

a very good morning to all of you we will continue with our discussion in magnetostatics let us recall in the last lecture we had looked at effects of magnetic field on moving charges

so we had calculated the trajectories of particles charged particles as they move in the presence of magnetic fields and electric fields and we had used you have seen some applications of this in the discovery of the electron by j j thompson then applications in mass spectrometry and also in particle accelerators like the cyclotron

so all the discussion was based on the effects of magnetic force and electric force on moving charges now today what i want to discuss is the effects of magnetic forces on current carrying conductors

so let us begin our discussions

so we have what we want to discuss is force on a current carrying conductor

so remember ah when a wire is carrying current these are actually charges moving in the wire primary electrons moving in the wire from one direction to one position another position that constitutes the current but conventionally we define the current as the direction opposite to the electron flow and

so the current is actually nothing but flow of electrons through a wire

so electrons are propagating through the wire ah particle charge particles are probably going through a wire now when you place this wire in a magnetic field we have seen that there is a lawrence force acting on each of these charges

so ah when the wire is placed in a magnetic field the magnetic field will have a force on each of these charges which then transfers itself into the wire and the wire gets pulled or repelled because of the presence of magnetic field

so magnetic forces on the charges have finally an effect on current carrying conductors

so our objective is to find out what is the force acting on a current carrying conductor

so for this let me assume that we have a a long straight wire of cross sectional area a of length l carrying current in this direction now for the moment i will assume that the current consists of positive charges moving up ah and i will show you later on that the force that we calculate will be the same ah as if the electrons are going down are the positive charges are flowing up it is the same

so i take a length l of the wire and place it in a magnetic field which is pointing into the plane of the paper which i draw through crosses now what this implies is now the current is flowing in the upward direction

so if i assume that the current is produced by positive charges each of these positive charges moving up

so when a positive charge is moving up in the presence of magnetic field pointing downward we we know that there is a lawrence force acting on this and that force is $q \mathbf{v} \times \mathbf{b}$

so if q is the charge v is the velocity of the of the particle and b is the magnetic field the force is $q \mathbf{v} \times \mathbf{b}$

so v is upward b is downward

so $v \times b$ is in this direction

so when a current is flowing through a wire in the upward direction then the magnetic field has a force on the wire which is towards the left as drawn here now i want to calculate what is the net force acting on this wire because of this charge motion we know that the magnetic force is equal to $q \mathbf{v} \times \mathbf{b}$ b is the velocity of the charge particle ah q is the charge of the particle and b is the magnetic field now as i mentioned i will assume that these are positive charges moving up in the wire constituting a current

so let me assume that the drift velocity of the charges is equal to b

so the force on each charge magnitude of the force is q times v times b it is in this direction it is in the direction shown here the magnitude is q times v by v b because velocity vector is perpendicular to the magnetic field direction so v cross b is nothing but v times b now that is the force on one charge so i need to calculate the force on the entire wire which has not just one charge but a large number of charges

so for this let me assume that the charge density which is charge per unit volume i need to calculate the charge density how many charges are there in this this thing

so let me assume that the charge density that means the number of charges per unit volume the number of charges will be equal to let me assume n

so there are n charges per unit volume in the wire which is flowing which are flowing upward in the direction of the current

so in this volume what is the volume of the wire volume is equal to the area times the cross sectional area is a and length of the wire is l

so the volume of this wire is a times l

so the number of charges present number of charges present in the wire of length l is equal to n times a times l a times l is the volume n is the number of charges per unit volume

so the total number of charges is this

so total charge is equal to $n a l$ times q each of the charge is has a magnitude q

so the total charge of this volume is $n a l$ into q

so the force is acting magnetic force is acting on all these charges

so the total force on the length l will be equal to $n a l q$ which is the charge times v times b the number of charges e charge is acted upon by force charge times $q b$

