

a very good morning to all of you we will continue with our discussion in the field of magnetostatics ah remember last time we started looking at magnetic fields and calculation of magnetic fields etcetera now before we started magnetostatics we had discussed electrostatics in which we said that ah a charge a stationary charge is affected by an electrostatic force

so if you have a charge then it creates an electric field surround in the surrounding region and if you put another charge here that charge is either attracted or repelled by this electric field

so depending on the type of charges you can either have attraction or repulsion and this force is along the line joining these two charges

so that is electrostatic force now in magnetostatics we look at magnetic field effects and these magnetic fields are generated by currents

so when you have a stationary charge it has no magnetic effects because the only effect is electric electric effects

so if you have a charge which is stationary even if there are magnetic fields there is no force on the charge the charge is affected only by electrostatic forces when the charge starts to move then apart from the electrostatic force there is another force which is called the magnetic force now if i have a charge which i make to move along certain direction i find that the force depends on the direction in which i am moving this charge

so suppose i take a positive charge and move like this there is a certain force acting on the charge if i move in another direction then the force is different

so what i do is i vary the direction of propagation and i find that along one direction of propagation there is no magnetic force

so if i vary the directions i will find one direction of propagation along which there is no magnetic force and that direction defines the direction of the magnetic field at that point and then if i vary the direction of my velocity vector i find that when the particle is moving perpendicular to this direction of zero force

so for example if the zero force was along this direction if i move perpendicular to that in any orientation i find that the force on the charge is maximum

so the force acting on this moving charge depends not only on the velocity this the speed of the particle but also the direction in which the particle is moving and

so we had defined the magnetic field through a relationship with the force just like electric field we are defined a magnetic field

so suppose you had a magnetic field which is represented by  $\mathbf{b}$  vector like this and if you move a charge in this direction then you find that the magnetic force magnitude is  $q v \sin \theta$

so we had defined  $b$  as magnitude of force divided by  $q v \sin \theta$

so this is 90 degrees and

so this is this is unit called tesla that is one newton per ampere meter

so tesla is a big unit and we had also introduced another unit called gauss which is  $10^{-4}$  tesla

so this is the force acting on the charge

so if the charge moves in different directions then the force changes and so we find that the force can be represented by a vector relationship  $\mathbf{F} = q \mathbf{v} \times \mathbf{b}$

so if i have a coordinate system like this

so  $x$ ,  $y$  and  $z$  suppose i have a magnetic field oriented like this and if my velocity of the charge particle is in this direction

so suppose let me assume there is a positive charge moving in this direction then the force is  $q \mathbf{v} \times \mathbf{b}$  and if this is angle  $\theta$  then the magnitude of the

force as you can see here the magnitude of the cross product is  $q v b \sin \phi$  depends on this angle and if  $\phi$  is zero then the force is zero as we discussed before that is the direction of the magnetic field if  $\phi$  is ninety degrees you get the maximum force  $q v b$  also note the direction of the force unlike electrostatic forces which were acting along the direction of the electric field either towards the direction of electric field or oppositely the magnetic forces are perpendicular to the magnetic field  $b$  and to the velocity vector

so you must have studied cross product before

so  $v \times b$  is a vector in this figure  $v \times b$  is a vector in this direction so if the charge is positive this force has the direction of  $v \times b$  and as i mentioned last time i must use the right hand rule

so i take my right hand and move my four fingers from  $v$  to  $b$  and the direction of the thumb represents the direction of the force

so i get this force along  $v \times b$  here and the magnitude of the force

so the unlike electrostatic forces magnetostatic forces are perpendicular to the velocity vector as well as to this magnetic field direction and that defines my force on the on this moving charge also note that if  $q$  is negative the force is opposite in the opposite direction is at the direction of  $-\mathbf{v} \times \mathbf{b}$  if  $q$  is negative after we had discussed this we introduced bio savart law which will exp which will tell us what is the magnetic field produced by current carrying conductors

so if you have a current carrying conductor like this suppose the current is propagating in the direction

so i take a small elemental length  $dl$  the direction of  $dl$  vector is along the current direction and if i have to calculate the magnetic field at this point i draw a vector joining these two points the current element  $idl$  and the position here  $r$  vector

so the magnetic field  $db$  because of this current element  $dl$  is  $\mu_0$  by four pi  $i dl \times r$  by  $r^3$  we had discussed this before in the last lecture that the magnetic field generated by this current element  $dl$  where real vector  $idl$  is the current element here at this position  $p$  whose coordinate with respect to this is  $r$  vector here then the magnetic field generated by this is represented by this equation  $\mu_0$  is the permeability of free space and as a value four pi ten to the minus seven tesla meter per ampere we are also seen that  $\epsilon_0 \mu_0$  is one by  $c^2$  where  $c$  is the velocity of light in free space speed of light in free space and

so  $\epsilon_0$  which is the dielectric permittivity of free space and  $\mu_0$  the magnetic permeability of free space are related to this equation  $\epsilon_0 \mu_0$  is one by  $c^2$

so this gives me the magnetic field generated by a small current element and just like electrostatic fields the magnetic fields also satisfy superposition principle

