

very good morning to all of you in the last lecture we started to discuss about the concept of displacement current

so i would like to recall some of the discussion that we had in the last end of the last lecture because this is a very very important concept that we must understand very clearly

so we showed that ampere's law in this form ampere's law which we had obtained before and used for calculation of magnetic fields produced by currents the amperes law in this form has some problems some inconsistency to show this what we had done was we had taken a pair of capacitor plates here and we look at charging of the capacitor

so there is a current flowing in as a function of time and charges the capacitor plates

so the objective is to find out what is the magnetic field say at this point

so what we did was with what what how do we normally do we draw a

a loop we take a loop of integration a circular loop around the axis and calculate the left hand side which is $\int \mathbf{v} \cdot d\mathbf{l}$ now in this example because of a straight wire because of symmetry as we have discussed earlier the magnetic field will be azimuthal and

so i can actually integrate this left hand side now what is the right hand side of this equation the right hand side is involves the current that is passing through the surface whose boundary is this curve please remember that on the left side we have an integration over a line line integral thats integration over a path the right hand side has the current crossing the surface of which this line is the boundary

so normally what we tend to do is to take the surface to be the plane surface crossing the wire and

so the right hand side simply becomes μ_0 naught times the current that is passing through the surface and we had used this to calculate the magnetic field surrounding a wire and obtained various different different magnetic fields now the problem is that in this integral on the right hand side if i see the current enclosed there is no necessity that i need to choose the surface all that is required is a surface whose boundary is this line

so i could have chosen for example if i draw the same same capacitor here is the plate capacitor plate here coming in here

so this is my loop which i have taken i could have chosen another surface the surface i could have chosen is like this this is the surface

so this is like a a box with a hole here in the center and this is my surface now the surface encloses the capacitor plates but does not cross the wire

so when i look at this problem it it seems that the current enclosed is zero because there is no current crossing the surface this surface is not crossing the wire the wire is not crossing the surface which means that there is no current crossing the surface

so with this argument it looks like the right hand side is zero

so obviously i cannot get two different results for the magnetic field depending on the surface which i choose for integration or for calculating the current enclosed

so there is a inconsistency in this

so we solve this problem or we try to analyze this by using the following argument now let me call these two surfaces

so let me draw the figure again here

so i have this capacitor plates

so i have this loop here

so let me call this surface s one and let me draw another surface i call this

surface s_2 the two surfaces which I take now for surface s_1 current enclosed is equal to i because that is the current which is crossing the surface and for s_2 the current enclosed seems to be zero

so that's the problem here

so we actually resolve this issue by doing a following calculation now look here for the surface s_2 there is a magnetic field which is within this area between the capacitor plates sorry electric field between the capacitor plates

so we calculate the electric flux through s_2

so electric flux is $\int \mathbf{E} \cdot d\mathbf{A}$ and as we showed last time it is electric field into area

so if I take a surface like this of this shape the electric field lines are going like this and if I neglect edge effects over the capacitor the electric field is uniform across the surface area of the capacitor plates and

so there is electric field in this area of the capacitor plates and the electric field into area and I know from an earlier discussion that the electric field is nothing but σ / ϵ_0 where σ is the charge density charge per unit area

so σ into A is nothing but q / ϵ_0 where q is the charge on the capacitor plates σ is the charge density charge per unit area multiplied by A the area of the plates gives me the total charge contained on the surface of the capacitor plates

so that is q / ϵ_0

so now I can calculate the current i is equal to dq / dt which according to this equation is nothing but $\epsilon_0 d\phi_E / dt$

so the current that is flowing into this wire into the capacitor plates is exactly equal to ϵ_0 times the rate of change of electric flux through the surface

so I can actually modify ampere's law to the following equation

so if we write $\int \mathbf{v} \cdot d\mathbf{l}$ is equal to μ_0 times now I will call this conduction current the current that is actually flowing through the wire I will call it as conduction current

so this I call as conduction current to differentiate from another current

so that is a conduction current that means current that is being that is flowing because of electrons movement plus I add another term which is $\mu_0 \epsilon_0 d\phi_E / dt$