so there are $q b$ there are

so many charges present

so the total force on the charge on the wire of length l is $n a l q$ into $v b$ now i want to relate this to the current flowing through the wire

so now what is the current to calculate an expression for the current let me take a wire the same wire of cross sectional area a of length v what is v v is the velocity drift velocity of the charges

so i take a wire of the same cross section a and of length v now because v represents the velocity of the charges in this direction remember all the charges contained in this volume will cross this area in a unit time

so as the charges are moving up all the charges because this distance is v the charges move at distance v in a unit time

so in unit time this surface will come to this surface

so all the charges present within this volume would have crossed the surface in a unit time

so this is the length v all charges contained in the length v will have crossed the upper surface in a unit time

so the current is nothing but current i is equal to the charge flowing per unit time which is equal to

so what is the volume of this a times v is the volume the number density is n

so n times a times v is the number of charges into q a times v is the volume n is the number of charges per unit volume

so the number of charges is this and

so the total charge is $n a v$ into q and that must be the current

so that is the current flowing through the wire

so i can replace $n a v$ times q by current i and i get the force which is equal to i times l times b

so here $n a$ into $q b n a q v$ is i remaining i have l and b
so the force is nothing but current multiplied by velocity length of the wire
multiplied by the magnetic field

so that is the force acting on the wire and

so if we draw the wire again here

so this is the wire carrying a current i and the magnetic field is pointing
into the page of the wire into the page here and these are positive charges
moving up

so the net force is in this direction $v \times b$ now this is a case where the
current is flowing perpendicular to the magnetic field but this may not be
always true i can have a current carrying conductor which not oriented
perpendicular to the magnetic field but at some angle to the magnetic field

so i want to calculate what is the force on a wire which is carrying a current
but that current is not flowing perpendicular to the magnetic field

so let me draw a figure here

so this is the a wire is oriented like this this carrying this is the current
carrying wire ah let me draw the axis this is z axis let me assume this is x
axis and the current is going like this and let me assume that the magnetic
field is oriented like this

so now you see i have an angle between the current carrying conductor wire and
the magnetic field and that angle is not its not 90 degrees its some arbitrary
angle ϕ the earlier example we had considered was a situation when ϕ was 90
degrees

so now i want to calculate what is the force on this

so ah what is the magnetic field magnetic field b vector is equal to b times
magnitude times \hat{a} magnetic field is oriented along the direction what is the
velocity vector of these charges velocity is moving like this

so it has both x component and z component

so x component is $v \sin \phi$ and

so $v \sin \phi$ is the x component and z component of $v \cos \phi$

so the velocity of the charge particle is given by $v \sin \phi \hat{i} + v \cos \phi \hat{k}$
and where v is the magnitude of the velocity and the magnetic field is
 $v \times k$

so force on every charge force on one charge $q v \times b$ which is equal to $q b$
 $\sin \phi \hat{i} + v \cos \phi \hat{k} \times b \hat{k}$ which is equal to $q b \sin \phi \hat{i} + v \cos \phi \hat{k} \times b \hat{k}$
 $\hat{k} \times \hat{k}$ is zero and $\hat{i} \times \hat{k}$ is $-\hat{j}$

so the minus $q v b \sin \phi \hat{j}$

so remember in this figure y axis is pointing up in the page because of the
right handed system $x y z$ and the force is in the downward direction as you can
see here $v \times b$ must be downward

so its $-\hat{j}$ direction

so that is the force on each and every individual charge passing through the
wire and i can relate this to the current and by calculating again the
total charge in a length l must be equal to $n a l$ times q has
before

so the total force on length l will be equal to $-\hat{j} n a l q$ that is the
charge into $v \times b \sin \phi \hat{j}$ and i know the current as before i is equal
to we had done before current is equal to $n a l$ times q that is the
current flowing through this