so if i want to calculate the total magnetic field generated by this entire element of current here i have to calculate the current i take individual current elements at different points and calculate the magnetic field generated by each individual current element add them up at this point vectorially and get the total magnetic field

so in fact last class what we have done is to calculate the magnetic field due to a circular loop of current

so let me recall we had taken a loop like this i can call this  $z$  this is  $x$  this is  $y$  and let me assume current is flowing like this

so ah we try to calculate the magnetic field along the axis to get a simplified expression we can get analytical expression for magnetic field along the axis by using bio sapher law

so we started looking at it

so what we need to do is if this is the point  $p$  i need to consider different elements of current here here here etcetera and integrate all the current elements the magnetic field produced by all current elements at this point and calculate the total magnetic field now we had used some physical arguments to show that for every element here there is another current corresponding element on the other side which produces a magnetic field whose  $x$  components cancel now i want to show this a little more rigorously here before we move on to the next problem and let me do the following

so let me draw a figure now which corresponds to the exact plane the plane  $xz$  and let me draw a figure here

so this is  $x$  axis here this is the  $z$  axis

so remember the current is coming out of the paper here and going into the paper on the other side

so if i extend  $x$  axis in the backward direction here

so the current is coming out from this direction and current is going back

so current is moving in the  $y$  direction here and minus  $y$  direction here

so i will draw corresponding arrows here

so this is a point in the center of a circle means the arrow is pointing up which means the current is coming out of the paper here at the same distance on the other side i will have i will plot the end of the arrow and that is like the current going towards the into the page of the paper and this is the radius of the radius of the current loop

so this is the radius of the current loop capital  $r$  and

so this is the exact plane and my problem is to find out the magnetic field at this point  $p$

so this if you as you can see here this has coordinates  $0 z$  and this has coordinates

so  $r \theta$  and this has coordinates minus  $r$  zero  $x$  coordinate is  $r$   $z$  coordinate is zero there is no  $y$  coordinate i am in the exact plane similarly the  $x$  coordinate is minus  $r$  here and the  $z$  coordinate is zero

so let me try to find out what is the magnetic field generated by this small current element at this point

so i have a small element of current coming out of the place of paper and i draw this vector  $r$  now as you know i have this equation biosaver law  $dB$  is equal to  $\mu_0$  by four pi  $i dl$  cross  $r$  by  $r^3$

so i need to calculate i need to know  $dl$  vector and  $r$  vector and the distance  $r$  to be able to estimate the magnetic field produced by this current element here and i will show you that if i calculate the magnetic field because of this current element and this current element one of the components gets cancelled which we argued last time through a discussion but i would i would like to show you explicitly now for this what is  $dl$  vector  $dl$  vector remember is pointing along the  $y$  direction

so the  $y$  direction is coming out of the pla out of the plane plane of the paper and

so  $dl$  vector will be  $ah j$  cap into  $dl$  small element and  $j$  cap because it is along the  $y$  direction and  $r$  vector is the coordinate of this because the vector joins from here to here  $r$  vector is the coordinate of this point minus the coordinates of this point

so i will have  $ah$  minus  $r i$  cap plus  $z k$  cap  $z k$  cap is the position of this point and minus  $r i$  cap  $ah r i$  cap is the coordinate of this point

so the difference is  $r$

so  $dl$  cross  $r$  will be equal to  $j dl$  cross minus  $r i$  cap plus  $z k$  cap which is equal to now