so I have just added this term into this equation to modify ampere's law

so this is called a modified ampere's law now what is this if I look at this equation if I take the surface s_1 for integration then the second term is zero and the first term is $\mu_0 i$ which is i which is current flowing through the wire if I take the surface s_2 the first term is zero and I can only contribution from the second term and second term is also $\mu_0 i$ the same the same term as the first term

so if I modify ampere's law to this equation then I find that whether I use surface s_1 or surface s_2 I get the same value of the right hand side and the analysis becomes independent of the surface that I choose to calculate current enclosed

so this was the modification that was made by James Clark Maxwell and this equation is the modified form of ampere's law modified form of ampere's law it includes two terms one is this term called the conduction current term and the second term is what is called as the displacement current

so I call the displacement current displacement current is i_d is equal to $\epsilon_0 d\phi_E / dt$

so that's a displacement current

so I will write this equation as $\int \mathbf{B} \cdot d\mathbf{l}$ is equal to μ_0 times

i conduction plus i displacement

so this modified form of ampere's law will help me resolve the problem and this is what james clerk maxwell did and he modified ampere's law to introduce this displacement current term and this displacement current is nothing but related to the rate of change of electrical flux through the surface now i must mention here that there is no displacement taking place it is just a definition here and in free space there is no displacement it is still called displacement current and that is modified form of ampere's law and using this i can use this law to calculate ah using any particular surface i can also define a displacement current density current density is the current crossing per unit area along a direction perpendicular to the area and that is i can define the displacement current density as $\epsilon_0 \frac{d\phi_e}{dt}$

so this is the displacement current per unit area perpendicular to the area that's flowing and that's called the displacement current density and just like conduction current density we have a displacement current density which is j_d is equal to $\epsilon_0 \frac{d\phi_e}{dt}$ of the electric field at that point

so the displacement current can vary between point to point because the displacement current density can vary from point to point because the electric field itself can be varying from point to point

so in general the electric field is not uniform the electric field is non uniform and non uniform electric field can give you a non uniform displacement current density and of course if i integrate over an entire area i will get the total displacement current

so we started looking at some prob some example an example we started looking for is a capacitor with circular plates

so that's a parallel plate capacitor with circular plates

so let me assume the radius is r and the electric field here

so the current is flowing like this flowing out from here and the electric field between the two current plates will be like like this in this direction

so if i draw ah in this direction if the electric field is pointing downward i choose

so that is electric field pointing downward this is the looking at the capacitor plates and this is the radius r ok

so i am looking at the capacitor plate from here

so the electric field is pointing downward and

so i want to calculate the following problem the following problem is because the electric field is changing with time here i want to calculate what is the magnetic field generated between the plates of the capacitor at this point for example at some distance from the axis this is the axis here some distance from the axis and outside

so we did this problem yes in the last class

so what i do is i take if the radius r is smaller than first let me take the situation where small r is less than capital r

so if this is my capacitor plate ah i take a point which is in between the capacitor plate of a distance small r that is capital r the electric field is pointing downward

so i take a loop of integration like this and i use this formula now between
so i have

so what i will do is i take a this is my path of integration and i take the surface the simplest surface again just like before i take the surface

so i take the surface

so because of symmetry b is azimuthal

so $\int b \cdot dl$ is nothing but b times $2\pi r$ and the electric flux is equal to the area of this πr^2 into electric field which is E is