so ah this is nothing but this force is equal to total force on length l is
equal to $-\hat{j} i b \sin \phi$ where l is a vector which is actually $l \sin \phi \hat{i} + l \cos \phi \hat{k}$
this vector in this direction it has a x component $l \sin \phi$
and a z component which is $l \cos \phi$ and this force is

nothing but $i \mathbf{l} \times \mathbf{b}$

so if you have a wire carrying a current i of length l placed in the magnetic field given by \mathbf{b} vector the total force acting on that length of wire straight length of wire is $i \mathbf{l} \times \mathbf{b}$ this is a uniform magnetic field and I have a force which is $i \mathbf{l} \times \mathbf{b}$

so I can actually if I do not have a straight wire and if I have an infinite decimal length force on an infinite decimal length $d \mathbf{l}$ wire current current wire is equal to $i d \mathbf{l} \times \mathbf{b}$ and this is equal to nothing but a force which I call $d \mathbf{f}$

so if I have if I have a wire of a certain shape I can consider each I can consider small small elements of $d \mathbf{l}$ vector along the wire and each of these real vector has a force acting on it which is given by $i d \mathbf{l} \times \mathbf{b}$ and from this I can calculate the net force acting on the on the total wire of any any shape etcetera by integrating over all the forces acting on each and every element of wire now I have assumed here that the current consists of positive charges flowing up but actually the current consists of electrons flowing down

so let me see what happens because of electrons going down we saw that if the positive charge is going up magnetic field is pointing inward the force is $\mathbf{v} \times \mathbf{b}$ which is in this direction instead of this if I have electrons going downwards constituting the same current $\mathbf{v} \times \mathbf{b}$ is now this direction but because the charges are negative the force is essentially back in the same direction

so independent of whether I consider positive charges moving up or negative charge is moving down the net force acting on the wire is essentially given by this direction here which is \mathbf{f} is equal to $i d \mathbf{l} \times \mathbf{b}$ on the on the current carrying conductors of infinitesimal length $d \mathbf{l}$

so I can use this to calculate the forces on various situations when I have current carrying conductors placed in magnetic fields now before we go to a more general situation I want to find out what is the force between two current carrying conductors

so let me take

so force between two current carrying conductors

so let me take two straight conductors this is carrying current i one this is carrying current i two for the moment I am assuming that the currents are going in upward direction parallel let me assume the distance is d now there are two wires carrying current in the upper direction parallel currents and I want to find out what is the force between these two wires now why there will be force remember this current kinetic conductor produces the magnetic field at the position of this wire

so current is moving up

so what is the directional magnetic field magnetic field on the second wire is pointing downwards and we know the magnitude of the magnetic field will actually calculate the total force with this

so current is going up the this wire is producing magnetic field in this wire and

so we have just now seen that if the current is going up and the magnetic field is pointing down there is a force in this direction

so this particular current produces a magnetic field on this wire which then exerts a force on this wire towards the crossfire what does the what happens with the first wire this second wire i two also produces the magnetic field in the position of the first wire what is the directional magnetic field because the current is going up here the magnetic field is pointing towards me remember the right hand rule a current going up produces a magnetic field like this

so on this on this side of the wire the magnetic field is going downwards but

on this side of the wire on this side of the wire the magnetic field is coming above the page current going up magnetic field pointing up

so what is the force $\mathbf{v} \times \mathbf{b}$ the force is like this

so this wire carrying current i_1 produces a magnetic field at the position of the wire carrying current i_2 and exerts a force towards the first wire the second wire produces a magnetic field in the position of the first wire and the force on the first wire is towards the second wire

so it is just like two charges these two current carrying conductors will attract each other when the currents are parallel

so i am going to calculate what is the force of attraction between these two

so remember ah to calculate the force

so let me calculate the force on on wire two due to wire one

so this let me call this one and let this let me call this two

so now for that i need to know the magnetic field here i know the current and i know the length of the wire

so let me let me take a length l

so what is the magnetic field produced by i_1 one at this point

so b_1 which is equal to we have already seen this before $\mu_0 i_1 / (2\pi r)$ this distance is d as i have written here

