

good morning to all of you we will continue with our discussion in magnetostatics you may remember the last lecture we had introduced bio savart law and from bio server law we calculated the magnetic field produced by a current loop and also the magnetic field produced by a infinitely long straight current carrying conductor

so let me recall

so if you have a infinitely long straight current carrying conductor with current i passing through the wire then we calculated the magnetic field at some point p at a distance x from here

so we call this as x axis and this is the y axis here and we calculated and showed that the magnetic field b is μ naught i by two π x minus sin k k and the magnetic field at this point is pointing inside the paper

so magnetic field is going into the paper here and we are calculated by using bios effort law by taking a small current element here then using that current element to calculate the magnetic field at this point and integrating over all current elements it is interesting to note that all current elements produce magnetic fields in the same direction

so all we had to do was to add the magnetic fields due to each element and get the total magnetic field now we also note that because of symmetry the magnetic field will be the same at all points which are at a distance x from here

so we can actually generalize this and write that if i have a current kinetic conductor like this and if i calculate the magnetic field at any point which lies on a circle of radius r with the wire at the center then b magnitude of b will be μ naught i by two π r and the directional magnetic field will be according right hand rule please note that if the current is going upwards with the right handed screw if i move the screw in this direction then the screw will move up

so if the current is going upwards the magnetic field has to curve around the wire like this in this direction

so this is magnetic field magnitude independent of distance independent of z axis along this length of the wire independent of angle and it only depends on the distance of that point from the wire and also note that magnetic field lines form closed lines

so if i were to draw the magnetic field from the ah if this is the current kinetic conductor carrying current towards me the magnetic field lines will look like this or closed loops around the current carrying conductor and again the direction of the magnetic field is determined by the direction of the current that is flowing through this because of the right hand screw rule the current magnetic fields are in the anti-clockwise direction when the current is coming towards me

so this

so this also implies that if you take any closed surface suppose i take a closed surface s as many field lines will enter the surface as will go out and you have this equation the gauss's law for magnetic fields integral b dot d a is equal to zero which essentially implies that there are no sources of magnetic field lines that magnetic field lines do not start from any point and at any other point they are they form close loops or they start from here and go to end at infinity

so this is contrary to the ah equation satisfied by electrostatic fields where the flux was equal to the charge net charge enclosed divided by epsilon zero from this calculation of the field produced by a current kinetic conductor we have derived an equation which is the ampere's law

so let me recall again

so we had this is the current kinetic conductor and if i take a circular loop

around this point and integrate $\mathbf{v} \cdot d\mathbf{l}$ around this loop i showed you last time that this is equal to $\mu_0 i$ that is the integral over the loop from across the in the circular arc here is equal to $b \cdot d\mathbf{l}$ integral b or $d\mathbf{l}$ is equal to $\mu_0 i$ that is called ampere's law now this law is always valid it is very similar to gauss's law energy statics it is always valid it is very useful as i will show you whenever you can take the magnetic field be outer outside the integral then you can actually use this integral formulation to calculate the magnetic field otherwise it is always valid

so what we did essentially was to integrate over the circular path
so let me recall

so what we did was we took a i taken small element here $d\mathbf{l}$ length here
so if this is angle is $d\phi$ and this red distance is r then $d\mathbf{l}$ vector magnitude is equal to $r d\phi$ and the magnetic field is also along the same direction as $d\mathbf{l}$ vector

so $b \cdot d\mathbf{l}$ is equal to b times $d\mathbf{l}$ which is equal to b times $r d\phi$ and magnetic field i just calculated $\mu_0 i$ by $2\pi r$ into $r d\phi$ which is equal to $\mu_0 i$ by 2π into $d\phi$

so if i integrate integral $\mathbf{v} \cdot d\mathbf{l}$ becomes equal to $\mu_0 i$ by 2π integral $d\phi$ integral $d\phi$ is the complete angle surrounding this point which is equal to 2π

so this gives me \mathbf{v} naught i that's what we had used to calculate this equation value for integral $b \cdot d\mathbf{l}$ and that happens to $\mu_0 i$ now this is a calculation assuming that the wire is at the center of the circular path now i want to show you that this value of the integral is always $\mu_0 i$ irrespective of the path that i take around the current carrying conductor

