

we have been discussing kirchhoff's laws i will quickly again ah recall for you that kirchhoff's laws consists of two laws one is called the junction rule where we said that at any junction which is defined by having at least three conductors joining at that point i must have the algebraic sum of all the currents arriving there equal to zero by algebraic sum i mean i take for instance those which are arriving to be positive those which are leaving to be negative or vice versa and a similar rule which says voltage rule that is if you go around any closed loop then your net drop of the voltage is equal to zero that you come back return back to the same point the point that i told you is that in if you move in the direction of the current and pass through a resistance then the potential will drop that is decrease and the other thing in the circuit which is a battery there when you travel from negative to the positive terminal then the potential will increase these are the two things that you have to remember and based on that i have these kirchhoff's laws last time we considered the problem of a cubical network of 12 equal resistances and obtained equivalent resistance between two diagonal coordinates now let us repeat the same problem but with a slightly different pair of points uh about which or i want the equivalent resistance between a different pair of parts now you will see immediately how the symmetry changes

so let me look at it this way it does not look like a cube but

so suppose i this time wanted that find the equivalent resistance between a point let us say a let me call this give them the name a b c d e f

so supposing this is what i wanted between ends of a two diagonal on the base now look at this problem the symmetry of the problem is not the same as before

so looking at this i can i can see the following i can immediately realize that the all the four base points are identical because with respect to a and e these base points are symmetrically placed

so therefore this path and that path are identical

so let us call the current v_i but no longer that is true of this path because this path which is here is not symmetric with respect to a and e is position but however something else is there

so what we said is that because of this symmetry i must have the following that the current in each one of these four paths will be identical

so i will write this way that way magnitudes will be the same now another point i can make is this i don't know how much is the current here will work out

so let's just call it something at the moment now but another point i can make is this that this top face each one of the sides is also identical with respect to these two parts

so therefore these currents their magnitudes will also be the same supposing i double prime comes out on this also there should be an i double prime this is must be another i double prime this must be another i double prime and notice something interesting what it tells me is this that there are two parts in this where there are no currents

so this conductor and this conductor no currents in this in other words if even if you removed them they will not participate in the circuit and

so you notice the major difference okay

so let us look at what is the situation here this is all i haven't still said what is this current as yet

so let us look at one of the things you immediately notice is this by junction rule this current which is coming into the point b must be equal to the sum of these two currents which are moving out

so therefore this must be equal to two i double prime earlier you remember in the previous case i had i i and i but this is not true now if i now take a path a b

so let me write down a b c d this path here here here then e f and a something
i look at look at that path this this this this this and that that's a closed
path and i apply the voltage rule there

so what am i getting

so notice this i have a $2i$ there times of course whatever is the resistance r
let me take it as 1

so that i do not have to repeat that r all the time then plus i double prime
plus another i double prime now this one because of the junction rule again must
be $2i$ double prime

so plus $2i$ double prime minus this time because i am traveling opposite this
is i

so minus i minus another i must be equal to zero actually the entire thing
there is a common factor r that has been removed because r is the resistance

so you notice immediately that what i am getting here is this $6i$ double prime
equal to $2i$ in other words my i double prime is equal to i by 3

so if i double prime is i by 3 let us look at what is the current that is being
supplied by the battery now notice my battery is supplying $2i$ plus $2i$ double
prime

so $2i$ plus $2i$ double prime

so let us call that i in that's $2i$ plus $2i$ double prime and that is equal to $2i$
plus 2 by $3i$ because i double prime is i by 3 which is equal to $8i$ by 3.

so therefore since v is the battery voltage

so i can write this as v equal to $8i$ by 3 times whatever is the equivalent
resistance now notice i can get an alternative expression by looking at for
example what is the potential drop between a and e that's what i am interested
in

so i have an ir there and an ir

so therefore this is also equivalent to $2ir$

so if you compare these two expressions you find r equivalent equal to three by
four r

so let me look at now some problems where the symmetry is not obvious or there
is a lack of symmetry

so let us look at another problem there is no resistance there okay let me take
these numbers 4 ohms this is a 10 volt let's suppose this is 1 ohm this is 4
ohms this is 2 ohms this is 2 ohms and that's a 5 volt battery this circuit
cannot be regarded as either a serial or a parallel combination of resistances
now let us first use the junction rule to find out what are various currents

so i will do the following since this is the bigger box bigger battery as i
have told you several times you do not have to but let me start by saying i_1
this is supply and let me say that on this branch i_2 goes out now clearly at
this branch i have i_1 minus i_2

so this i_2 goes there and let us say that this battery suppose is giving out a
current i_3

