

hello good morning to all of you towards the end of the last lecture i had started talking about what are known as kirchoff's laws basically before we did that what we realized is there are several combinations of resistances which can be reduced to series or parallel combination just the way we did it for the capacitors but of course there is a slight difference that we pointed out that the formula for the series resistances go the way the formula for the parallel capacitances go and vice versa however it is very unusual to have situations where circuits are simple enough to be reduced into equivalent parallel or series combinations and in general we have a set of two laws known as the kirchhoff's law which can be used to find out the currents in such complicated circuits

so there are basically a set of two laws the first what we did is to define what is meant by a junction

so what we said is a junction is a point where three or more conductors meet

so there is a rule regarding the junction law

so first law as we talked about last time is called a junction rule

so johnson rule simply says that if you assign a sign to the current which is coming in you can do it anyway supposing the current which is coming in is taken to be positive and the current which is leaving that junction is taken as negative then the algebraic sum of currents at a junction

so which is what i write like this $\sum_i i = 0$ this is algebraic sum of current at a junction let me also state the second law and then will do a slight discussion of both and this is what is known as the voltage law

so this you might call it a current law

so voltage rule

so voltage rule basically says that the algebraic sum of voltage difference around any closed loop

so $\sum_i v_i = 0$ around any closed loop

so these are basically the two things which we are concerned with and i will in this lecture discuss the applications of these laws by giving you several examples

so let me ah talk about junction ruling

so basically the origin of junction rule lies in the fact that electric charges do not accumulate there is a continuity of electric charges

so whatever is coming into a junction have to go out and that's that's what we meant by saying that the algebraic sum of currents because as you know current is nothing but rate of change of charge and

so therefore the charge cannot accumulate at a junction that's the simple reason why a junk cell rule is valid

so let me illustrate this i am not giving any circuit but let me say that i have a junction of this type let me just draw some points here then i will

so what i have done is this supposing this is i_1 let me say this is 4 amperes this is 3 amperes this is minus 2 amperes

so something's like this 4 amperes let's call this i_2 which i don't know this is 2 amperes this is i_3 let us say and this is 2 amperes

so look at what i have done i have several junctions in this circuit

so here is a junction this junction this is a junction this is a junction any point which has three or more resistances or conductors going out that is what it is

so let us look at how i can use junction rule for this

so i will assume that a current which is coming in is positive let us say there is nothing special about this assumption if you wanted you could have assumed that the current which is going out of a junction is positive in which case that commit current that is coming into a junction will be negative

so let us look at this first junction
so i have i_1 coming in
so that's positive i have 4 amperes coming in again
so that's positive and there are two of them going out
so i put a minus sign there i've got minus 3 and you have minus 2 because there is a 3 ampere going out here 2 ampere going out
so if you look at this
so this quantity is equal to 0 okay i have written i_1 here with a minus 2 what it actually means is that the current in this branch is actually going in but that's easily remedied by writing this as minus of minus 2
so let me take it as plus 2.

so let me make a note here that this is shown as going out but a negative current going out why am i doing this see very often in a circuit you have no previous knowledge of the direction in which the current is going
so you assume some direction and if the result turns out to be negative then you know your original assumption was wrong and the current actually flows in in the direction opposite to what you have assumed
so there is no problem i have minus 2 going out which also means plus 2 the direction is actually opposite
so that is what i have taken care of by writing this as minus of minus 2
so look at this this tells me that i_1 is equal to this is 4 plus 2 is 6
so it is minus 3 appears
so what am i trying to say here what we are saying is that what you have done is probably wrong it's not wrong but i_1 is turning out to be negative in other words my actual direction of current here should have been like this
so you look at how this actually works this is minus 3 which means 3 amperes going out this branch has a 3 amperes going out this branch has a 4 amperes coming in this branch has a minus 2 going out means plus 2 coming in
so 6 going out and 6 coming in which is what we expect and and like this you can also for instance look at what is happening to this second branch a second junction

so in let's talk about junction b let's say
so this was junction a now what happens in junction b is this that in junction b i have said i_2 coming in

so i have got i_2 is positive there is a 1 ampere coming in there is a 3 amperes coming in and there is a 2 amperes going out that's equal to 0

so i_2 plus 2 is equal to 0

so i_2 is equal to minus 2 amperes

so once again the minus sign simply indicates that the direction as we have assumed in our example should have been actually opposite but that does not matter because i have got the right sign there

so let me look look at now the voltage actually junction rule is very easy to implement it is a voltage rule which you have to be a little careful nothing

so special but you have to be a little careful basically the origin of voltage rule comes from the fact that in a static field my integral for a static field this we have repeatedly talked about that closed integral of $\mathbf{e} \cdot d\mathbf{l}$ is equal to 0.

so the net emf if you recall integral $\mathbf{e} \cdot d\mathbf{l}$ was defined as my emf

so the net emf in a closed loop must be equal to 0 now in doing

so i need some working method and this working method is the following that once again these are simple convention by which you could do your problem you could decide that you want the opposite convention nothing would go wrong

so let me look at this supposing i have a current which is flowing through a resistance and let's suppose this resistance there and suppose the current is entering in here now the part the end of the resistance where the current enters in remember current is the direction in which the positive charges move

so therefore this is at a higher potential and at this point where the current is actually moving out that's at a lower position

so therefore if you are moving in the direction of current then the potential actually drops as you move

so moving in the direction of current change in the voltage is neg it drops

so Δv is negative and how much is this this is simply equal to ir

so drop is ir

so this is a drop now since it's a drop when you write down in an equation you would put a minus sign in front of it that is will work this out now there is another thing the circuit in addition to resistances also has seats of emf the battery or things like that now again there we know that when a positive charge moves from negative terminal to the positive terminal it gains energy