so $\mu_0 i_1 / (2\pi d)$ and that is going into the page

so this is the magnetic field produced by this wire on this ah second wire and so the force f_{21} force on wire two because of i_1 is $i_2 l$ and because the force is because the current is carrying current is perpendicular to the directional magnetic field the force is in this direction

so $i_2 l$ into magnetic field which is $\mu_0 i_1 / (2\pi d)$

so the magnetic the force is nothing but $\mu_0 i_1 i_2 l / (2\pi d)$

so length l of the wire has a force f_{21} towards the first wire which is $\mu_0 i_1 i_2 l / (2\pi d)$

so i can write the force per unit length force on wire two per unit length f_{21} is equal to $\mu_0 i_1 i_2 / (2\pi d)$

so that is the force on wire two because of wire one now what is the force on wire one because of wire two for this i need to know the magnetic field because of i_2 at this plane and knowing the length l here

so if i have a length l again here i can calculate

so b_2 which is the magnetic field produced by wire two at the position of i_1 is equal to $\mu_0 i_2 / (2\pi d)$ and the force on wire one will be f_{12} which is equal to current into ah length into magnetic field which is $\mu_0 i_1 i_2 l / (2\pi d)$ which is equal to $\mu_0 i_1 i_2 l / (2\pi d)$ into length

so force per unit length of wire one is equal to $\mu_0 i_1 i_2 / (2\pi d)$ the same force as the f_{21}

so this wire attracts this wire with a certain force f_{21} this wire attracts this wire by the force f_{12} which is equal to f_{21} and

so the two wires get attracted to each other

so this is nothing but essentially uh description of newton's law that you have this particular wire attracts this four wire by a force f_{21} this wire attracts this force this wire by force f_{12} and f_{12} is equal to one and they are opposite directed

so both get attracted to each other and that is the force of attraction between two wires which are carrying parallel currents now if the currents were anti parallel

so if the two wires had were carrying wires into opposite directions

so if this was i_1 and this is i_2 here now i_1 produces again a magnetic

field pointing away from the paper here and this particular current is now going downwards

so you can see the force on this will be in this direction this wire produces a magnetic field which is going into the page here you can use right hand rule again and you can see the force on this is now again $\mathbf{i} \times \mathbf{B}$ is in this direction

so the forces are now repulsive and

so what we find is parallel currents attract each other anti parallel currents repel each other

so parallel currents attract each other anti parallel currents repel each other so let me consider an example

so let me assume a current i_1 is equal i_2 equal currents of five amperes flowing through two wires and let me take a separation d of one centimeter which is ten to the minus two meters

so the force of attraction will be equal to $\mu_0 i_1 i_2 / 2\pi d$ which is equal to $4\pi \times 10^{-7} \text{ into } 5 \times 5 \text{ divided by } 2\pi \text{ into } 10^{-2}$ which is equal to five to the minus four newtons per meter

so the force of attraction between these two parallel currents will be 0.5 million newtons per meter and if the currents were opposite they are directed the same force will be a repulsive force between the two currents

so what we see is a current constitutes charges moving in wires and these charges moving when they are placed in the magnetic field are there is a magnetic force exerted on these charges and

so current carrying conductors also have forces exerted on them by magnetic fields and we have been able to calculate what is the magnetic field on a current carrying conductor and if you take an infinite length l carrying a current i the force is nothing but $i \mathbf{dl} \times \mathbf{B}$

so if you are given a current carrying circuit of an arbitrary shape you can break this up into small elementary lengths dl vector calculate the force on each of these dl vectors and add them up and to calculate the total force now I want to have an application of this to the calculation of a torque on a current carrying loop torque on a current carrying loop placed in a uniform magnetic field

so I have a current carrying loop placed in a uniform magnetic field and I want to calculate the torque on this