so that the current which is flowing through this is i_2 plus i_3 everywhere i am
using simply junction rule

so notice what happened now at this junction i_2 plus i_3 comes in and another i
goes out i_1 goes out

so therefore at this junction it is i_2 plus i_3 minus i_1 and that is going out
algebraic sum is still 0 and clearly this current is also i_3 i have got all the
currents in there now since i have got 3 unknowns there i_1 i_2 and i_3 i

require 3 loop equations remember i have already exhausted my junction equations
so let me look at the first the right hand side loop

so right loop

so you have to simply go like this

so minus because i am going in the direction of current i_2 plus i_3 into 2 that is the resistance minus again i_2 plus i_3 minus i_1 into 2 and i am climbing up the potential hill

so therefore plus 5 is equal to 0.

let us combine them if you look at them this becomes $2i_1 - 4i_2 - 4i_3$ is equal to 5.

so that's your first equation let us look at this upper left loop

so i_2 into 4 well minus because it is drop minus i_2 plus i_3 into 2 then minus i_1 into 1 because that's i_1 plus 10 equal to 0 simplifying them you get $i_1 + 6i_2 + 2i_3$ equal to 10.

and finally the lower left if you do lower left this is again minus i_1 minus i_2 into 4 this time i am going against the direction of current

so therefore plus i_2 plus i_3 minus i_1 into 2 minus i_1 into 1 and that is equal to actually plus 10 equal to 0

so minus 10 and you simplify them you get $7i_1 - 6i_2 - 2i_3$ equal to 10.

so these three equations provide you the necessary requirement for solving uniquely for the three variables i_1 , i_2 and i_3 i am not going to show you the algebra but you can trivially show that i_1 is 5 by 2 amperes i_2 is 5 by 8 amperes and i_3 is 15 by 8 amperes let me consider another circuit where we can use symmetry to our advantage

so let us look at this circuit and we are supposed to find out what is the equivalent potential across the point a and the point e

so look at this now let's assume that at the point a i have a current i_1 going on this branch and a current i_2 going on this branch now by symmetry i know that this and this side also must have i_1 and i_2 only thing is we have to decide which one should be uh having i_1 and which branch should be having i_2 now in order to do that you notice the following that this branch which is having i_1 is in series with this branch bc now

so therefore the current i_1 flows through two resistances here and if i am looking at the symmetry on this side then it is this at the point e this red resistance and these two resistances are connected

so therefore this must be i_1 because here two series resistances are there and therefore this one must be i_2 now let's look at the junction rule at the point c

so here i have a current i_3 which is equal to $i_1 - i_2$

so basically you notice that by use of symmetry i have reduced the number of unknowns to only 2 i_1 and i_2

so let us look at what we can say about i_1 and i_2 first let us look at this loop bc d a b

so if i do the loop rule here i get minus $i_1 r$ minus $i_1 r$

so this is minus $2i_1 r$ then minus again $i_1 - i_2$ into r and this time plus i_2 into r because i am traveling in the direction opposite to the current and since there is no source in that branch that is equal to 0

so if you simply simplify it you will find i_2 is equal to $3i_1$.

now let us look at the other loop namely say remember i had only two unknowns there

so therefore i need the second loop with the v in it

so if you look at this loop here that i have a f

so let me design that specify that look a d f e x y a

so i've got minus i_2 into r then i have got $i_1 r$ and another $i_1 r$

so $2i_1$ into r plus v equal to 0

so that is equal to v

so therefore i have $i_2 r$ plus $2i_1 r$ equal to v but i already know what is i_2

so if you look at that then i get i_1 is equal to $\frac{2}{7} \frac{v}{r}$

so therefore i_1 plus i_2 is already calculated there

so that will work out to $\frac{5}{7} \frac{v}{r}$ because i_2 is $\frac{3}{7} \frac{v}{r}$ and into $\frac{2}{7} \frac{v}{r}$ and suppose the equivalent resistance of this circuit is r_{eq} then this is also equal to v divided by r_{eq} because i_1 plus i_2 is the current that is supplied to the this circuit

so that tells me immediately that r_{eq} should be equal to $\frac{7}{5} r$ as another example let me look at this circuit and once again i will be using the symmetry of the problem to my advantage

so let us suppose this is i_1 and this is i_2 now one thing i noticed is these two branches with respect to the point b is symmetrical as these two branches are with respect to the point a

so therefore one of these branches must supply i_1 and the other branch must supply i_2 that is one of these branches must take in i_1 and another i_2 the only thing that we have to decide is which one carries i_1 and which one carries i_2

so you notice one thing that this is actually obvious that the resistance of here is connected to the point b and to the point a through one resistance here