so let me again draw a figure here

so this is my current coming out of the plane of the paper

so i take some arbitrary path like this around this current kind of conductor

so let me try to draw a figure here

so for example at this point uh \mathbf{b} vector is perpendicular to this line this is the line joining the center to this point \mathbf{b} vector is like this and $d\mathbf{l}$ vector is here

so let me call this angle in between as θ

so let me draw another line here

so what is $b \cdot d\mathbf{l}$ $b \cdot d\mathbf{l}$ is equal to $b d\mathbf{l} \cos \theta$ θ is the angle supplemented between \mathbf{b} vector and $d\mathbf{l}$ vector

so $d\mathbf{l}$ vector is along the path which is not necessarily circular with the wire at the center

so $d\mathbf{l} \cdot \mathbf{b}$ is $b d\mathbf{l} \cos \theta$ and $d\mathbf{l} \cos \theta$ is this length

so $d\mathbf{l} \cos \theta$ is this length and if i call this angle $d\phi$ and this distance $r d\mathbf{l} \cos \theta$ happens to be equal to $r d\phi$ $r d\phi$ is this distance this distance times the angle $d\phi$ and that is also $d\mathbf{l} \cos \theta$

so $b \cdot d\mathbf{l}$ becomes nothing but $b r d\phi$

so for this case b i know is $\mu_0 i$ by $2\pi r$ into $r d\phi$

so that gives me $\mu_0 i$ by 2π into $d\phi$

so integral $b \cdot d\mathbf{l}$ would be equal to $\mu_0 i$ by 2π integral $d\phi$ which is again 2π because the entire angle conv covered by ϕ which is nothing but $\mu_0 i$

so if i even if i have a path of integration which is not circular with the wire at the center what i have shown is that this integral $b \cdot d\mathbf{l}$ is always equal to $\mu_0 i$ times the current in the conductor which is enclosed by this loop of integration

so although \mathbf{b} and $d\mathbf{l}$ are not parallel to each other $b \cdot d\mathbf{l}$ happens to be $b r d\phi$ and when i integrate i just get $\mu_0 i$ now what will happen if i

so how did i choose the direction of integration here that loop of integration is such that it happens to be along the magnetic field because for a current carrying conductor with current coming up towards me the magnetic field is anti-clockwise i can also do an integration in the clockwise direction

so for example if i have a current kind of conductor like this and if i have a loop like this with integration in the reverse direction then $\int \mathbf{b} \cdot d\mathbf{l}$ will be minus $\mu_0 i$ here this is over curve c two and the same current carrying conductor if i have another path with c one like this $\int \mathbf{b} \cdot d\mathbf{l}$ is equal to $\mu_0 i$

so it depends on the path that you are taking around the current kind of conductor if it happens to be along the direction of the magnetic field satisfying the right hand rule or the reverse direction you can have plus $\mu_0 i$ or minus $\mu_0 i$ this also tells you that i can actually if i have not just one conductor but suppose i have more than one conductor carrying current

so let me assume that i have a current carrying conductor with current i_1 another with i_2 and i form another some loop like this

so $\int \mathbf{b} \cdot d\mathbf{l}$ will be equal to $\int \mathbf{b}_1 + \mathbf{b}_2 \cdot d\mathbf{l}$ because magnetic fields satisfy superposition principle

so the total magnetic field at any point is the sum of the magnetic field because of i_1 and magnetic field because of i_2

so this is nothing but $\int \mathbf{b}_1 \cdot d\mathbf{l} + \int \mathbf{b}_2 \cdot d\mathbf{l}$ and this is nothing but $\mu_0 i_1$ this is the current this is equal to μ_0 times the current carried by conductor one and plus μ_0 times i_2

so this is nothing but μ_0 times $i_1 + i_2$

so what i can show by choosing many many currents is that $\int \mathbf{b} \cdot d\mathbf{l}$ is equal to μ_0 times i enclosed here which i have shown here now what happens to conductors which are carrying current but which lie outside the path of integration