so for this I want to take an example of a rectangular current carrying loop of sides a and b carrying current i a loop is placed in a magnetic field B

so now let me draw a figure to show you the geometry of the problem

so let me draw the axis first

so I have x, y, z I assume a planar loop placed in the x, y plane carrying a current like this

so it's a planar loop of side a by b

so let me call this side a this will be a rectangular loop of a cross b carrying a current i and placed in a magnetic field B is a planar loop and placed in the x, y plane now the magnetic field would have some arbitrary direction but let me as an example here consider the magnetic field to be in the vertical plane pointing at some angle like this now let me call this angle ϕ

so this is a planar loop like this it's a loop placed like this in a magnetic field in a x, y plane and is a magnetic field pointing in some direction like this in this direction here make an angle ϕ with a vertical

so the current carrying conductor is a rectangular loop like this carrying a current and the magnetic field is necessarily not perpendicular to this plane or parallel to this plane it's some angle making an angle ϕ with the

normal to this plane and lying in the y z plane

so i want to calculate what is the net force acting on this current loop and what is the net torque acting on this current tube

so for this let me write down the magnetic field of this given by given in this figure here magnetic field is has a component which is along y and a component along z

so i have $b \sin \phi \hat{j}$ cap plus $b \cos \phi \hat{k}$ cap it has a component $b \sin \phi$ along the y direction and a component $b \cos \phi$ along the z direction i am assuming the magnetic field to be lying in the y z plane

so this magnetic field will now have a force on each of these elements and according to the formula we have derived earlier we can use what is the force on this element what is the force on this element what is the force on this element and what is the force on this element

so let me ah call this paths one two three and four

so one is this length here two is this one three is this one and four is this one

so i want to calculate the forces on all these four ah current elements constituting the loop

so let me start by force on one that is this force on this element now for this i need to know i know the force is $i \mathbf{l} \times \mathbf{b}$ l is the length of the wire and i is a current and b is the magnetic field

so i need to know that current is i and i need to know what is l vector now this is a line this is a current carrying conductor pointed along the x direction

so l vector for this will be l vector will be simply and of length b look at this this figure here this length is b point along the along the x direction current is flowing along the x direction

so l is actually nothing but b times i cap b is the length of this vector l this is the current kind of conductor here length is l the length is b here and it is flowing along the x direction

so l vector is nothing but b i

so force f one will be equal to $i \mathbf{l} \times \mathbf{b}$ which is equal to $i b i \text{ cap} \times b \sin \phi \hat{j}$ cap plus $b \cos \phi \hat{k}$ cap which is equal to $i b b \sin \phi i \text{ cap} \times j \text{ cap}$ is $k \text{ cap}$ and then i have $i \text{ cap} \times k \text{ cap}$ is minus $j \text{ cap}$

so $i b b \cos \phi i \text{ cap} \times j \text{ cap}$ is $k \text{ cap}$ $i \text{ cap} \times k \text{ cap}$ is minus $j \text{ cap}$

so that is the force f one acting on the current element this part of the current loop now let me calculate the current the force on this part of the current which is which i call f two this is of length a and pointing in the y direction the current is flowing in the y direction into the length a

so force on two i want to calculate

so for this l will be equal to a times j cap length is a and the current is flowing along the y direction the the current the current element is along the y direction

so l vector is nothing but a times j cap

so f two force is equal to again $i \mathbf{l} \times \mathbf{b}$ which is equal to $i a j \text{ cap} \times b \sin \phi \hat{j}$ cap plus $b \cos \phi \hat{k}$ cap which is equal to now $j \text{ cap} \times j \text{ cap}$ is zero $j \text{ cap} \times k \text{ cap}$ is $i \text{ cap}$

so $i a b \cos \phi i \text{ cap}$ $i a b \cos \phi i \text{ cap}$ remember this force is has only x component

so this must be acting like this this force has both y and z components

so it must be force its acting like this it has a positive z component and a negative y component