so therefore this is not cement the identical to this resistance here which is connected to the point a through these two resistances or any other two resistances

so therefore this one must be i_1 and this one must be i_2

so i can use the junction rule here now let me call this one as i_3

so that this is i_2 minus i_3 and you can see here since i_1 is coming in i_1 is going out

so therefore this one must be i_3 which was coming in from this branch

so let me now try to find out the relationship between them suppose i take this central loop i already have used $abcd$ let's call it ef

so let me take $e f o e$ branch

so what do i have i have an $i_3 r$ minus $i_3 r$ another minus $i_3 r$ plus i_2 minus i_3 into r equal to 0 you simplify this that gives you i_3 equal to i_2 by 3 now let me look at for instance the left loop which is $e o a e$ now what do i have there i have minus $i_2 r$ minus $i_3 r$ plus $i_1 r$ equal to 0

so i already have a relationship between i_2 and i_3 i_3 is i_2 divided by 3 put that back in into this equation you get a relationship between i_1 and i_2 and i_2 turns out to be $\frac{3}{4} i_1$

so therefore the current supplied by the battery at the point a or flowing into the battery at the point b which is equal to i_1 plus i_2

so i_1 plus i_2 is equal to $\frac{7}{4} i_1$ but look at what statement i can make regarding how much is i_1

so notice if you look at this square loop then what i have is $i_1 r$ or rather minus $i_1 r$ minus $i_1 r$ which is minus $2i_1 r$ plus v equal to 0

so which tells me that $2i_1 r$ equal to v

so that i_1 is equal to $\frac{v}{2r}$

so this if you put it in i_1 plus i_2 you get $\frac{7}{8} \frac{v}{r}$ which must be identical to $\frac{v}{r}$ equivalent

so that gives me r_{eq} to be equal to $\frac{8}{7} r$

so in the last few lectures including this one we have talked about direct current circuits including kirchoff's law which is very useful in solving currents given voltages i will end this series of lectures on direct current

circuits or current electricity with couple of laboratory applications the first one is known as a wheat stones bridge this is a circuit which is used to measure resistances how do i measure resistance of an unknown sample in the laboratory and in fact this usually allows you to measure resistances in the range one ohm to one mega ohm with a one percent accuracy as a rather simple circuit and the circuit works like this you have a quadrilateral of resistance circuit

so let us assume these two r_1 and r_2 are given

so r_1 and r_2 are known resistances now r_3 is a resistance which can be varied there are sliders ah variable resistances you can find

so r_3 can be varied known of course and let me say r_x this is the unknown resistance to be measured now the arrangement is like this the circuit is made such that there is a ammeter or a galvanometer in the circuit and across these points which i will call as a and b i connect a battery

so let us look at what happens

so there is a current which is coming in here the current goes out like that and that

so let us call this i_1 let us call this i_2 as usual we have been doing then this current would normally go through this and that it will divide here and the circuit will be complete now what we do is this we adjust the resistance r_3

so let me because it is an important point let me talk about adjust r_3 the variable resistance till such time that there is no current passing through the emitter this is generally known as a null deflection that is galvanometer or the ammeter doesn't show any deflection

so that's null deflection now you see what happens is this when there is null deflection this current i_1 which is coming from here sorry this is this i_1 equal to $i_1 + i_2$ that will all pass through this one and likewise this i_2 which is coming in here will go through this

so if you now look at the this circuit

so notice $i_1 r_1 = i_2 r_2$

so $i_1 r_1 = i_2 r_2 + i_3 r_3$ minus $i_2 r_x$

so therefore $i_1 r_3$ is equal to $i_2 r_x$ when null deflection takes place

so that this section doesn't contribute

so if you look at the resistances there then you will immediately find out by dividing one equation with the other that $r_1 / r_3 = r_2 / r_x$ now

so therefore when values of r_1 r_2 are known and r_3 we determine experimentally by varying the resistances till i get a null deflection i can calculate r_x by the formula that r_x is equal to $r_2 / r_1 \times r_3$

so this is the principle of wheat stones bridge in which you have a bridge in which through which this connecting the bridge it doesn't give you any deflection suppose i have r_1 equal to 6 ohms r_2 equal to 1.

5 ohms and let's suppose my null deflection has been achieved by having r_3 equal to 8 ohms then of course r_x will turn out to be $r_2 / r_1 \times r_3$ which is equal to 2 ohms is trivial but you this suppose my r_x is slightly different supposing r_x is let's say 2.