nothing but σ by ϵ_0 σ is nothing but the surface charge density which is q by area of the plates which is q by πR^2 so this is equal to πr^2 by ϵ_0 1 by πR^2 q and that's equal to q times r^2 by $\epsilon_0 R^2$ that is the electric flux passing through this this surface which I have taken and so $d\Phi_e$ by dt is nothing but r^2 by $\epsilon_0 R^2$ dq by dt and dq by dt is nothing but the current which is flowing through the wires so $\epsilon_0 R^2$ into i so the rate of change of electric flux through the surface is nothing but πr^2 by $\epsilon_0 R^2$ into i and so if I substitute into the ampere's law so for this loop for example for this surface there is no conduction current this is the surface which is taken between the plates of the capacitor so there is no conduction current passing so I have the two plates of the capacitor and my area of integration is here and the current is flowing through a wire from here and getting out from here so there is no conduction current there is only displacement current so if I use this formula so in the presence of only displacement current so I have actually $\mu_0 \epsilon_0$ times i_c plus $\mu_0 \epsilon_0$ $d\Phi_e$ by dt for this loop which I have taken this is equal to zero so this becomes simply $\mu_0 \epsilon_0$ $d\Phi_e$ by dt which is equal to $\mu_0 \epsilon_0$ d of $d\Phi_e$ by dt have just now calculated r^2 by $\epsilon_0 R^2$ into i and the left hand side I have done as b into $2\pi r$ so b becomes equal to $\mu_0 \epsilon_0 r$ by $2\pi R^2$ into y this is for r less than R so if the distance from the axis of the circular plate capacitor is less than R which is the radius of the capacitor plates and I am inside in the space between the capacitor plates there is a magnetic field associated with the changing electric field and that magnetic field comes out to be $\mu_0 \epsilon_0 r$ by $2\pi R^2$ into y now I leave this as a problem to you that you can show that if you take a conductor of radius R that is the axis and if you calculate the magnetic field at a distance r from the axis within the conductor you will get exactly the same expression as this so I leave this problem to you to show that this is exactly the same as if there was an actual current conduction current flowing through a wire of radius R and you are calculating the magnetic field at a distance r from the axis of that conductor so that is the magnetic field inside the capacitor plates and for r greater than R which means this is my capacitor plates electric field is pointing downward here again and I take a path outside so this is my distance r so again now Φ_e is equal to now please remember the electric flux is only present in the radius R so πR^2 into σ which is equal to πR^2 into σ by ϵ_0 and σ into πR^2 is nothing but q because πR^2 is area between the plates of the plates times the charge density is the total charge so $d\Phi_e$ by dt for this case becomes equal to 1 by ϵ_0 dq by dt which is nothing but 1 by ϵ_0 times i so if I use ampere's law I will so if I use this law $b \cdot dl$ is equal to $\mu_0 i_c$ plus $\mu_0 \epsilon_0$ $d\Phi_e$ by dt this is again zero there is no conduction current passing through this area and the second term is the displacement current which I will

get and if i use that i will get b into $2\pi r$ is equal to $\mu_0 \epsilon_0 \frac{dq}{dt}$ which is equal to $\mu_0 i$ because $\frac{dq}{dt}$ is the current that is passing through

so the magnetic field happens to be $\mu_0 i$ by $2\pi r$ this is for r greater than r

so if i had this capacitor plate like this here ah for points between these within this region b is given by this value $\mu_0 r$ by $2\pi r^2 i$

so the magnetic field on the axis is zero small r is zero and as you move away from the axis the magnetic field increases linearly with the distance until it reaches the distance capital r beyond capital r the magnetic field decreases as 1 by r

so if i were to plot magnetic field as a function of position between the capacitor plates this is r this is b and suppose this distance is capital r till here the magnetic field increases linearly and then decreases one by r and please note that the magnetic field is continuous at small r is equal to capital r

so magnetic field at small r is equal to capital r is equal to $\mu_0 i$ by $2\pi r$ also note that

so this is the magnetic field as i have calculated say at this point at a distance small r from the axis this is also the same as the magnetic field at a distance small r from the axis above the conducting wire because what you would have done you would have taken an amperian loop around this the the current that is passing through is purely conduction current which is i and you would have got exactly the same result

so magnetic field whether you calculate using this ah conduction current passing through the surface or the displacement current passing through the surface you get the same value and

so this is that additional term which has been introduced by maxwell is a very very important term as it makes the ampere's law consistent with no matter what surface you take to calculate the current enclosed by the surface

so the current enclosed could consist of either conduction current or displacement current and

so i have to take into account both these currents now i just want to look at look at this continue to look at the same problem

so i had this capacitor plates and i was trying to find out the magnetic field at this point which is within the area of the capacitor plate this is the capacitor plate here and that is my ah