so it is a force acting like this

so i have calculated the force on this current element and on this current element similarly i need to calculate the force on this element and on this element

so let me calculate f_3

so for f_3 i must write again \vec{l} now \vec{l} for f_3 is nothing but look here in the figure this is a current flowing in the minus x direction of length b

so \vec{l} vector will be equal to minus b times \hat{i} and f_3 will be equal to $\vec{l} \times \vec{B}$ is equal to minus b \hat{i} cross $B \sin \phi \hat{j} + B \cos \phi \hat{k}$ which is equal to now i can cross \hat{j} is \hat{k}

so minus $b B \sin \phi \hat{k} - b B \cos \phi \hat{i}$

so plus $b B \cos \phi \hat{j}$ now its interesting to note that this force f_3 is exactly minus of this force this is $b B \sin \phi \hat{k}$ this is minus $b B \sin \phi \hat{k}$ is minus $b B \cos \phi \hat{j}$ plus $b B \cos \phi \hat{j}$

so this force is exactly opposite of this force and that's expected because this current carrying conductor is parallel to this current carrying conductor and the current is flowing in the opposite direction

so the force if this force is acting on this like this the force on this must be like this direction similarly i leave it u to calculate force on four

so \vec{l} vector for this will be equal to minus a \hat{j} and f_4 will come out to be minus $a B \cos \phi \hat{i}$ ah just which is just the minus of this force here it was $a B \cos \phi \hat{i}$ it will be minus $a B \cos \phi \hat{i}$

so there are four com four parts to this current this circuit or this current carrying conductor and i have calculated the forces on each of these parts

so i can from here calculate the total force on this current kinetic conductor

so total force $f_1 + f_2 + f_3 + f_4$ and you can see here f_1 and f_3 are exactly equal and opposite to each other f_2 and f_4 this is this is f_2 this is f_4 they are exactly equal and opposite to each other similarly f_2 and f_4 are exactly equal and opposite to each other

so f_1 and f_3 cancel of f_2 and f_4 cancel of and the total force is zero

so current carrying loop placed in the uniform magnetic field has no net force acting on the current on the loop and the net force is zero now although the net force is zero because of these two forces there is a torque acting on this system and we need to calculate this torque

so let me draw the figure again here

so i have this current kind of conductor here

so there is a force in the direction there is a force in this direction

so let me look at

so this is let me draw the plane again here this is y this is z this is x

so let me draw the figure in the bizarre plane to be clear

so this is y said

so there is a conductor here there is a conductor here this current is flowing here like this

so this current is flowing towards me and this current is flowing away from me and this force is like this and this force is like

so this is f_1 and this is f_3 please note that f_2 and f_4 do not create any torque on the loop because f_2 is like this and f_4 is like this they are exactly equal and opposite and acting through the origin here

so effectively they are parallel to the x axis and there is no net torque but these two forces can create a torque around this i can calculate the torque around this point o remember this distance is a

so i can calculate the torque ah because of these two forces

so let me calculate the torque ah now

so let me have the figure here

so torque due to f_1 about O

so ah let me call this τ_1 is equal to $r_1 \times f_1$ and where r_1 is this vector here this vector is $r_1 \times f_1$ is a force now r_1 is has a length a by 2 and is oriented along $-\hat{y}$ direction

so $-\frac{a}{2} \hat{j} \times f_1$ we have calculated before f_1 's expression is here

so let me substitute the expression for f_1 $i b b \sin \phi \hat{k} - i b b \cos \phi \hat{j}$ $r_1 \times f_1$ $\hat{j} \times \hat{k}$ is \hat{i}

so this is $-\frac{a}{2} b^2 \sin \phi \hat{i} + \frac{a}{2} b^2 \cos \phi \hat{j} \times \hat{k}$ is \hat{i} $\hat{j} \times \hat{k}$ is zero