so let me take an example here

so i have a current kind of conductor here and i take a path like this

so what happens now what i do is let me draw a line here

so let me say that this corresponds to angle ϕ_1 this corresponds to angle ϕ_2

so $\int \mathbf{b} \cdot d\mathbf{l}$ i need to calculate please remember i had just now shown you that for an arbitrary path $d\mathbf{l} \cos \theta = r d\phi$

so $\mathbf{b} \cdot d\mathbf{l}$ is nothing but $b r d\phi$

so i will get this is $\int b r d\phi$ which is equal to $\int \mu_0 i$ by $2\pi r$ into $r d\phi$ which is equal to $\mu_0 i$ by 2π $\int d\phi$ r cancels off which is

so let

so this is equal to $\mu_0 i$ by 2π $\int \phi_1$ to ϕ_2 $d\phi$ plus

so i go from ϕ_1 to ϕ_2 along say c one and then i come back

so i go from here to here along this curve and i come back along this

so ϕ_2 to ϕ_1 $d\phi$ which is nothing but $\mu_0 i$ by 2π ϕ_2 minus ϕ_1 plus ϕ_1 minus ϕ_2 which is equal to zero

so $\int \mathbf{b} \cdot d\mathbf{l}$ along this closed path which does not enclose the current kinetic conductor happens to be zero

so any current element lying outside the loop of integration does not contribute to $\int \mathbf{b} \cdot d\mathbf{l}$ and that is why i can actually write

so if i have multiple current carrying conductors

so i can write $\int \mathbf{b} \cdot d\mathbf{l}$ is equal to μ_0 times i interest that's ampere's law now couple of things i must mention i have been drawing curves

which lie in a plane the curve of integration the path of integration may not lie in a plane

so i can have a wire carrying current like this

so i can integrate some arbitrary path like this and i will get still get μ naught times the current enclosed of course whether it is plus μ naught i or minus μ naught i depends on whether i integrate the direction of integration whether it corresponds to the right hand rule with respect to current carrying conductor or not

so i can have an arbitrary path some arbitrary path which may not line a plane but in the figures which i am drawing here the curves seem to be lying on a plane

so this is a very general result

so i could for example i can draw a figure in which i can say that i can have current carrying conductor like this i one another current carrying conductor i two and another one for example i three

so i can have a loop of integration which may be going behind coming like this so although this currents are not the curve is not in the plane containing the current i still have in this integral $\mathbf{v} \cdot d\mathbf{l}$ is equal to now in this case as you can see here this direction corresponds to positive direction with respect to this current this μ naught i one minus i two minus i three

so and if i have another current current conductor here for example i_4 i_4 does not contribute to this integral or because it is like outside the loop of integration just like i had mentioned in electrostatics i must mention here that the magnetic field at every point is determined by all current carrying conductors just like in the electrostatics case in gauss's law the electric field is determined by electric field produced by all charges while the flux over a closed surface depends only on charges inside similarly the magnetic field produced at any point for example in this figure magnetic field produced here is because of current i one i two and i three and i four but in when i integrate $\mathbf{v} \cdot d\mathbf{l}$ the only currents that contribute to the integral value are the three currents enclosed by this loop

so please do not forget that magnetic fields at any point are generated by all currents in ampere's law in integral $\mathbf{p} \cdot d\mathbf{l}$ only the currents which are contained within the loop contribute to this integral value

so integral $\mathbf{v} \cdot d\mathbf{l}$ suppose i find in a situation integral $\mathbf{b} \cdot d\mathbf{l}$ is equal to 0 this does not imply magnetic field is zero as we just now saw if the loop of integration exists outside the current carrying conductor $\mathbf{b} \cdot d\mathbf{l}$ is zero although the magnetic field at every point was not zero now i want to leave a problem here for you to think

so let me look at it from above

so i have a current coming towards me five amperes i have another current here going inwards five amperes another current which is coming towards me ten amperes

so let me consider two loops one this one and one this one

so find the value the values of integral $\mathbf{b} \cdot d\mathbf{l}$ for paths c one and two draw paths for which integral $\mathbf{b} \cdot d\mathbf{l}$ is maximum and positive and maximum and negative and finally draw another path having the same value of integral $\mathbf{b} \cdot d\mathbf{l}$ as over c one