so minus  $\hat{r} \times \hat{l} \times \hat{j}$  cross  $\hat{i}$  is minus  $\hat{k}$   
 so this complex  $\hat{k} \times \hat{j}$  cross  $\hat{k}$  is  $\hat{i}$   
 so plus  $\hat{i} \times \hat{z}$   $\hat{i} \times \hat{z} \times \hat{l} \times \hat{j}$  cross  $\hat{i}$  is minus  $\hat{k}$  with a minus sign  
 because plus here  $\hat{j} \times \hat{k}$  cross  $\hat{k}$  is  $\hat{i}$  which is  $\hat{z}$   
 so because of this current element this one the magnetic field produced at this point  $db$  let me call it  $db_1$   
 so this is point one and this is point two  
 so i want to calculate what is the magnetic field here because of the small current element at this point one and what is the magnetic field here because of the small current element two  
 so because of one i have  $\mu_0$  by four  $\pi$   $\hat{i}$   
 so  $\hat{d} \times \hat{r}$  is  $\hat{r} \times \hat{l} \times \hat{k}$  plus  $\hat{z} \times \hat{l} \times \hat{i}$  by  $r^3$  or is this distance from that point to this point  
 so that is the magnetic field produced at this point its a magnetic vector field as you can see here it has both the  $\hat{z}$  component and the  $\hat{x}$  component both are positive  
 so it must be pointing like this this  $\hat{b}$  vector must be perpendicular to this  $\hat{d}$   $\hat{l}$  vector and the  $\hat{r}$  vector because this is the equation  
 so  $\hat{b}$  vector is proportional this is the  $\hat{r}$  vector  $db_1$   
 so  $\hat{d} \times \hat{v}$  one vector is perpendicular to  $\hat{r}$  vector and to the  $\hat{r}$  vector now let me calculate because of the second element  
 so let me draw the figure again here  
 so i have this element here this element here and this is the point  $p$   
 so now i have to draw this vector this is my  $\hat{r}$  vector now and now i must  
 so again i must use this equation  $\hat{d} \times \hat{b}$  vector is equal to  $\mu_0$  by four  $\pi$   $\hat{i}$   $\hat{d} \times \hat{r}$  by  $r^3$  now  $\hat{d} \times \hat{l}$  vector is equal to now please remember the current is flowing into the page  
 so this is my  $\hat{x}$  axis this is my  $\hat{z}$  axis  
 so  $\hat{y}$  axis is coming out of the plane and the current is going into the plane  
 so this is minus  $\hat{j} \times \hat{d} \times \hat{l}$  here it was plus  $\hat{j} \times \hat{d} \times \hat{l}$  because the current is coming out in the  $\hat{y}$  direction here the current is going in in the minus  $\hat{y}$  direction  
 so  $\hat{d} \times \hat{l}$  vector is this and  $\hat{r}$  vector is equal to again the coordinates of this are zero  $\hat{z}$  and the coordinates of this are minus  $\hat{r}$  and zero  
 so  $\hat{r}$  vector will be  $\hat{k} \times \hat{z}$  plus  $\hat{r} \times \hat{i}$   
 so  $db_2$  will be equal to  $\mu_0$  by four  $\pi$   $\hat{i}$   
 so now that i must calculate  $\hat{d} \times \hat{r}$   
 so let me calculate  $\hat{d} \times \hat{r}$  separately  
 so  $\hat{d} \times \hat{r}$  is equal to minus  $\hat{j} \times \hat{d} \times \hat{l}$  cross  $\hat{k} \times \hat{z}$  plus  $\hat{i} \times \hat{r}$  which is equal to  
 so minus  $\hat{j} \times \hat{k}$  cross  $\hat{k} \times \hat{z}$  is plus  $\hat{i}$   
 so minus  $\hat{i} \times \hat{d} \times \hat{l}$  into  $\hat{z} \times \hat{j}$  cross  $\hat{i}$  is minus  $\hat{k}$   
 so plus  $\hat{k} \times \hat{r} \times \hat{d}$   
 so let me wrap this up  
 so i have  $\hat{d} \times \hat{r}$  vector minus  $\hat{j} \times \hat{d} \times \hat{l}$  cross  $\hat{j} \times \hat{k} \times \hat{z}$  plus  $\hat{i} \times \hat{r} \times \hat{j}$  cross  $\hat{k}$  is plus  $\hat{i}$   
 so with the minus sign here and  $\hat{j} \times \hat{k}$  cross  $\hat{i}$  is minus  $\hat{k}$   
 so that becomes plus  
 so that is  $db$   
 so i can calculate  $db_2$  the magnetic field produced by the second element  
 so  $db_2$  is  $\mu_0$  by four  $\pi$   $\hat{i}$  into minus  $\hat{i} \times \hat{z} \times \hat{d} \times \hat{l}$  plus  $\hat{k} \times \hat{r} \times \hat{d} \times \hat{l}$  divided by  $r^3$  and let me recall ah what we had for  $db_1$   
 so  $db_1$  vector is  $\mu_0$  by four  $\pi$   $\hat{i} \times \hat{z} \times \hat{d} \times \hat{l} \times \hat{i}$  plus  $\hat{r} \times \hat{d} \times \hat{l} \times \hat{k}$  by  $r^3$   
 please remember that small  $r$  is the distance of this point from the current

element and because I am on the axis of the current loop this distance is equal to this distance

so small  $r$  is the same both in the  $db_1$  formula as well as  $db_2$  formula the only difference between these two is the current element here it is coming up here the current element is going down the  $r$  vector is here here and in the other case the  $r$  vector is the it is  $r$  vector

so  $r$  vectors are different in the two cases now you can clearly see

so let me draw the figure again

so I have this  $z$   $x$

so this is coming out this is going in

so at this point

so this is one  $r$  vector this is another  $r$  vector here

so as you can see here  $db_1$  is the magnetic field produced by this current element at this point  $db_2$  is the magnetic field produced by the diametrically opposite current element at the same point and you can see here the  $x$  components are exactly equal and opposite and they cancel off and  $x$  component is nothing but the component perpendicular to the  $z$  axis  $z$  components add up and  $x$  components cancel out this is precisely what we had discussed in the last class I had said that this produces a magnetic field like this  $db_1$

this one produce the magnetic field like this  $db_2$  both of them are at the same angle they have the same magnitude of  $x$  component but opposite and

so cancel off and the  $z$  components add and you can see here by a simple calculation with a very simple calculation using vectors we can we have found out that the  $x$  components cancel off and the  $z$  components add and

so I will get the total magnetic field that is generated at that point by the two elements  $db$  vector is equal to  $db_1$  vector plus  $db_2$  vector

so  $db_1$  vector is the magnetic field produced by one current element  $db_2$  is because of the other current element