so electric field

so current is flowing like this here current is flowing out from here and the electric field lines are like this

so i do an integration like this now as i told you this is the ampere's law simply tells me $\oint \mathbf{b} \cdot d\mathbf{l}$ is equal to $\mu_0 i_c$ plus $\mu_0 \epsilon_0 \frac{d\phi_e}{dt}$ now i had taken this area for my integration i taken this area between the the circular area that is lying ah between the within the loop but again as before i am not constrained to take that area i could have taken another area which looks like this

so i could have taken the surface area to be outside

so this is this is like a cylinder this is like a cylindrical surface here and this is the cylinder here

so its a cylinder with a hole here

so i could have chosen this surface area rather than the surface area which is flat surface area which contains the circle as its boundary just like in my earlier discussion remember in my earlier discussion i said that when i have to calculate the current enclosed by this i can take this surface area this surface

area or the surface area and i got the same result

so here also i can do the same thing i can take this surface area which is the flat surface area to calculate the magnetic field at this plane at this point between the pair of capacitor plates or i could have taken a surface area outside

so i want to check whether i get the same result and you will see that i will get the same result because the equation is right

so now what happens in this case is there are both currents present in my problem because this surface now includes this surface in which the conductor is passing

so there is current i entering the surface here and there is a current there is a

so there is a conduction current entering the surface this volume and there is a displacement current leaving this volume

so if i do my integration like this please remember i have always mentioned that in this integral the area over which

so how do i define whether the current crossing the area is positive or negative

so if i integrate like this in my loop of integration according to right hand rule this implies this is my right hand direction

so current enclosed will be positive if it enters like this and current will be negative if it enters like this if my loop of integration is like this

so remember if i if i integrate like this then the one which is coming towards me is a positive current the one which is going away from me is a negative current on the other hand if i integrate like this

so if my line integral is taken like this then the positive current implies current going towards you and negative current implies current coming towards me because of the right hand rule

so i must be very careful

so here because i am integrating in this figure in this direction

so positive surface area positive area will be away from this

so here this area vector the normal to the area is actually like this because of the because of the area that i have taken because of the closed loop that i have taken the area integral which that means they whether the current entering is positive or negative depends on the direction of the normal or the area and that normal i must use judiciously

so now what happens in this problem is there is on the surface area there is conduction current entering from this point there is no current anywhere except in this region in the cylindrical region between smaller and capital r

so there are two currents now conduction current i_c is equal to i and displacement current between r and $r + dr$

so sorry between r and r

so between this radius

so if i look from the from the side if i look

so that is my capacitor plate and that is the distance which i am calculating here is where i am calculating magnetic field

so this is small r and

so the area actually consists of

so the area let me draw here the area consists of a plane going outside the plates

so this is the total area the area of integration is here

so as you can see here that is the loop and which i am integrating

so if the electric field is pointing downward

so in this case if i if i plot the if i if i am drawing like this the electric

field is pointing towards me in this region only this is capital r

so the flux that is actually responsible or the current which is entering is only in this area because that is my area of integration that is the area which is this which is contained in the surface that because i am i am integrating over this and the surface which i have chosen is not the standard surface which is the flat surface of which this is a boundary i have taken a surface which is outside and

so there are two kinds of currents in this problem now there is a conduction current entering from here and there is a displacement current that is that is that is passing through the surface between the radius small r and capital r

so i have to consider both currents in this equation in this equation which is $\oint \mathbf{B} \cdot d\mathbf{l}$ again let me write is equal to $\mu_0 i_{\text{conduction}} + \mu_0 \epsilon_0 n \frac{d\phi}{dt}$ i must consider both of them

so in this in this for this surface i_c is equal to i and i must calculate the displacement current i_d and displacement current is nothing but $\epsilon_0 \frac{d\phi}{dt}$

so i_d is equal to $\epsilon_0 \frac{d\phi}{dt}$ now here is where the problem appears that because of the direction of integration the normal is in this direction and the electric field is pointing away from the surface outside the surface and the area vector is towards the surface