so you calculate integral $\mathbf{b} \cdot d\mathbf{l}$ for c one and c two draw paths for which integral $\mathbf{b} \cdot d\mathbf{l}$ is maximum and positive and maximum negative and then you have already calculated for path c one draw another figure another curve for which integral value of $\mathbf{b} \cdot d\mathbf{l}$ is the same as for c1

so just give some thoughts to this problem and it will help you to understand better the application of ampere's law ok

so i want to apply ampere's law for certain situations and just like we did for gauss's law for gauss's law we obtained gauss's law and applied gauss's law to calculate electrostatic fields over charged distributions now remember what we found is gauss's law is always valid it is useful in certain situations where there is symmetry because in symmetric situations i can take the electric field out of the integral in gauss's law and that will help me to calculate the electric field distribution here ampere's law is always valid amperes law is useful whenever i can take the magnetic field outside the integral by some symmetry arguments and use that to calculate the magnetic field

so we will start looking at some examples the first example i want to look at is an infinitely long and straight current carrying conductor

so this is my current current conductor now first thing i notice is because of symmetry magnetic field cannot depend on z this distance it has to be the same here here here everywhere it is infinitely long wire it cannot depend on this angle because if you have a current kind of conductor this point is the same at this point i mean it has to be the same it cannot have an angular dependence the only dependence can have it can have is a r dependence that is the distance from here and the magnetic field although magnetic field has an r dependence it has three components it can have three components it can have this component it can have this component and it can have the perpendicular component a component parallel to the wire a component perpendicular to the wire and a component parallel to the wire in the other direction now i can for example think in terms of bio server law and see that if i have any every current element along the wire will produce a magnetic field which is along this direction no element in this current carrying conductor will ever produce any magnetic field along this direction or along this direction because please remember the magnetic field direction is

so this is $d\mathbf{l}$ vector and this \mathbf{r} vector

so $d\mathbf{l} \times \mathbf{r}$ in the magnetic field direction that is always perpendicular to $d\mathbf{l}$ and \mathbf{r} vector

so it lies like this

so the magnetic field has to be azimuthal

so if i look from the top in my current carrying conductor the magnetic field can have only this component a component but

so if i have a magnetic field here can be only like this here it will be like this now i can also use some symmetry arguments to show that magnetic field cannot have this component this component but here i am just using a bioseveral law to to convince you that the magnetic field whatever exists has to be in this direction now once i know the direction of the magnetic field and once i know that the magnetic field does not depend on this angle i am going to use this equation

so this is my bios ampere's law

so what i do is i take a circular path along around this wire with the wire at the center with a distance r

so remember at every point $d\mathbf{l}$ is like this and \mathbf{b} is also like this

so at any point \mathbf{b} is parallel to $d\mathbf{l}$ vector and

so $\mathbf{b} \cdot d\mathbf{l}$ is nothing but $b dl$ at this point \mathbf{b} is like this $d\mathbf{l}$ is like this at this point \mathbf{b} is like this $d\mathbf{l}$ is like this

so i'm please remember for this integration i can choose any path just like gaussian surface i can choose any gaussian surface i can choose any curve which i want in this integration

so my choice is a circular path around the wire with a wire at the center

so that it will help me to integrate the left hand side and what is $d\mathbf{l}$

so $d\mathbf{l}$ for example if this angle is $d\phi$ and this is $r d\mathbf{l}$ is nothing but $r d$

phi

so $\mathbf{b} \cdot d\mathbf{l}$ is $b r d\phi$ and

so ampere's law gives me $\int \mathbf{b} \cdot d\mathbf{l}$ is equal to μ_0 times the current enclosed which is just the current i carried by the conductors which is $\mu_0 i$

so this is nothing but $\int b r d\phi$ is equal to $\mu_0 i$ now b is independent of ϕ b is the same here here here here everywhere b is the same because i am taking a circular path with this y at the center

so b here b here everywhere is the same

so i can take b out of integral and of course r does not depend on ϕ

so b are $\int d\phi \mu_0 i$ which is nothing but $b r \int d\phi$ is the total angle subtended by the circle at this point which is 2π