so if I add these two quantities  $x$  components cancel  $z$  components will add up and I will get  $\mu_0$  by four  $\pi$   $I$  into two  $r$   $dl$   $k$  cap by  $r^3$

so you can see the magnetic field is along the  $z$  axis now if I go back here and look at this

so what I have shown is the magnetic field produced by this current element and this current element are cancelling of their components perpendicular axis similarly a magnetic field generated by this element and the diametrically opposite element on the other side will have cancelled their components perpendicular to the  $z$  axis and

so on

so all these components will cancel off resulting in a total magnetic field only along the  $z$  axis

so I can calculate total magnetic field is equal to  $\mu_0$   $I$  by four  $\pi$  into two  $r$  by  $r^3$  now ah small  $r$  is this distance smaller is this distance this is capital  $r$  this is  $z$

so small  $r$  is nothing but  $r^2$  plus  $z^2$  the square root

so this is  $r^2$  plus  $z^2$  raise to power  $3/2$  into  $k$  cap into integral  $dl$  now I have to be little careful because in deriving this equation I have taken a count of both these elements diametrically opposite elements

so integral over  $dl$  must be in a semicircle because the upper half semi circle and the lower half semi circles are exactly cancelling cancelling their normal components

so this will be in a semi circular arc only this is semicircle and over a semi circle the length is nothing but

so  $\mu_0$   $I$  two  $r$  by four  $\pi$  into  $r^2$  plus  $z^2$  is power three by two into this is  $\pi$   $r$  into  $k$  cap

so this is nothing but  $\mu_0 i r^2$  by two times  $r^2 + z^2$  square three by two  $k$  cap

so that is the magnetic field and if you go back to my last lecture you will find that we have derived the same equation for the magnetic field of a circular loop of coil circular loop of current carrying conductor along the axis this is along the axis please remember this not at arbitrary points

so let me draw the figure again

so this is my loop current carrying current like this this is  $z$  axis  $x$  and  $y$  so this along this here the magnetic field is along this direction and here the magnetic field is along the same direction here please remember the magnetic field is along  $k$  cap direction and this along the axis and as this equation shows the maximum magnetic field appears along at the point  $z$  is equal to zero where you get the maximum magnetic field and last time we are drawn a figure showing the magnetic field variation with position and it goes like this this is magnitude of magnetic field versus  $z$  and that's a magnetic field

so at this point  $b_{max}$  is given by  $\mu_0 i r^2$  by two into  $ah$   $i$  put  $z$  is equal to zero

so you get  $r^3$  which is equal to  $\mu_0 i$  by two  $r$  that is the magnetic field at the center of the circular loop of current and  $ah$  if  $i$  put vectors here  $k$  cap  $sk$  cap here ok

so let me draw  $ah$

so this was the magnetic field along the axis we have not been calculating magnetic field elsewhere but let me just draw a figure which you will which if one way to calculate magnetic field  $ah$  at all points you will get a figure something like this

so  $i$  have the current carrying conductor here circular loop

so  $i$  have one magnetic field line coming like this there is another line coming like this and going like this then another line coming like this going here here another line coming like this going off closing off

so you have magnetic field lines which are  $ah$  going in one direction and they form circular loops

so these loops actually go for a long distance and come back and close on each other and

so this magnetic field distribution because of a current loop is very different from  $ah$  charge distribution electric field produced by charge distribution also notice that we have to use the right handed screw rule to find out the direction of the magnetic field

so the current carrying conductor is carrying a current like this

so as we have seen last time if the current is probably flowing like this then the right handed screw will move towards me and

so the magnetic field direction is towards me

so a current going like this will generate a magnetic field like this a current going like this will generate a magnetic field in the opposite direction now there is something which is interesting which  $i$  can which  $i$  can get from this equation

so let me re recall this equation here the magnetic field equation let me read rewrite this equation  $b$  is equal to  $\mu_0 i r^2$   $k$  cap by two times  $r^2 + z^2$  square three by two

so let me look at distances which are much greater than the diameter of the loop

so  $b$  will be  $\mu_0 i r^2$   $k$  by two  $z^3$

so  $i$  multiply and divide by  $\pi$

so  $i$  can write this as  $\mu_0 i \pi r^2$   $k$  cap by two  $\pi z^3$  by multiply and divide by  $\pi$  now what is  $\pi r^2$   $\pi r^2$  happens to be the area of

this loop  $r$  is the radius of the loop and  $\pi r^2$  is the area of the loop and the loop is carrying a current like this and this is my directions remember ah some lectures ago we had introduced the concept of vector area

so if i have an area i can define a vector area and here i define the vector area according to the right hand screw rule

so if i have a current carrying conductor like this then the vector area is here

so i define the vector area as  $\vec{A}$  is equal to  $\pi r^2$  area into  $\hat{k}$  cap

so this is the  $z$  direction which i have chosen that is the vector area

so i can define the magnetic field as  $\vec{B} = \frac{\mu_0 I \vec{A}}{2\pi z^3}$  now while doing electrostatics i had introduced the concept of electric dipoles