so i get a negative sign in the integration

so what i will get is this is equal to $\epsilon_0 \frac{d\phi}{dt}$ of minus of electric field into this area i am assuming electric field to be uniform between the capacitor plates and there is no electric field outside

so electric field is uniform and the area is nothing but $\pi R^2 - \pi r^2$

so this is equal to $\pi R^2 - \pi r^2$ which is equal to $\epsilon_0 \frac{d\phi}{dt}$ of minus $\sigma \pi R^2 - \pi r^2$ which is equal to $-\frac{dq}{dt}$ now σ is $\frac{q}{A}$ into $\pi R^2 - \pi r^2$ which is equal to $-\pi \frac{dq}{dt} \frac{R^2 - r^2}{R^2}$ which is equal to $-\pi \frac{dq}{dt} \frac{R^2 - r^2}{R^2}$ π cancels off and i get $-\frac{dq}{dt} \frac{R^2 - r^2}{R^2}$

so there is a displacement current of $-\frac{dq}{dt} \frac{R^2 - r^2}{R^2}$ by capital R^2 there is a displacement current of $-\frac{dq}{dt} \frac{1 - \frac{r^2}{R^2}}$ by capital R^2 that is crossing this part of the surface there is no other current in any other surface except from here there is a conduction current entering

so the total current that is entering now consists of these two parts

so if i now use integral i need to use now the amperes law $\oint \mathbf{B} \cdot d\mathbf{l}$ is equal to $\mu_0 (i_{\text{conduction}} + i_{\text{displacement}})$ and

so $\oint \mathbf{B} \cdot d\mathbf{l}$ into $2\pi r$ is equal to $\mu_0 i$ now that was the conduction current and displacement current is this thing

so $-\mu_0 i \frac{R^2 - r^2}{R^2}$ by capital R^2 which is equal to $\mu_0 i \frac{R^2 - r^2}{R^2}$ plus $\mu_0 i \frac{r^2}{R^2}$ by capital R^2

so this cancels off right this becomes equal to $\mu_0 i \frac{r^2}{R^2}$ by capital R^2

so B becomes is equal to $\mu_0 i \frac{r^2}{R^2}$ by capital R^2 into $\frac{1}{2\pi r}$ which is equal to $\mu_0 i \frac{r}{2\pi R^2}$

so let me compare this with what we had obtained earlier for the for the position $r < R$ and that that's the formula here $\mu_0 i \frac{r}{2\pi R^2}$ exactly the same equation

so irrespective of what surface i choose i must get the same value of magnetic field and i have shown that through this example that it is not necessary that i

must choose a surface which only conduct any conduction current i could choose a surface which only carries conduction current i could choose a surface which only carries displacement current or i could choose a surface which carries both conduction current and displacement current and

so in this example it shows that if i take a surface which i have now taken in this example this example in this example the current that is entering the surface or crossing the surface contains both conduction current and displacement current and as i showed you i must be very careful in taking the right signs for the currents because whether the current is entering or leaving the surface depends on the direction of the area of the surface and this one must choose appropriately and carefully in this calculations

so this was an example which i wanted to discuss to show you that it is possible in problems to have both kinds of current both conduction and displacement current densities

so let me take an example here

so let me take a capacitor with r is equal to one centimeter carrying a current of one amperes at any given time there is a current of one ampere flowing through the capacitor plates to the to the capacitor

so for r less than r let me calculate

so r is equal to let me take point five centimeter ah magnetic field is given by $\mu_0 i$ by $2\pi r^2$ into i

so this is equal to $4\pi \times 10^{-7}$ into small r is point five ten to minus two meters into current is one ampere divided by 2π into ten to the minus four r^2 and that it comes out to be ten to the minus five tesla

so there is about this is ten micro tesla micro is 10^{-6} thats 10^{-5} micro tesla that is the magnetic field at a distance of 0.5

centimeters from the axis of the capacitor plates

so please see that although i am only passing a current and generating an electric field between the capacitor plates the changing electric field is creating a change in electric flux and that changing electric flux creates a magnetic field and that magnetic field happens to be about 9×10^{-6} micro tesla here now if i want to calculate for a point outside the capacitor plates