so 2π is equal to $\mu_0 i$

so magnetic field i have got is $\mu_0 i$ the same as before

so ampere's law although please remember i have obtained ampere's law by deriving the magnetic field due to infinitely long current carrying conductor the mag ampere's law is a very general law it's valid for all situations and i am again using the ampers law to calculate the field due to current carrying conduction infinitely long current carrying conductor and in the finite long current carrying conductor i can use some symmetry arguments to find out the direction orientation of \mathbf{b} vector and the dependence of \mathbf{b} vector on the distance of the wire the position along the wire the angle with respect to wire etcetera all this i can two symmetry arguments i can find out and then i choose an appropriate path of integration which will help me to take b out of the integral this is what i have done

so i have taken a circular path around the wire if i take some arbitrary path i will not be able to do this

so i must to choose an appropriate path a judiciously chosen path of integration and here my duration is judiciously chosen path is a circular path around the wire and because i have chosen that part b happens to be parallel to $d\mathbf{l}$ at every point

so i can write $\mathbf{b} \cdot d\mathbf{l}$ as $b r d\phi$ and b happens to be independent of ϕ

so i can take b out of integral i could not have done this if b was a function of ϕ

so i am able to take b out of the integral and immediately integrate and get the magnetic field

so that is a very interesting example which tells me that the magnetic field of a infinitely long current carrying conductor is nothing but b is equal to $\mu_0 i$ by $2\pi r$ which we had obtained before by using bio server law now i want to con you take another example ah current distributed uniformly over the cross section of a cylindrical wire of circular cross section and infinitely long

so something like this i have a thick current carrying conductor its current is flowing like this in the carrying conductor

so let me assume the radius is r

so the top view will be looking like looking like this i have a circular wire

so current is flowing towards me at every point equally distributed and

so i have to find the magnetic field of this both inside the wire as well as outside the wire now i can use the same argument as for a infinitely long thin current kinetic conductor and say that the magnetic field cannot have a dependence on this position because infinitely long

so this point this point this point all these points are exactly equivalent

so magnetic field cannot have a dependence on this coordinate because it's a cylindrical cross section of a circular cross section wire magnetic field cannot

have a ϕ dependence and dependence on angle that means it will be the same everywhere at the as a function angle if i take certain distance and calculate the magnetic field at any point along the circle it has to be the same because there is no difference between this point this point at this point

so it has to have it cannot have a ϕ dependence

so magnetic field can only have an r dependence the radius the distance from the center of the wire it can only depend on r now again what i find is because these are i can consider this to be a large number of thin current elements going along the direction we will all produce a magnetic field which is azimuthal and which is along this direction and i can use this immediately to calculate the magnetic field of the current kinetic conductor

so this ampere's law tells me $\oint \mathbf{B} \cdot d\mathbf{l}$ is equal to $\mu_0 i_{\text{enclosed}}$

so if this is my current conductor obvious r i take a path inside of radius r now one can show through similar argument symmetry arguments that the magnetic field has to be along the direction this azimuthal direction which is the direction which is tangential to a circular path around the center

so if i take a path this path integral $\oint \mathbf{B} \cdot d\mathbf{l}$ is equal to μ_0 times i_{enclosed} now i am having a total current i passing through an area by r^2 the total y area of the wire is πr^2 and the current is distributed uniformly across the wire

so the current i is carried over a wire of cross sectional area πr^2

so i can define what is called as the current density which is current per unit area which is i by πr^2

so if you take a unit area perpendicular to the wire i will find a current which is passing i by πr^2

so the current enclosed by the path c one is equal to current density into area of c one which is nothing but i by πr^2 into πr^2 i by πr^2 is current density multiplied by the area of this enclosed by the circular path here this area and i get this

so the current enclosed is i into r^2 by r^2 ok

so the current enclosed is i times small r^2 by capital r^2 now as i as i told you the symmetry arguments tell me that the magnetic field is in this direction along the circular earth arc

so $\oint \mathbf{B} \cdot d\mathbf{l}$ will nothing be nothing but $\int \mathbf{B} \cdot d\mathbf{l}$ will again be equal to $\int B r d\phi$ and because B is independent of the angle the same at every point at different angles this is nothing but B times r times $\int d\phi$ which is nothing but B times r times 2π