so $-\frac{a}{2} b^2 \sin \phi \hat{i}$ that is the torque of this force around this origin O i can similarly calculate torque due to f_3 about O now this is r_3

so i can write τ_3 is equal to $r_3 \times f_3$ and r_3 is as a magnitude a by 2 and directed along y

so this is equal to $\frac{a}{2} \hat{j} \times f_3$ vector f_3 vector we have again calculated before and f_3 vector is here

so $-\frac{a}{2} b^2 \sin \phi \hat{i} + \frac{a}{2} b^2 \cos \phi \hat{j} \times \hat{k}$ $\hat{j} \times \hat{k}$ is \hat{i} $\hat{j} \times \hat{k}$ is zero

so i get again with the minus sign here $-\frac{a}{2} b^2 \sin \phi \hat{i}$

so the torque due to force f_1 around this point is this this torque due to f_3 around this point is also the same and they are same orientation

so the total torque τ_1 plus τ_3 which is nothing but $-\frac{a}{2} b^2 \sin \phi \hat{i} - \frac{a}{2} b^2 \sin \phi \hat{i}$ i want to write this as follows

so i want write this as $-\frac{a}{2} b^2 \sin \phi \hat{i}$ is nothing but

so $-\frac{a}{2} \hat{i} \times \hat{j} \times \hat{k}$ plus \hat{k} is \hat{i}

so $-\frac{a}{2} \hat{i} \times \hat{j} \times \hat{k}$

so τ i want to write in terms of a vector like this now i am doing this we will just just become clear

so this is my torque as calculated from the calculation of the total force force is acting on the four components of the current element current circuit and

so remember we had introduced the concept of magnetic dipoles and i had introduced what is called as a dipole moment is equal to current into area and area has is a vector

so in my case ah look at the figure here this z direction this is y direction this is x direction current i this length this length is a this length is b

so now remember we have defined an area vector here if i use the right hand rule this area vector has a magnitude the area of this current loop which is a times b and according to right hand rule its orientation is along the z axis

so area of this will be $a b \hat{k}$ and current is i

so for this current loop m is equal to $i a b \hat{k}$ m vector that is the magnetic dipole moment of this is nothing but $i a b \hat{k}$ now we already know the direction of the magnetic field b is nothing but $a \sin \phi \hat{j} + b \cos \phi \hat{k}$

so let me calculate $m \times b$ $i a b \hat{j} \times b \sin \phi \hat{k} + b \cos \phi \hat{k} \times \hat{j}$ $\hat{j} \times \hat{k}$ is \hat{i} and i have $i a b b$ just one second sorry this is $i a b \hat{k}$ sorry this is \hat{k}

so $\hat{k} \times \hat{j}$ is $-\hat{i}$

so let me write this as $b^2 \sin \phi$ into $\hat{k} \times \hat{j}$ $\hat{k} \times \hat{j}$ is $-\hat{i}$ this is $\hat{k} \times \hat{j}$ is $-\hat{i}$ this it is not zero $\hat{k} \times \hat{k}$ is zero

so i get $i a b^2 \sin \phi \hat{i}$ plus \hat{j}

so let me compare this with the expression for tau we have obtained $i a b \sin \phi$ into capital b into $\sin \phi$ k capital j cap

so this torque is nothing but $\mathbf{m} \times \mathbf{b}$

so i get a torque on this current loop tau is equal to

so if i have a current loop like this the two forces acting like this as the magnetic field oriented in some direction and this will tend to

so this is x direction this is y this is z direction this is y direction the m vector is pointing up magnetic field is pointing like this

so $\mathbf{m} \times \mathbf{b}$ gives you the direction of rotation that the torque and this torque will tend to orient the the dipole in such a fashion that torque finally becomes 0.