so let me take for example r is equal to five centimeters the magnetic field b is equal to

so that i must use the other formula now $\mu_0 i$ by $2\pi r$

so thats the formula which i must use now

so thats $\mu_0 i$ by $2\pi r$

so this is equal to $4\pi \times 10^{-7}$ into 1 ampere divided by 2π into 5 into 10^{-2} that comes out to be ah four micro tesla ok

so thats thats the magnetic field address in the five centimeters on the capacitor plates you can also calculate at a distance of five centimeter from the wire that is charging the capacitor and you will get the same magnetic field outside the wire at a distance of 5 centimeters from the wire

so this example tells me that i can actually use this to calculate the magnetic field between the capacitor plates now please remember i could calculate the magnetic field simply because of symmetry this equation is always valid this equation modified form of ampere's law is always valid in situations where there is symmetry i could actually i can actually calculate the left hand side and take magnetic field outside the integral and obtain the magnetic field value but if there is no symmetry i would have to do an integration over an appropriate path to actually calculate the magnetic field

so please remember that this equation is always valid it is very useful in situations where there is symmetry in the problem and i can calculate the magnetic field

so let me leave a problem to you to work out a parallel plate capacitor air-filled is getting charged and the current at a specific time is 0.45 amperes if radius of the plates r is equal to five centimeters calculate the total displacement current between the capacitor plates we calculate the displacement current density and see calculate the magnetic field b at r is equal to 2.

5 centimeter and r is equal to 10

so please try out this problem a parallel plate capacitor which is air-filled is getting charged and at any given instant of time the current is about point four five amperes and the radius of the capacitor plate is given

so please calculate the displacement total displacement current passing through the plates the displacement current density and we calculate the magnetic field at a distance two point five centimeters from the axis and at a distance of ten centimeter from the axis now let's recall we have now discussed almost all the basic requirements in electromagnetism now before we move on I just want to recall the Faraday's law of induction and the Ampere's law

so in Faraday's law we obtain this equation $\int \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$ the rate of change of magnetic flux this is equal to minus $\frac{d}{dt}$ of $\int \mathbf{v} \cdot d\mathbf{a}$ a time rate of change of magnetic flux leads to an electric field modified Ampere's law

so let me look at a situation where there is no conduction current there is a region of space where there is electric and magnetic fields

so when there is a magnetic field in a region then the rate of change of magnetic field leaves gives you an electric field and with i am looking at a region with conduction current is equal to zero I will get $\int \mathbf{B} \cdot d\mathbf{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$ which is equal to $\mu_0 \epsilon_0 \frac{d}{dt}$ of $\int \mathbf{E} \cdot d\mathbf{a}$ a rate of change of magnetic flux leads to electric field rate of change of electric flux leads to magnetic field

so you see Maxwell's addition of this term into this equation has coupled the electric and magnetic fields if you have a magnetic field and in a region of space which is changing with time it will lead you to an electric field which may be varying with time and if the electric field varies with time then it will lead to a magnetic field

so this magnetic field couples to the other earlier magnetic field and we get a set of coupled equations

so electric field time varying electric field generating magnetic fields time area magnetic field generating electric field and

so the electric and magnetic field get coupled through these two equations

so the addition of this term was extremely important and what has happened now is it has become symmetric there is a bit of symmetry in these equations now because changing magnetic fields produce electric fields changing electric fields produce magnetic fields and this symmetry is beautiful in these equations and as we will see that this presence of this term leads to a very very important prediction from here which is the existence of electromagnetic waves

so Maxwell when he obtained when we put in these equations found that these equations which we which I will write down little later show the existence of new types of waves called electromagnetic waves which are nothing but waves of electric and magnetic fields now before we do that let me try to draw a figure representing these two equations

so if I take a region of space for example here the magnetic field is say pointing downward uniform and magnetically pointing downward then if I take a loop of loop like this suppose the magnetic field is increasing with time