so let us recall that if you have a negative charge and a positive charge we can define a electric dipole moment which is  $q$  times  $d$  and it is in the direction of this from negative to positive

so let me call this

so so this is  $z$  axis this is  $z \hat{k}$  that was the electric dipole moment i can also define a magnetic dipole moment  $\vec{m}$  by current into area vector

so you have current carrying loop here

so this is area vector

so this is the magnetic dipole moment current into area vector is called the magnetic dipole moment and

so if i use that equation here i will get the magnetic field produced by this current tube on the axis far away is  $\vec{B} = \frac{\mu_0 \vec{m}}{2\pi z^3}$

so this is for  $z$  much greater than  $r$  remember we had also calculated the electric field produced by an electric dipole far away from the dipole and we had obtained an equation for the electric field

so electric dipole  $E$  is equal to  $\frac{p}{2\pi\epsilon_0 z^3}$  for much greater than  $a$

so i will call this plus and this is minus and this we are called two  $a$  and  $a$   $p$  was the dipole moment and this is for distance is large

so this is  $z$  axis this was that distance is large compared to the size of the dipole and for the magnetic dipole moment for the magnetic field we have a similar relationship except addition of one by  $2\pi\epsilon_0$  we have  $\frac{u_0}{2\pi}$  here and instead of  $d$  electric dipole moment your magnetic dipole moment here and instead of  $a^3$  both of them go down as  $z^3$

so the field reduces at the cube of the distance from the dipoles

so we will come back to looking at magnetic dipoles and torques and forces on magnetic dipoles but before that i just want to draw a figure to show you the difference between the two dipoles dipole fields

so let me draw an electric dipole here

so if i have a plus charge here and a minus charge the field lines will be looking like this we had defined we had seen this before the field lines start from positive charge and negative on the end of the negative charge

so all the fields are starting from the positive and negative ending on the negative for the magnetic dipole the field lines are very different

so magnetic dipole i have a loop of current

so i am taking a loop like this

so magnetic field lines will be this field line will start from here

so look at the dipole fields are very different here all the electric field lines are starting from the positive charge and ending on the negative charge here the field lines have no beginning or end

so they are loops they are continuous loops and they do not have start from anywhere and end anywhere

so that is the reason why there are no corresponding magnetic charges unlike electric charges we have electric charges positive and negative and you can find an individual separate charge you cannot find an individual magnetic charge and there are no magnetic field lines starting from some point and then ending at another point all the field lines are closed on each other and this leads to as we have seen before um a gauss's law for a magnetic field which is  $\oint \mathbf{B} \cdot d\mathbf{a}$

so the flux of magnetic field through any closed surface will be zero

so if you take any surface here suppose i take a surface like this as many field lines will enter as will get out from here because there are no individual charges there are no starting points and ending points here there will be no flux of total flux of magnetic field will be zero

so that's the gauss's law for magnetic fields and this is the corresponding difference between electric's electric field and magnetic fields

so please note that the two field lines are very different here one is starting from positive ending or negative the other one are closed loops ah

so that is a magnetic dipole here and that's the electric dipole

so now i want to look at another problem another example from where we will derive a very important relationship later on

so this is an infinitely long straight current carrying conductor

so i want to find the magnetic field due to an infinitely long straight current carrying conductor

so i have a a current carrying conductor like this and i want to find the magnetic field at this point some point p and

so this is my point p where i want to find the magnetic field and that's an infinitely long current carrying conductor remember in electrostatics we had also calculated the electric field due to an infinitely long line charge similarly i have an infinitely long current carrying conductor from which i should i want to find what is the magnetic field at the point p

so i will use bio separate law write down the electric field at p due to a small element of current and using superposition law principle of superposition i will add the magnetic field due to all current elements at this point p and get the total magnetic field

so for this what i do is let me assume that this is my x axis and this is my y axis and i take a small current element here  $dl$  and this this is my point here and i join this

so let me assume that this distance is  $x$  and this distance is  $y$

so i take one current element at a distance  $y$  from this perpendicular point here

so that's my  $x$   $y$  axis and i want to calculate the magnetic field at this point because of  $dl$  again i will use bio several law  $dB$  is equal to  $\frac{\mu_0}{4\pi} \frac{dl \times \mathbf{r}}{r^3}$  now you see here the  $dl$  vector is always equal to  $dl$  along the  $y$  direction the current is flowing like this i am assuming current is flowing along the  $y$  direction

so  $dl$  vector is  $dl$  times  $\hat{j}$  cap its along the  $y$  direction

so all current elements wherever you take along the straight path of the current it is always  $dl$  prime times  $\hat{j}$  cap and  $\mathbf{r}$  vector is equal to the coordinate of this point minus coordinate of this point

so this point is has coordinates  $x$  and zero and this point has coordinates  $0$   $y$

so i will have the this will be  $x\hat{i} - y\hat{j}$   $x$  is the or this vector from here to here is  $x\hat{i}$  and from here to here vector is  $y\hat{j}$