so i get using appears law B times r times 2π which is equal to μ_0 naught times i_{enclosed} which is equal to μ_0 naught $i r^2$ by capital r^2

so this tells me B is equal to μ_0 naught $i r^2$ by r^2 into one by two πr which is nothing but μ_0 naught i into r by two πr^2

so magnetic field is now proportional to small r it has a dependence like μ_0 naught $i r$ by two πr^2

so the magnetic field at r is equal to zero the magnetic field is zero the magnetic field increases as you go away from the center and this is only valid this formula is valid for a path lying inside the conductor

so that's small part c one

so that's for r less than r because our path is inside the conductor now what happens to a path outside the conductor that is r greater than r

so that's my conductor r and i could take a circular path outside

so the current is two coming towards me now the same arguments tell me that the magnetic field has to be along the direction of this circular path because the circular path has this point at the center

so the magnetic field along the circular path that again tells me $\int \mathbf{b} \cdot d\mathbf{l}$ is equal to $\int \mathbf{b} \cdot r \, d\phi$ which is equal to $ah \, b \times r \times \int d\phi$ which is equal to $2\pi b \times r$ and what is current enclosed is nothing but i the total current carried by the conductor

so i get $2\pi b r$ is equal to $\mu_0 i$ or b is equal to $\mu_0 i$ by $2\pi r$ and this is the same as the magnetic field produced by a current carrying conductor of carrying current i

so it does not depend on the size of the conductor outside the conductor the magnetic field is as if the entire current was passing through the center of the current current conductor

so i have got two expressions here

so let me write it down

so b is equal to $\mu_0 i$ by $2\pi r$ square r or $r < r$ is equal to $\mu_0 i$ by $2\pi r$ for $r > r$ notice that b at r is equal to from this equation $\mu_0 i$ by $2\pi r$ and is the same as from this equation

so b at r is the same

so magnetic field is continuous across the boundary

so if i draw a figure here

so this is my current current conductor

so this is r

so look at this formula magnetic field at $r = 0$ is 0 .

so this magnetic field here it increases linearly with r

so it goes like this up to the point r and then it decreases 1 by r

so so if this is 0 to r magnetic field increases linearly this is some constant error outside and then it decreases 1 by r outside the wire and

so that is the distribution magnetic field for a current kind of conductor of radius r and the directional magnetic field can be obtained by simply looking at the right hand screw rule and finding out the direction and in this case the direction is if the current is coming towards me the direction of the magnetic field is anticlockwise now we would like to look at another example coaxial conductor

so the problem is the following

so i have a one conductor here and another one which is outside

so this current flowing like this in this conductor and current flowing backwards and this contact outside

so the cross section will look something like this

so current is flow coming towards me for example here and currently is flowing away from me here it's uniformly distributed across the cylinder here same current

so a current eye flows in from here and flows back from here it's coaxial conductor because there are two conductors one lying coaxially within the side this is on the axis of the outer cylindrical conductor

so what is the magnetic field now please note that again because of symmetry magnetic field cannot have a dependence on this position along the conductor it cannot have a dependence on the angle

so it has to be the same at all points along this angle it can only have an r dependence where are the distance from here you only have an r dependence and my objective is to find the magnetic field between the points the radius a and b now

so what i do is i take a path of integration

so this is my inner conductor that's outer conductor here the current is coming towards me here

so here is away from me and in the central conductor is towards me

so i take a circular path here now as you can see here the this current is not enclosed by this path

so $\int \mathbf{b} \cdot d\mathbf{l}$ is nothing but $\mu_0 n i$ the current carried by the inner conductor or the current carrier by outer conductor but it only has one i and again because of symmetry you can show that this is $2\pi r$ into b and magnetic field happens to be $\mu_0 n i$ by $2\pi r$ this is for r greater than a less than b

so this is this radius is a this radius is b now i leave it you have to find out what is the magnetic field outside

so at this point for example outside the coaxial conductor what is the magnetic field

so i leave with you please try to use ampere's law and to find out what is the magnetic field outside the coaxial conductor pair it's an interesting problem and you appreciate this the coaxial conductors are used in many electro electronics experiments and these are very important components of electrical engineering and electronic instrumentation now i want to look at another device which is a very very important device another example solenoid

