $i_1$  if I pass a current  $i_1$  through  $S_1$  the flux magnetic flux enclosed by  $S_2$  is  $M_{21} i_1$   $M_{21} i_1$  into  $i_1$  if I brought the same current through  $S_2$  the flux enclosed by  $S_1$  is the same because the proportionality constant  $M_{21}$  and  $M_{12}$  are exactly equal

so let me reply repeat if I pass a current through  $S_1$   $S_2$  encloses the flux which is proportional to the current passing through  $S_1$  and the proportionality constant I called it  $M_{21}$  if I brought the same current through  $S_2$   $S_1$  now encloses a flux and that flux is proportional to current

$i_1$  and  $i_2$  passing through  $S_1$  and  $S_2$  and the proportionality constant is called  $M_{12}$  and  $M_{21}$  and both these are exactly equal

so  $M_{12}$  is equal to  $M_{21}$

so that's a very important relationship now although I have shown this for this example of a pair of solenoids one can in general prove this result that the mutual inductance between these two coils is a fixed quantity and

so I have this proportionality constant is called  $M$

so I can also write  $M_{12} = M_{21} = M$  it does not depend on whether I pass a current through  $S_1$  and look at  $S_2$  or I pass the current through  $S_2$  and look at  $S_1$ .

so this is a very very useful result

so let me give you an example where this equality becomes very useful

so let me look at an example

so let me assume I have a solenoid like this a very long solenoid and I place inside this a short solenoid now my objective is to calculate the flux enclosed by the outer solenoid when I pass a current through the inner solenoid

so I pass the current through the inner solenoid

so I want to calculate the mutual inductance

so I want to find out what is the flux enclosed by the outer solenoid when I pass a current  $i$  through the inner solenoid now you see here the problem becomes quite complicated because this solenoid is not an infinitely long solenoid it will generate its magnetic field like this

so each loop of the outer solenoid is enclosing different amounts of magnetic fluxes and

so on

so this becomes a very complicated problem but I can use this relationship that  $M_{12} = M_{21}$  and this I will get the same relationship if I pass the same current through the outer solenoid and calculate what is the flux enclosed by the inner solenoid must calculate the flux enclosed by the outer solenoid when I pass a current through the inner solenoid that problem is a little complicated

so I can use the relationship that  $M_{12} = M_{21}$  and I pass the same current through the outer solenoid and I calculate the flux enclosed by inner solenoid because that is much much easier

so for example if I pass a current  $i$  through the outer solenoid and if I let me write this as this is  $n_1$  number of turns per unit length and this is  $n_2$  number of turns per unit length of the shorter solenoid let me assume this length is  $l_2$  and let me assume that this radius is long solenoid is  $r_1$  and the short solenoid has a radius  $r_2$

so the number of turns per unit length of the shortest solenoid is  $n_2$  then radius of the inner solenoid is  $r_2$  and  $r_1$

so the magnetic field for a current  $i$  through the outer solenoid the magnetic field is equal to  $\mu_0 n_1 i$  times the number of turns per unit length times the current

so the magnetic flux

so this please remember now this magnetic field is uniform within the outer solenoid and this inner solenoid occupies a certain area of this and

so the flux through the shorter solenoid the smaller solenoid is equal to the magnetic field into the area into the number of turns which is  $n_2$  into  $l_2$

so this is the magnetic field that is the area this magnetic field this is the area and the number of turns total number of turns please remember that this flux is per turn and there are

so many turns

so this is equal to  $\mu_0 n l n s \pi r^2$  into  $l$  into  $i$   
so i get an expression for mutual inductance between these two  $M$  is equal to  $\mu_0 n l n s \pi r^2$

so this relationship is very useful here because if i were to calculate the mutual inductance between these two solenoids when i pass a current  $i$  through the inner shorter smaller solenoid it would have been very difficult for me because the magnetic field is not uniform and different loops of the outer solenoid have different fluxes passing through them and that would have been a very complex problem

so i have solved the problem by using the fact that  $M_{12}$  is equal  $M_{21}$  and that has given me a very simple solution to this problem

so this is a very important relationship in mutual inductance and can be very useful in certain situations

so when there is a flux enclosed when there is a flux enclosed by the two by one circuit because of the other

so let me recall this

so when i have whenever whenever flux passing through this

so according to faraday's law a changing current in one of these coils should induce an emf

so changing current through say loop one will induce an emf in loop two

so if i loop one here another loop two and if i change the current here as a function of time that will induce an emf in the second loop and that induced emf will be equal to minus  $d\phi_2$  by  $dt$  which is equal to minus  $M$  times  $d i_1$  because  $\phi_2$  the magnetic flux is equal to  $M$  into  $i_1$  where  $M$  is a mutual inductance

so if i have a a loop here another loop here if i change the current in this loop

so if i have a current  $i_1$  passing through this and if i change the current if i change the current then this current actually leads to a flux magnetic flux of  $M$  times  $i_1$  in this loop and when i change the current that induces an emf in the second loop and that is given by minus  $M$   $d i_1$  by  $d t$  and that's a very important relationship when i um when i look at circuits in which there are multiple loops present

so the individual emf is a relationship that we must remember this is equal to  $M$  times minus  $M$  times  $d i_1$  by  $d t$  and there are two loops placed close by now having seen mutual inductance we see that there is another important concept which is also called self inductance