01 you can use the same circuit but you will now find that there will be a current in the through the emitter it will be small because my deviation here is small now an interesting application of this is the following let me look at a slightly different circuit let me give some name suppose each one of them is r the problem is find the equivalent resistance between a and b once again the circuit is neither a series combination nor a parallel combination and it's very difficult to do it by that method

so remember again what we said we said that in which case you should imagine between a and b you have a battery i could do that but let me let me make each one of them the same and let's suppose this i call it as an r prime now

something interesting actually happens that if you look at it carefully you will find that this circuit is nothing but a wheat stones bridge and in order to do that you have to recollect back how the connections are made

so let me give it some numbers let us call this c let us call this d

so this point is b this point is a because this is the same point

so look at what are the various connections my point a is connected to c through a resistance r and to d through a resistance the point b is connected to c through a resistance r again it is connected to d through a resistance r and cd are connected by a resistance r prime now compare this compare this with the circuit that i had this was my a

so this is c this is d

so notice a is connected to c and 2 d b is connected to c and 2 d and c d is connected through the ammeter and through any resistance that may be there

so with these ideas let me redraw that circuit again

so look at what will happen if i drew the circuit again this is the way our connections are in that circuit now but if you look at it since this by this is this by that this is the balanced fifth stone switch which implies that no current is flowing through the branch cd no current through cd

so therefore what i can now do is this since there is no current in cd and i am required to find out the effective resistance between the points a and b i can now redraw that circuit as follows that i have these points a and b and if this is basically absent what i have is a series resistance r plus r namely two r between a and b and another pair which also works out to 2r from the lower branch

so this is 2r this is 2r which tells me that the equivalent resistance is simply r

so this problem is interesting because on the first look it doesn't look like a white stone's bridge but on the other hand once you have realized what is connected to hot it can be shown to be equivalent to a white stone's bridge one of the applications of wheat stones bridge is what is known as a meter bridge

so this is the diagram of a meter bridge

so basically the reason why it is called a meter is the following that this has a wire of length one meter uniform cross section which is stretched and is fixed to the points a and b now these connectors they are low resistance connectors there are two gaps in these low resistance connectors across one there is a wire of resistance r and across the other there is a known resistance s and the two ends a and b are also connected to a battery with a key now from the

so this is basically a white stones bridge and what one does is that one point of the one end of the galvanometer is connected to the middle point here and the other one can slide on a b and one slides it till you receive or till you get a null deflection

so no current through galvanometer now notice one thing if rho is the resistivity of the material of the wire a b and suppose i have achieved a null deflection when this length is l let's let's take l centimeters

so that this one is 100 minus l centimeters

so according to the principles of principle of white white stones bridge that we did when there is null deflection i get r by s must be balanced by resistance of the section a d divided by the resistance of the section db since resistance of the section a b a d is the resistance of the wire of length l centimeter we write this as rho l 1 by rho l by a where the unit of rho is in ohm centimeter and l1 is in centimeter or l is in centimeter and this divided by rho times 100 minus l divided by a which is nothing but l by 100 minus l

so by finding out at what length the null deflection is achieved using a meter bridge an unknown resistance r can be determined yet another application of the

principle of wheatstone's bridge is in what is known as a potentiometer which i have shown here in the diagram

so potentiometer has basically two utilization in the laboratories and the first one is to compare emf of two or more cells now the arrangement is something like this

so there is this main circuit with a source of voltage v and there is a variable resistance here which uh the is varied

so that the deflection of the galvanometer at the two ends comes within the range of the galvanometer there is a switch which can be closed and thereby enabling the source v here to send a constant current through the circuit now what happens is this that this part a b these are all resistance less wires

so between the point a b is a long uniform wire uh which usually has several turns as is shown here but for convenience i have simply put it stretched between the points a and b of the circuit now the point a is connected to the two sources of emfs which we want to compare the emf source number one is connected to a three-way switch to let us say point number one and similarly the source two is connected to the second point of the three-way key and there is a common point three which is connected to a galvanometer and to a wire which can slide over a b the basic principle is the following that we let us say we connect one two three meaning thereby that the emf source e_1 is in the circuit but e_2 is not this switch s is kept closed now what we do is this we slide this wire which is connected to the other end of the galvanometer such that a null deflection is obtained

so when the null deflection is obtained we immediately see that in this situation a p e 1 r 1 which is the internal resistance the point 1 the point 3 g which is the galvanometer and let us say the null deflection is obtained at the point n 1 n 1 this section has no current because there is no deflection in the galvanometer however because the v sends a constant current through the section ba there is a potential drop across the wire a b

so there is a potential drop in the wire a b and since the null deflection is obtained at the point n 1 it tells me that the potential drop in the section a n1 is balanced against the emf that is supplied by the battery now note one thing that because there is no current in this part of the circuit the internal resistance does not play a role because internal resistance would have provided a further potential drop if there were a current in this part of the circuit

so therefore the potential drop across a n1 is the potential drop across p e 1.