so this vector minus this vector gives me this vector

so this  $\mathbf{r}$  vector is like this

so  $d\mathbf{l} \times \mathbf{r}$  is equal to  $dl \hat{j} \text{ cap} \times (x\hat{i} \text{ cap} - y\hat{j} \text{ cap})$  which is equal

to

so  $\mathbf{j} \cap \text{cross } \mathbf{i} \cap$  is minus  $\mathbf{k} \cap$  minus  $x \, d \, l \, \mathbf{k} \cap$  and  $\mathbf{j} \cap$  of  $\mathbf{j} \cap$  is zero

so  $d \, l \, \text{cross } r$  is minus  $x \, d \, l \, \mathbf{k} \cap$

so  $d \, y$  sorry  $d \, a \, h \, d \, l$  ok thats fine

so  $d \, l$  is nothing but small element  $d \, y$

so let me write this as minus  $x \, d \, y \, \mathbf{k} \cap$

so please note here that every current element no matter what value of  $y$  you take is producing a magnetic field along the  $z$  axis minus  $z$

so the  $z$  axis here you see  $i$  must use the right handed coordinate system

so  $x$  is here and  $y$  is here

so  $x \, z$  is coming out of the base out of the paper and what this says is  $d \, l \, \text{cross } r$  is minus  $x \, d \, y$

so that means the the magnetic field must be point into the board and that is expected because remember right hand rule if my current is flowing like this it will generate a magnetic field in this direction

so it just

so happens that all current elements along the length of the wire are producing a magnetic field all pointed along the  $z$  direction

so  $i$  can simply add the total magnetic the magnetic field produced by all small current elements to get the total magnetic field

so let me write an expression for  $d \, b$  is  $\mu \, \text{naught}$  by four  $\pi$   $i \, d \, l \, \text{cross } r$  by  $r^3$  which is equal to  $\mu \, \text{naught}$   $i$  by four  $\pi$

so  $d \, l \, \text{cross } r$  is minus  $x \, d \, y \, \mathbf{k} \cap$  by now what is the magnitude of  $r$   $x^2$  plus  $y^2$  is  $r^2$

so this is nothing but  $a \, h \, x^2$  plus  $y^2$  raised to the power three by two

so the total magnetic field will be equal to minus  $\mu \, \text{naught}$   $i$  by four  $\pi$  now  $x$  is independent  $x$  is this distance from here to here that is independent of my integration variable

so  $x$  comes out  $x \, \int \frac{d \, y}{x^2 + y^2}$  raised to the power three by two and into  $\mathbf{k} \cap$

so if  $i$  had a current carrying conductor from a coordinate  $y_1$  to  $y_2$   $i$  can find out the magnetic field produced by a finite length wire and then let the limits go to infinity

so let me assume that  $i$  am taking a finite length of wire lying between  $y_1$  to  $y_2$   $y_1$  is the coordinate of this end  $y_2$  is the coordinate of this end and this is the length  $y_2 - y_1$  is the length of the wire and  $i$  want to find the magnetic field produced by this this length this small length of the wire and

so  $i$  will have an integration from  $y_1$  to  $y_2$  and thats a simple integration all you need to know do is uh to use change of variables if  $i$  call this as  $\phi$  you note here that  $y$  is equal to

so  $y$  by  $y$  by  $x$  is  $\tan \phi$

so  $y$  is equal to  $x \, \tan \phi$   $d \, y$  is equal to  $x \, \sec^2 \phi \, d \phi$   $x^2 + y^2$  is equal to  $x^2 (1 + \tan^2 \phi)$  which is equal to  $x^2 \sec^2 \phi$

so  $i$  can substitute all this into this equation and find an expression for the current

so  $b$  is equal to minus  $\mu \, \text{naught}$   $i$  by four  $\pi$   $x \, \int \frac{\sec^2 \phi \, d \phi}{x^3 \sec^3 \phi}$  remember this is  $x^2 + y^2$  raised to three by two here

so  $i$  have  $x$  cube

so this is equal to minus  $\mu \, \text{naught}$

so there is a  $k \mu_0 i$  by  $4\pi$  there is  $x^2$  and i get  $x$   
 here integral  $\int \frac{1}{\sec^2 \phi} d\phi$  is  $\cos \phi d\phi$   $k \mu_0 i$  which is equal  
 to  $\mu_0 i$  by  $4\pi$   $x$  this is  $\sin \phi$  ah between the two limits  
 so let me call the two angles  
 so let me call the two limits as  $\phi_1$  and  $\phi_2$   
 so  $\sin \phi_2 - \sin \phi_1$  now what is  $\sin \phi$   $\sin$  of the  $\phi$   $\sin$   
 $\phi$  is nothing but  $y$  divided by this distance  
 so  $\sin \phi_1$  is nothing but  $y$  divided by  $x^2 + y^2$   
 is per half  
 so  $\sin \phi_2$  is nothing but  $y$  by  $x^2 + y^2$  raised power  
 half and  $\sin \phi_1$  the other limit is equal to  $y$  by  $x^2 + y^2$   
 square half  
 so these are the two limits and i can get  $x$   
 so this is the  $k \mu_0 i$  here  
 so the magnetic field is nothing but given by  $\mu_0 i$  by  $4\pi$   $x$   
 so  $y$  by square root of  $x^2 + y^2$  square minus  $y$  by square  
 root of  $x^2 + y^2$  square  $k \mu_0 i$   
 so that's a general expression for the magnetic field  
 so i have a current kinetic conductor  
 so this is  $ah$   
 so this at some point i am calculating  
 so this quart  
 so this is my  $y$  axis here  $x$  axis here here i am calculating at this point  
 so this has coordinates by 2 this is coordinate  $y$  1.