so let me again take a solenoid a long solenoid and i pass a current through the solenoid okay now when i pass a current through the solenoid there is a magnetic field generated by the solenoid and for example this these loops are also encircling those magnetic fields

so the magnetic field generated by the solenoid is also forming a part of the flux that is passing through the same solenoid

so suppose let me assume that the number of turns per unit length is equal to  $n$  and current is equal to  $i$  then the magnetic field is equal to  $\mu_0 n i$

so each loop of solenoid will have a flux  $\mu_0 n i$  into if i assume the radius of the solenoid to be  $r$   $\pi r^2$

so each loop of the solenoid will have a flux passing through this

so much

so total flux total magnetic flux through a length  $l$  of the solenoid is equal to  $\phi$  times the number of turns which is  $n$  times  $l$

so let me calculate this again

so this is the total flux first thing now is equal to  $\phi$  times  $n$  times  $l$  which is equal to  $\mu_0 n i \pi r^2$  into  $n$  times  $l$  which is equal to  $\mu_0 n^2 i \pi r^2 l$

$n^2 \pi r^2 l$  into  $i$  now I call this as  $L$  times  $i$  where  $L$  is equal to  $\mu_0 n^2 \pi r^2 l$  in this example this is called self inductance it is called self inductance because it is the flux the flux through a coil flux through the solenoid the magnetic flux passing through the solenoid because of a current passing through the same solenoid is nothing but proportional to the current passing into the solenoid and that proportionality constant is called self inductance and in this case it comes out to be  $\mu_0 n^2 \pi r^2 l$  and this is a very very important part of a coil like this and this is the  $L$  this self inductance defines how much is the flux passing through this through the solenoid if I have if I pass the current through this through the solid say same solenoid now if I change the current in the solenoid if I change the current passing through the solenoid then we see that this if I change  $i$  with time this will induce an emf in the coil itself because when I change the when I change the current through the solenoid I will change the flux passing through the same solenoid and that change of flux will lead to a an emf and the emf generated  $\mathcal{E} = -L \frac{di}{dt}$  is equal to minus  $L \frac{di}{dt}$

so that is induced emf

so whenever I try to change the current through the solenoid whenever I try to change the current through the solenoid there will be an induced emf generated in the solenoid and by Lenz's law that induced emf will try to generate a current to oppose this change and

so for example if this current passing through the solenoid produces the magnetic field pointing upwards and the current is increasing the magnetic flux in the upward direction is increasing with time

so the induced emf is will be such as to generate a current in the reverse direction to reduce the flux to reduce the change in flux to reduce the increase in flux

so it will try to oppose any increase in flux and its going to be resistance to the change of flux it is it is giving a resistance to my increasing current if I try to increase the current it sort of tries to maintain the current as before similarly if I try to reduce the current I will reduce the flux pass into the same solenoid and then use current will be such as to oppose this change will try to maintain the flux as before

so this induced emf is also called as back emf because it actually tries to reduce the changes that you are trying to impose on the circuit

so if you are trying to change the current in a circuit that will induce an emf in the circuit a back emf and that back emf will be in such a direction or that will generate a current in such a direction that it will try to oppose this change and

so will not allow this change to take place

so this is a very important part of circuits and this is called inductance and so inductance is like capacitance the capacitance was the device in electrostatic circuits in electrical circuits and inductance is a is a device which is used in magnetic material magnetic part of the circuit and this is always a positive quantity is always a positive quantity and self inductance plays the same role as mass in mechanical systems it started with inertia it gives an inertia

so larger the value of larger the value of  $L$  harder it is to change the current

so whenever we have a coil like this and when we try to change the current in the coil that will give a back emf and that back emf is opposing the change that you are trying to introduce and its like an inertia on the system and it acts like mass in mechanical systems depending on if the heavier the object the greater the force you need to move it and

so similarly in the case of inductance it is a its like an inertia and it sort of opposes any change that you are trying to do

so let me look at an example

so take a long solenoid of

so having 100 turns per 10 centimeter let me take a radius of 1.6 centimeter

so i want to calculate self inductance per unit length

so we had calculated a self induction inductance here this is this is the equation per unit length will be  $\mu_0 n^2 \pi r^2$

so  $L$  per unit length will be  $\mu_0 n^2 \pi r^2$

so this is equal to four pi ten to the minus seven into hundred tons per centimeter is per meter is ten power four

so that print was eight into pi into one point six whole square to ten to minus four into length is your one meter and this is comes out to be is equal to approximately equal to 0.

1 now let me define this this  $H$  refers to henry the unit of inductance is henry and one henry is one tesla meter square by ampere remember the inductance is defined through this equation where the flux is  $L$  times  $i$

so unit of inductance is flux divided by current flux is magnetic field into area

so inductance must be magnetic field into area by current which is tesla meter square per ampere

so thats called henry  $H$  and this is unit and

so this particular self inductance which have about point one henry per meter and if  $i$  were to change the current if  $\frac{di}{dt}$  is equal to 10 amperes per meter per sorry per second then the induced emf will be equal to minus  $L \frac{di}{dt}$  which is equal to minus point one into ten which is equal to one volt per one sorry one volt

so that induced an emf of one volt in the in the solenoid and that will act against your applied source that you are doing

so that you are passing the current through

so that is an example of a self inductance

so i will stop my lecture here and we will continue with the discussion on mutual inductance in the next lecture thank you very much you