now suppose the length for which we obtain the null deflection is given by l_1 then what we find is this i times the resistance of the section a n1 is equal to e_1 but how much is this this is equal to ρ the length l_1 divided by a and that's equal to u_1 .

now that tells me that the i could repeat the same thing for the sec the when the emf source e_2 is connected namely instead of 1 3 being connected if i connect it to 3 now in that case suppose the null deflection is obtained at a point l_2 which is a length l_2 there then what i have is a very very similar expression which i will write here that i times $r_{a n_2}$ which is equal to ρl_2 by a and that must be equal to e_2 the reason is again the same that because there is no current in that section of the circuit the internal resistance does not play any role

so therefore the ratio of e_1 to e_2 is simply equal to l_1 by l_2 and this is just the ratio of the lengths of the wire for which the null deflection is obtained yet another application of the potentiometer in the laboratory is to determine the internal resistance of a battery the circuit is more or less similar to what we showed in the previous case and what we have here is this that as before i

have a main circuit consisting of the potentiometer wire which as I have said earlier actually consists of several loops of wire but I have just shown it as stretched between the points a and b. V is an external source which supplies me a constant current through the wire a b and this is a key k_1 which will remain closed. There is a variable resistance r_v as before which is adjusted

so that the galvanometer reading comes between the section a and b. Now there is a modification in this part of the circuit. This is the emf whose internal resistance we are interested in determining.

So what happens is this. In this part of the circuit there is another resistance, usually a resistance box and a key k_2 which is adjusted which can be either closed or open.

So in the first part of the operation we have the key k_1 closed of course. k_1 will remain closed for even the second section and k_2 is kept open. Now when k_2 is open, this part of the circuit c d r k_2 etc. that part of the circuit doesn't participate. Repeat when k_2 is open, there is no current in this part of the circuit and if we adjust the sliding wires position

so that null deflection is obtained, let us say it is obtained at a distance l_1 from a.

So this distance is l_1 now because there is no current in this part of the circuit, the potential drop because of the emf is balanced by the potential drop for the section a times n_1 .

So suppose in this situation null deflection is obtained at n_1 . Then what happens is this. This potential drop across a n_1 balances the emf e . Now as I have said in the other part also that since there is no current through this section cd, let us say the internal resistance does not play any role in this process. Now let us look at how much is the drop. Now notice that this battery is providing a constant current i .

So current through the wire a b is given by $i = \frac{V}{r_v + r'}$ where r' is the resistance of the entire length a b of the wire of length l . Because the wire is of uniform cross section, there is a constant potential gradient along the wire and that is easily calculated by finding out since I know the current.

So the resistance is r' for the entire wire, the potential gradient which I will represent by ϕ is this current i which is $\frac{V}{r_v + r'}$ multiplied by r' of course. This is essentially the potential drop per unit length. Now since the balance has been obtained at a length l_1 and it has been balanced against the emf e , I have for the first part of the operation when the null deflection is obtained $e = \phi l_1$. Let us emphasize by saying for null deflection now in the second part of the circuit what I will have is I will also close k_2 . Now let's see what happens when k_2 is closed. Now when k_2 is closed, this source of emf e sends a current through this part of the circuit. Now at that stage what happens is this that I again adjust the length

so that the null deflection is obtained at a point l_2 . Let's say that the length that length is l_2 but this time when I receive a null deflection because there is a current through this part of the circuit, it means that that current which we will just now write down, how much will also give a drop across the internal resistance r and let's see how it works.

So with k_2 closed, the cell with emf e sends a current and how much is that current? Let's call it i' and that is obviously equal to $\frac{e}{r + r'}$ where this r is the resistance which is in this part of the circuit.

So as a result what happens is the potential drop across that cd which was earlier e when k_2 was open, now is reduced by an amount $i \times r$ that smaller.

so therefore the potential drop across cd is $e - i' r$ and that must be equal to the gradient ϕ multiplied by l_2 the distance at which i obtained a null reflection and since this is equal to $e - r$ divided by $r + r$

so that is equal to ϕl_2 the reason is that i have this amount $e - i' r$ is the potential drop across these two points

so therefore whatever is the current multiplied by this resistance r gives me the potential drop across this two or that is the potential drop across cd

so that's my ϕl_2 but we had seen that e was equal to ϕl_1 now you simply substitute for either and equate it to ϕl_2 and that enables you to determine r in terms of l_1 l_2 and the register with this we come to the end of our series of lectures on direct current circuits you