so finite length wire  
 so there is a wire finite length carrying a current like this current  $i$  now i  
 can take the limit  
 so this is for a finite length of wire i can actually if the wire becomes  
 infinitely long  $y_1$  tends to minus infinity and  $y_2$  tends to plus infinity  
 so what i will get is this  
 so this will become  
 so as  $y_2$  tends to infinity i can neglect  $x$  in terms in comparison to  $y_2$   
 square  
 so i get  $y_2$  by  $y_2$  which is one and here i get  $y_1$  tending to minus  
 infinity  
 so the two two of them add up and i will get  $b$  is equal to  $\mu_0 i$   
 so that becomes a factor of two here by  $2\pi$   $x$   $k \mu_0 i$   
 so magnetic field at this point if this distance is  $x$   
 so  $x$  is the perpendicular distance from here to this point  
 so then the magnetic field at this point is pointing into the paper here  
 because the current is moving up the  $z$  axis is coming out of the plane of paper  
 the magnetic field is  $\mu_0 i$  and if you go to minus  $x$  direction here  
 somewhere here because  $x$  is negative here  
 so field is coming up  
 so the field is coming up from here and going into this clean the vapor  
 so magnetic field is curved like this and please remember that because this is  
 has cylindrical symmetry there is a wire like this  
 so because there is a wire like this at this point the mag some of the current  
 is going up like this  
 so the magnetic field at this point is like this magnetic this point is like  
 this magnitude at this point is like this magnetic field at this point is like  
 this  
 so all that at every point the magnetic field is perpendicular to the current

and to this line

so its like this here like this here like this here

so it's like like a circular arc around this current current conductor

so if i call r as the distance from the conductor perpendicular distance  
actually then i can replace x by r and

so if i have my current carrying conductor if i the distances are b vector b  
vector magnitude will be nothing but  $\mu_0 i$  by  $2\pi r$  and i must know the  
direction of the current magnetic field by knowing the direction of the current  
and using the right hand screw rule

so the magnetic field will be going into the plane of paper here because the  
current is going up coming out of the planar paper here

so i can actually draw magnetic field lines

so the magnetic field lines will look like this and here

so that's my current carrying conductor here

so if i look at the top view

so if my current carrying conductor is coming if the current is coming towards  
me i will have

so please remember current is coming towards me

so i will have the current magnetic field like this that is my current kinetic  
control

so magnetic field lines are circular circles around this current carrying  
conductor and the magnetic field depends only on this distance and it goes down  
as one by r you may recall what we had done for an infinitely long linear charge  
distribution we also calculated the electric field there and you you can compare  
this expression with the expression for electrostatic field of an infinitely  
long current carrying conductor

so magnetic field lines as you can see here from closed loops

so let me try to draw a comparison between ah a line charge distribution and a  
line current

so if you have a line charge distribution for example this charge distribution  
is positive

so i have a line charge an infinitely long line charge coming out of the plane  
of the paper here and the positive

so there is no direction it is all positive charges

so i will have magnitude the electric field lines will be as you have seen  
coming out like this radially out of the charges on the other hand if i had a  
current carrying conductor with current coming up i have field lines which are  
closed a very different distribution of magnetic field and this is e field and  
this is b field here if you take a close surface which encloses the charge  
suppose i take a close surface like this i will get a finite flux if you take  
any close surface here you get zero flux because as many lines are crossing in  
as they are getting out of the surface and

so the the magnetic flux net magnetic flux is always zero and that is gauss's  
law that is because there are no individual magnetic charges

so here you have ah integral  $\mathbf{e} \cdot \mathbf{t} \, a$  is equal to q enclosed by epsilon zero  
and here you have integral  $\mathbf{b} \cdot \mathbf{d} \, a$  zero there are no magnetic fluxes

so let me take an example

so suppose i have a current carrying conductor here and suppose i assume 5  
amperes of current flowing like this and i want to find the magnetic field at a  
distance of 10 centimeter from the current carrying conductor

so i have a wire which is carrying 5 amperes of current and i am at a distance  
of 10 centimeters

so b is equal to magnetic field  $\mu_0 i$  by  $2\pi r$  that is the equation we  
had derived just now

so this is equal to four pi ten to the minus seven into five amperes divided by two pi into point one

so this factor of two here

so that is ten to the minus five tesla and compare this with  $b_{\text{earth}}$  is approximately three to the minus five tesla and