so the magnetic flux is increasing with time in this direction

so what will be according to Lenz's law there is an electric field which is

induced which will be like this the current will be current like this
 so that it opposes
 so this is the direction
 so this is these are the magnetic field lines this is b field and this is the e field
 so if the magnetic flux is increasing with time pointing downward and increasing with time because of the minus sign here because of the negative sign here the this the induced electric field will be in this direction to oppose the change in the magnetic field if i take a corresponding problem and if i had electric field pointing downward and the electric field
 so this is the electric field and the electric field was changing with time and if i take another loop like this the direction of induced electric field will be like this
 so that's the magnetic field sorry this is the magnetic field
 so magnetic field pointing downward increasing with time leads to an increasing magnetic flux in this loop and because the magnetic field field is pointing downward then use electric field will be anti-clockwise here if there is an electric field pointing downward and electric field is increasing with time then the induced magnetic field will be clockwise
 so there is a small difference in these two and that difference comes in primarily because of the presence of this negative sign in this equation there is no negative sign in this equation of course there are extra terms sitting here but there is no negative sign here and there is a negative sign here and that leads to two different situations here of oppositely directed electric field generated by magnetic field changing magnetic field and the corresponding magnetic field generated by electric fields now i want to take an example i want to show an example of a comparison between conduction current and displacement current now in an earlier class you must have studied about conduction through wires and you have studied about rc circuits and
 so so on
 so we define remember at that time we had defined a conduction current density j_c is equal to σ times e the conduction magnitude of the correct conduction current density is given by σe σ is called the conductivity
 so σ defines the conductivity of the medium and the conduction current density is proportional to the electric field and that's the σ is conduction current density we had obtained in the last in this lecture a displacement current density j_d of $\epsilon_0 \frac{d e}{d t}$
 so that is free space now without going into discussion i would like to mention here that if there is a medium then the displacement current density becomes $\epsilon \frac{d e}{d t}$ i replace the permittivity of free space ϵ_0 by permittivity medium which is ϵ and ϵ is nothing but ϵ_0 into dielectric constant remember this is the ah $\epsilon = \epsilon_0 k$ ϵ is equal to ϵ_0 into dielectric constant k
 so if there is a medium the displacement current density in the medium is given by j_d is equal to $\epsilon \frac{d e}{d t}$ the conduction current density is given by σ times e
 so i could have media in which there is partly they are partly conducting they are not perfect conductors they are they are conducting and they also have a displacement current
 so i could have situations where the medium carries both displacement current and conduction current
 so let me look at an example
 so as an example first
 so if i look at the ratio of these two i want to look at the ratio of these two

so let me take an electric field which varies as say $e_0 \cos \omega t$
so i have an electric field which is oscillating with time at frequency ω
so the conduction current density will be $\sigma e_0 \cos \omega t$ which is equal to $\sigma e_0 \cos \omega t$
the displacement current density is equal to $\epsilon_0 \frac{d e_0 \cos \omega t}{dt}$
which is equal to $-\epsilon_0 \omega e_0 \sin \omega t$

so i differentiate this with respect to time i get $-\epsilon_0 \omega e_0 \sin \omega t$

so that is the displacement current density the conduction current density the first thing that you notice is the conduction current density and the displacement current density are not in phase there is a minus sign here and this is cosine of a cosine function of time this is a sine function of time

so if i plot for example as a function of time

so let me plot for example first the conduction current density

so the conduction current is $\cos \omega t$

so one cycle if i plot that's the conduction current

so displacement current is minus this thing

so let me these are the values here

so what this will be this will be going like this this is the displacement current this is cosine cosine function of time this is minus sine function of time

so you can see here that there is a phase difference between the conduction current and the displacement current and this becomes an important consideration in some advanced courses that you will be studying a little later in your carrier

so this is the displacement current density and that is the conduction current density

so i can actually calculate what is the maximum value of conduction current density and then compare it with the maximum value of displacement current density

so the maximum value of current conduction current density $j_c \max$ is equal to σe_0 and $j_d \max$ is equal to $\epsilon_0 \omega e_0$ maximum value of conduction current density will appear when $\cos \omega t$ is one that is σe_0 and maximum value of displacement current density will happen when $\sin \omega t$ is minus one and that is $\epsilon_0 \omega e_0$