so solenoid is a device which has usually a structural cross section and it has current carrying wire wound around it

so let me draw a these are is it like a coil these are usually very closely bound coils and i can have a current either flowing upwards or downwards for example i can have a current which is flowing downward here in all this

so this the wire which comes from here it goes around and finally comes out from here

so this current carrying conduct currents are equal current is passing through all the wires

so i take a long wire and wrap it around the cylinder very closely around cell wires tightly bound and this is called a solenoid and this is used to create magnetic fields strong magnetic fields and usually we define the number of turns per unit length which means i take a small length unit length of this and calculate the number of turns

so that is a quantity which i would know which i would need to know because that will define as we will see the magnetic field

so if it is closely bound its as if each was a circular loop actually the structural loops are going like this like helix but if they are very very closely bound i can assume that each each winding is a is a closed loop like this and these loops are all carrying currents and all of the loops are carrying the same current

so my problem is to find out what is the magnetic field produced by this and i would like to take an infinitely long solenoid infinity log essentially implies that the if the radius is a and the length is l l is that is much much greater than a

so my length of the solid is very large compared to the dimensional solenoid

so if i have a solenoid i'll be looking somewhere near the center

so for effectively the end effects just disappear remember in a capacitor we had the same problem we had a capacitor with a finite size plates and we assumed that the plates are infinite extent otherwise i have to look at some end effects and here

so i i don't bother myself with end effects i have an infinitely long solenoid and i want to find the magnetic field within the solenoid

so i want to use ampere's law

so to use this law i must find out what will be depend on what par what coordinates and what will be the direction of \mathbf{b}

so ah let me draw the solenoid here

so this is my solenoid now first i take a surface like this now first thing to notice is that again because of its infinitely long the magnetic field cannot have a dependence on this coordinate it has to be the same every point along this and because it is azimuthally symmetric i am assuming that is very closely bound coil it cannot have a fine dependence if at all it can only have a dependence on r if at all it cannot have a dependence on z it cannot have it dependent on ϕ

so let me take a surface like this

so this is the top surface

so this the surface is cutting through the this is the closed surface and i know magnetic field satisfy this equation the constant law for magnetic fields is $\oint \mathbf{B} \cdot d\mathbf{a} = \mu_0 I_{enc}$ ok now

so this is my lower surface here and upper surface is here

so let me call this s_1 this s_2 .

now please note that because the normal to this surface is like this and the normal to the surface is like this the $d\mathbf{a}$ vector here on this surface is winding up the $d\mathbf{a}$ vector on the surface is pointing down remember recall gauss's law when you do an integration like this the $d\mathbf{a}$ vector is an area vector with outward normal

so $d\mathbf{a}$ vector here upward $d\mathbf{a}$ vector here is downwards magnetic field is independent of this distance

so you can immediately understand that the flux that is exiting from the upper surface must be exactly equal to the flux that is entering from the lower surface because remember the normals are oppositely oriented

so if the magnetic field happens to be pointing upwards then as much flux is entering here as is getting out here because magnetic field here and here are the same areas are the same

so they have to cancel each other

so the integral area over s_1 and s_2 just cancel off whether the magnetic field is pointing downward or upward or any angle because there is no dependence on this position as much flux is entering or leaving the lower surface as much as leaving or entering the upper surface

so the only integral which will be left will be over s_3 and because there is no dependence of magnetic field along the direction

so if i call for example suppose let me look at the top surface here

so this is my solenoid and i am taking this a path like this this is my part of the surface

so this is the \hat{r} direction a normal here is like this

so what i will get is $\mathbf{B} \cdot d\mathbf{a}$

so $d\mathbf{a}$

so $d\mathbf{a}$

so $d\mathbf{a}$ vector $d\mathbf{a}$ vector is also in the same direction

so this will be $\mathbf{B} \cdot \hat{r}$ into $2\pi r$ into $d\mathbf{a}$

so \hat{r} is this component $d\mathbf{a}$ is the same component

so at this point \hat{a} is $d\mathbf{a}$ is like this and \hat{r} is this this direction

so at this point $d\mathbf{a}$ is here and \hat{r} is the direction and \hat{r} is independent of this