so you are producing magnetic fields at a distance of 10 centimeter from the current carrying conductor which is carrying 5 amps current you have some kind of a magnetic field generated about 10 to the minus 5 tesla as you get closer and closer to the wire the magnetic field will increase but far away from the wire the magnetic field will keep on decreasing and you can sort of estimate the magnetic fields under for example ah high voltage lines which are carrying currents what kind of magnetic fields will be existing under currents current carrying conductors huge current carrying conductors its an interesting problem to understand

so now i want to introduce a very very ah important concept in magnetostatics and that is ampere's law a very important quantity ah concept of ampere's law remember in electrostatics we had first introduced coulomb's law which told us the electric field generated by a point charge then we use superposition principle to calculate the electric field produced by any charge distribution then we defined the quantity called electrostatic flux and then we derived gauss's law gauss's law which relates the electric flux to the charge enclosed by that surface now in magnetic fields there are no magnetic fluxes the magnetic flux is always zero the net flux through any surface is always zero closed surface please remember i am looking at a closed surface the entire magnetic flux which is entering magnetic field line entering is also leaving there are no magnetic charges there are no individual magnetic poles

so what we say is there are no magnetic monopoles there are only magnetic dipoles and higher order poles but not magnetic monopoles

so we cannot derive there is no derivation of another gauss's law for ampere for the currents because the magnetic flux through a closed surface is always zero

so we have another another kind of law which is called the ampere's law in which we do not have area integrals but line integrals

so now let me look at this problem which we have discussed an infinitely long current carrying conductor let me assume that the current is coming up

so ah i know that the magnetic field at any distance  $r$  is given by

so let me just write the magnet magnitude  $\mu_0 i$  by two pi  $r$  and i know the magnetic field is like this

so if i if i will draw the the magnetic field lines will be like this everywhere it is like this everywhere it is perpendicular to this line here it is perpendicular to this line at this point perpendicular to this line

so it is circling around the wire and it has the same magnitude all across now let me calculate this quantity  $b \cdot dl$  over the closed loop

so i start from some point calculate for the entire loop now please note that the magnetic field is always parallel to the  $dl$  vector

so  $dl$  vector here is like this  $b$  is parallel real vector here  $b \cdot dl$  vector is like this  $b$  is parallel  $dl$  vector here  $dl$  vector is like this  $b$  is parallel vector

so this is nothing but  $b \cdot dl$  and  $b$  is nothing but  $\mu_0 i$  by two pi  $r$  into  $dl$

so as you vary the point of integration  $r$  remains constant

so i will get nothing but  $\mu_0 i$  by two pi  $r$  into integral  $dl$  integral  $dl$  is the total length of this path which is nothing but two pi

so this is equal to two pi the ah two pi  $r$  circumference of the circle

so this is nothing but  $\mu_0 n i$   
 so what i have shown is for this case  $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 n i$   
 so integral of  $\mathbf{B} \cdot d\mathbf{l}$  gives me  $\mu_0 n i$  and this is for a path which have taken a circular path around the current carrying conductor  
 so i have taken infinitely long current kinetic conductor then i calculate the i have calculated the magnetic field and then i calculate an integral  $\oint \mathbf{v} \cdot d\mathbf{l}$  around this current carrying conductor on a circular path with the current kinetic conductor to be the center of the circle and i find n value  $\mu_0 n i$   
 so what happens if i have another path some other path which is not circular around this current kinetic conductor but some arbitrary path  
 so for example i will take ah  
 so magnetic field is always perpendicular to this but its not along  
 so here magnetic field may be in this direction here the magnetic field is like this a different point magnetic field is always perpendicular to the line from this point to this point but the  $d\mathbf{l}$  vector is like this here now and  $\mathbf{v}$  vector is here and if this angle is  $\phi$   
 so let me draw a figure again here  
 so this is ah at this point the current element is like this the magnetic field is here  $d\mathbf{l}$  vector is here this is  $\phi$  ok  
 so i need to calculate i want to calculate this quantity  
 so i will show you that this is still equal to  $\mu_0 n i$  irrespective of the shape of the curve which is encircling this current carrying conductor and i will do this in the next class i will show you that the total integral  $\oint \mathbf{v} \cdot d\mathbf{l}$  over a closed path is always equal to  $\mu_0 n i$  where  $i$  is the current enclosed by this path and we will generalize this to more interesting problems and that is this is what is called as ampere's law now before i finish i want to just give you a problem i will just leave a problem here  
 so consider two parallel infinitely long current carrying conductors  
 so you have one current carrying conductor like sorry one conductor like this  
 so let me assume the currents are in opposite direction same current but in opposite directions  
 so i want you to find ah  
 so ok  
 so let me draw like this  
 so if i look at from the top i have this current carrying conductor here another current carrying conductor  
 so i want you to calculate the magnetic field at this point p and other point q on the equatorial plane  
 so what is the magnetic field  
 so this is this is not the point here b at p and q  
 so use the use the formula we have derived you can calculate the magnetic field because of this current carrying conductor you know the magnetic field because of this current carrying conductor  
 so calculate the magnetic field because of these two use superposition principle and calculate the net magnetic field here and here  
 so the two wires are like this one carrying current up and the other carrying current down and  
 so the problem is to calculate the magnetic field here and elsewhere on this equatorial plane thank you very much you