so the ratio of uh this conduction current to this or displacement current conduction current is equal to the maximum value $\epsilon_0 \omega e_0$ by σe_0 which is equal to $\epsilon_0 \omega$ by σ

so thats the ratio of the displacement current to conduction current and ω is actually in terms of frequency i can write this $2\pi \nu \epsilon_0$ by σ where ω is equal to $2\pi \nu$ ω is the angular frequency ν is the frequency and ω is the angular frequency

so let me take two examples one i take a good conductor

so in a good conductor the conductivity is approximately 10^7 mos per meter it's a large conductivity

so that's what's called a conductor it's a very large value and if i take a frequency of say one gigahertz remember we had introduced this 10^9 per meter the power nine which is called a giga gigahertz then and for conduct good conductors ϵ_0 is approximately equal to 10^{-12} and i can calculate j_d by j_c which is equal to $2\pi \cdot 10^9$ into ϵ_0 which is eight point eight five ten to the minus twelve divided by σ which is 10^7 to the power 7 and that comes out to be 5.6 into 10^6 to the power minus 9.

so you can see here that for a good conductor the majority of current is

conduction current the displacement current is negligible in compared to the conduction current density

so the current that is flowing through a conductor is primarily conduction current and there is hardly any displacement current and that is why it is called a good conductor it is a conductor because much of the current that is flowing through this medium is because of conduction current and not displacement current let me take a power conductor such as seawater

so sea water has epsilon is equal to eighty one times epsilon zero and sigma is approximately four mohs per meter and

so j_d by j_c is equal to

so this is 2π into frequency ten per nine hertz into epsilon which is eighty one times eight point eight five ten to the minus twelve divided by sigma which is four and that is about one point one

so at the frequency also the frequency which i am taking is ten point nine hertz

so at this frequency sea water when you propagate this frequency of a wave through a sea water there is almost equal contribution of conduction current and displacement current passing through sea water please note that this ratio depends on frequency

so at higher and higher frequencies this term can start to increase and a lower and lower frequency this term will start to decrease

so depending on this ratio of displacement to conduction current you can have different situations

so if you have a situation where sigma is much much greater than $\omega \epsilon$ when sigma is much much greater than $\omega \epsilon$ then conduction current is much greater than displacement current then this behaves as a conductor and if sigma is much less than $\omega \epsilon$ then this behaves as a dielectric

so depending on the frequency and the properties of the medium in terms of conductivity in epsilon a medium could behave as a conductor where the conduction current is much much bigger than displacement current or behave like a dielectric in which the conduction current is negligible compared to displacement current

so i can have two both these limits and it depends on frequency

so i leave it to you to look at the same problem please calculate this ratio at frequency say 1 megahertz which is 10^6 hertz and say 100 gigahertz which is much higher frequency

so you will see the difference in this ratio because this ratio is approximately 1 and on 1 gigahertz

so as you will see for higher lower and higher frequency the same medium can behave either as a conductor or the dielectric

so this is a very very important consideration of these two

so let me just write down before we close the four equation that we have obtained till now which are maxwell's equations $\oint \mathbf{E} \cdot d\mathbf{a} = \frac{q_{enc}}{\epsilon_0}$ $\oint \mathbf{B} \cdot d\mathbf{a} = \mu_0 (I_{enc} + \epsilon_0 \frac{d}{dt} \int \mathbf{E} \cdot d\mathbf{a})$ $\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{a}$ $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I_{c,enc} + \epsilon_0 \frac{d}{dt} \int \mathbf{E} \cdot d\mathbf{a})$ the four very very important maxwell's equations i will stop my lecture here and what we will do in the next class is to look at these equations and i will show you that these equations predict the existence of what are called as electromagnetic waves and that was a very very important discovery and a very important contribution of james clark maxwell when he showed that these equations predict the existence of electromagnetic waves and light is a form of electromagnetic wave and

so these are called maxwell's equations

so i will stop my lecture here and we will continue with the discussion in the next lecture thank you you

Prutor@IITK