so what i will get is \hat{r} integral $\mathbf{B} \cdot d\mathbf{a}$ over surface s_3 will be equal to zero that is \hat{r} into $2\pi r$ which this length l this implies \hat{r} is equal to zero there can be no radial component of the magnetic field magnetic field cannot have a component pointing away from the solenoid

so i have used gauss's law for magnetic fields to show that magnetic field cannot have a radial component for the solenoid it is closely bound infinitely long solenoid please remember i am calculating the magnetic field for an

infinitely long closely bound very closely bound solenoid now let me do take another integration

so this is my solenoid and i take a circular path like this and i want to use this equation ampere's law

so from the top if i look from the top that's my solenoid and i am taking a path circular path like this now that's a solenoid ok that's a solenoid and my path is like this and this is r now

so because my path is like this if i call this as a component b_ϕ this is $d\phi$ b_ϕ is the ϕ component of magnetic field which is the azimuthal component which is along the tangent to the circle now please note that in my solenoid i am assuming very tightly bound solenoid

so the coil is like this this all each

so my curve my curve goes like this and if you look at this curve there is no current entering or leaving this path because the current is lying inside here and the other currents are not at all crossing the path there is net current which is entering or leaving this path circular path which is perpendicular solenoid solenoid is like this now and my path is like this

so the right hand side current enter will be zero and because the integration is along this curve i will get b_ϕ into $2\pi r$ must be equal to zero $2\pi r$ is the circumference of the circle

so $b \cdot dl$ becomes b_ϕ into $r d\phi$ and i get $b_\phi r$ this implies there can be no azimuthal component there can be no component which is

so if the solenoid is like this first thing i showed you is there can be no component of magnetic field like this i have also shown you there can be no magnetic field component in this direction

so you can know magnetic field component like this in fact this you can actually use biosaver claw with current elements in different directions to find out again using symmetries to tell you finally that magnetic field has to have only component v_z component

so z axis is like this now

so for for the solenoid it cannot have a component like this it cannot have a component like this sorry sorry ah sorry yeah

so it cannot have a component it cannot have a component like this which i showed you first here i showed you that it cannot suppose this is my solenoid my solenoid is like this

so it cannot have an r component it cannot have a component away from this solenoid it cannot have a component along the azimuth direction the only component left is this exact component that's the only component which can survive now once having obtained this now i can use ampere's law to find out the magnetic field of a solenoid

so let me draw the solenoid again here

so this is a section of the solenoid ah

so the current is coming towards me and this is entering here these are the coils now first thing i do is take a loop outside

so please remember

so z axis b can only have a z component and it can only depend on the radius r

so now i use this ampere's law here for this loop $\int \mathbf{b} \cdot d\mathbf{l} = \mu_0 I_{enc}$

so this curve c now this loop does not enclose any current

so this must be equal to zero

so $\int_a^b \mathbf{b} \cdot d\mathbf{l} + \int_b^c \mathbf{b} \cdot d\mathbf{l} + \int_c^d \mathbf{b} \cdot d\mathbf{l} + \int_d^a \mathbf{b} \cdot d\mathbf{l}$ plus integral d to a because magnetic field has only a z component b_c this integral must be zero and this term of zero because remember the path of integration is like this and the magnetic field can only have a z component

so these two integrals are zero and magnetic field does not depend on the

position at all

so if i call this r_1 this is r_2 b at r_1 first integral is times if this length is l plus b at r_2 now please note the direction of integration opposite

so minus v r_2 into l must be equal to 0 and this implies b at r_1 must be equal to b at r_2 .

so magnetic field seems to be independent of the distance from the axis so that is another exam another result we have got

so what i will do is i will stop my lecture here in the next class i will continue with this discussion and we will then calculate with all these arguments what is the magnetic field inside the solenoid and outside the solenoid and before i leave let me see here that i know that the magnetic field at infinite distances from the solenoid must be becoming zero

so if i tend r_2 to infinity it must be becoming zero

so b outside the solenoid is zero

so b outside the solid

so that's something we have obtained today in the next class i will calculate i will take another amperion loop and i will calculate and show you that the magnetic field within the solenoid is uniform and will calculate the magnitude of the magnetic field you