so this is the torque acted upon on this current loop having a magnetic dipole moment \mathbf{m} and this is very similar if you recall our discussion in electrostatics this is very similar to the torque on an electric dipole which was $\mathbf{p} \times \mathbf{e}$ the electrostatic torque on an electric dipole is $\mathbf{p} \times \mathbf{e}$ \mathbf{p} is the electric dipole moment and each electric field here the torque is magnetic dipole moment across the magnetic field \mathbf{b} it is the same similar expression and it gives you the torque on the current element

so the torque becomes 0 when \mathbf{m} and \mathbf{b} become parallel

so the magnetic field tends to align the dipoles along the directional magnetic field

so that \mathbf{b} be the top becomes zero in fact when \mathbf{m} is anti-parallel to \mathbf{b} also torque is 0 but \mathbf{m} parallel to \mathbf{b} is a stable equilibrium position the opposite direction is an unstable equilibrium position

so you can work it out to show that when \mathbf{m} and \mathbf{b} are parallel you have a stable position when \mathbf{m} and \mathbf{m} are anti parallel you have an unstable position of equilibrium

so this will give me a torque and

so if i have if the loop contains n turns closely bound then the dipole moment is nothing but n times i times area times

so the torque gets multiplied by the number of turns in the coil and

so you have a higher torque if you have more turns in the coil than if you have a single turn in the coil

so the the torque not only depends on current it also depends on the area of the loop it also depends on the number of turns

so this this torque is used in many electrical instruments for example motors and generators and many other types of instruments what i would like to study in the course here is an application to a current measuring device which is called the moving coil galvanometer

so whenever you place a current carrying loop in a magnetic field there is a torque acting on this if you have if you have uniform magnetic field the net force is zero but if you have a

so but there is a torque acting on this which tries to align the magnetic dipole with the magnetic field and this torque can be used to make instruments

so here i want to consider what is called as a moving coil galvanometer

so let me draw the construction it consists of a a pair of permanent magnets here this is the north pole this is south pole

so magnetic field is going from n to s and in the center you have a a coil in a wound on a soft iron core and this coil is carrying current

so let me the coil goes like this n number of coils going here

so the coil is like this and this is connected to a spring and which spring has a as a needle pointing

so this spring is fixed if you try to twist this the spring acts gives a

restoring force and tries to bring it back

so restoring force is created by the spring and remember there is a magnetic field because of the shape of pole pieces here the magnetic field is pointing like this from the north pole to the south pole

so this is the this is one the directional magnetic field is from this point to this point

so you have almost a radial magnetic field

so now let us see what happens when i pass a current through this coil when a current passes through this coil these two are current carrying conductors placed in a magnetic field

so torque acts on this current carrying coil trying to rotate this coil around this axis

so when the current is acted upon by a force and the coil turns to rotate the spring provides it a restoring force

so if you pass a certain current of a certain magnitude the coil will rotate and stop because at that point the torque provided by the magnetic field is balanced by the torque provided by the restoring spring

so the needle will rotate and that will be an indication of the current passing through the coil if you change the current the torque changes and the deflection of the needle will change

so the deflection of the needle becomes proportional to the current passing through the coil and

so the deflection of the needle is an indication of the current that is passing through the coil

so you can get an indication of the current passing through the coil by simply looking at the deflection of this needle and that is called a moving coil galvanometer and

so let me calculate what is the deflection of this of this needle

so the torque due to current i

so let me call this τ current which is equal to number of loops n into i into area into magnetic field a is the area of the current loop i is the current property getting through the loop there are n loops and b is the magnetic field

so a is area of loop n number of loops and i is the current and b is the magnetic field

so this will produce a deflection and the restoring force provided by the spring will be proportional to the displacement angular displacement where k is the spring constant

so i will stop here what we will look at is the next class how to make this galvanometer into an ammeter which is an instrument to measure current propagating through a circuit or converted a voltmeter to measure potential differences across terminals in a circuit

so this moving called galvanometer is a very interesting example of using the torque because of the magnetic field in measuring currents thank you you