so therefore across a battery Δv is positive namely the potential rises in going from a negative terminal to positive terminal

so these are the two points you need to remember once again it doesn't matter whether you know the polarity or not now if you know the polarity then of course you know you have a priori idea of which direction the current is flowing and this will be easy to use but it is possible you do not know the polarity in which case assume any of the ends should be positive proceed using the same thing you will turn up with a negative sign at the end of your calculation in which case it will help you in fixing up what you actually want to do

so let me again without going to a specific circuit give you an example of how it works

so let me let me just draw i am not putting in any items here all that i am doing is this i am putting some blocks it simply says it could be anything it could be a resistance it could be a seat of emf and things like that all right

so what i do is this

so let me just draw this

so i will i will put some a priory signs there supposing i have this as minus this as plus and this is 8 volts this is plus this is minus let us call it some v v_1 let us say this is plus this is minus this is 8 volts this is plus this is minus this is again 8 volts the numbers i have taken to simplify my calculation and then now i am not indicated what these are now how does one find out what is v_1 and that will also tell you why i did not put anything here

so the way it works is this that i have to identify a loop and go around that loop and the net voltage difference once i come back to the point where i started that will be zero

so let me say that i start here

so when i cross there from this point to that point my voltage rises by 8 volts

so i have written plus 8 here this end is plus this end is minus

so therefore it drops minus v_1

so what i am doing is this i am not going to go around this that is because this data is not known to me but what i will do is this this loop law is valid for any closed circuit

so notice this supposing i start at this point is a b c and d now this is a closed loop i just go around in that

so if i do that i got next one again from plus to minus

so therefore this is minus eight once again from plus two minus another minus eight then i come like this minus 2 plus

so therefore it is plus 6 here again plus 10 no details of the circuit i have

written down if it is a battery then the positive the rise in the potential is when i go from its negative terminal to the positive terminal if the element there happens to be a resistance then in the direction in which i am going in this case i was i decided to go like this that is the assumed direction of the current then there is a potential drop as i go through a resistor but here i haven't necessarily assumed what type of things have that whether it's a resistance or it's a battery i have had way of handling it

so look at what this tells me this simply tells me v_1 is equal to while you add them up

so this is 16 minus 8

so that's equal to 8 volts

so let us start with a simple problem supposing this is 12 volts this is plus this is minus this is 4 volts this is plus this is minus this is a 1 hour resistance this is a 3 ohm resistance now what do i do see the thing is this this circuit doesn't have any junction rule as a simplest circuit you can think of and

so therefore you assume there is no junction

so only voltage rule is there you can decide which way to go there is a positive here there is a positive there you could have decided to go like this or like that but the it is totally immaterial how i want to go actually

so let us suppose i go like this and the reason is very simple because this is the positive end of the battery of a bigger battery than this

so therefore presumably the current will go like this and let the current be i then you notice what's happening i have started from here

so i when i go on the resistance last wire there is no drop of potential but there is a drop here because i have assumed the current to be in this direction

so there is a drop of i into 1

so therefore i will write this as minus i into 1 once again there is a drop here

so minus i into 3 this is going from positive terminal to the negative terminal one more drop

so therefore minus 4 and here i go from negative terminal to the positive terminal before i come back to this point

so therefore there is a plus 12 and that must be equal to

so this tells me that i into 4 $4i$ is equal to 8

so current i is equal to 2 amperes this is a situation where i have a single branch let me increase the branches a little bit say 2 ohm this is 12 volts this is 6 volts then this is let's take this as take since i am just illustrating something let us take this symbol okay look at this situation here i have two batteries i have three resistances once again you realize there is no way of reducing this circuit to a parallel or a series combination circuit

so what do i do now i have two loops here now let me say i want to go like this but before i do that let me first use see there are many there is a junction here there is also a junction here there are two junctions but these two junctions i will write down by assuming sum

so this is first junction

so let me assume this current that is coming out is i_1 one coming into that junction is i_1 one and let's suppose that this is going like this to i_2 and let me call this i_3 but i notice this i_3 must be equal to i_1 minus i_2 because i_1 coming in i_2 going out

so i have got net coming in is i_1 minus i_2

so net going out through this must be i_1 minus i_2 ok having done that i have got 2 unknowns which are i_1 and i_2 are my 2 unknowns

so i_1 and i_2 are 2 unknowns and i_3 is known already because it is nothing but

i_1 minus i_2

so therefore let us look at the left loop

so what i get is this that since coming in is i_1 the the amount of current that is going in going out from here is i_1 what is coming inside must be also equal to i_1

so let us do that i got minus 2 into i_1 minus i_2 supposing i started from here then i got minus 2 i_1 this is this one then plus 12 that's equal to 0 as one equation this second equation you do it from this loop now once again it doesn't matter how you assumed it

so let's suppose we go like this

so if i do like that i have a minus 6 there minus 2 times i_2 but this time since this loop has been taken like this it will climb up

so therefore it will be plus 2 times i_1 minus i_2 equal to 0

so i have got 2 equations in 2 unknowns i will not be unnecessarily solving it here the idea is to tell you how to solve this equation is a trivial simultaneous equation and you can yourself solve it

so two equations to unknown

so two equations now notice the second equation i need not have done it on this loop i could have also done it in the bigger outside view and there will be it will be an independent equation let me let me take some more we have given two loops now let me give you three loops

so let me give you some numbers let it be 6 ohms that is also be 6 ohms 3 ohms here and 3 ohms there 6 volts there in this way and a 12 volts there

so once again what do i do i can assume directions but look at the following first that this section of the circuit is a parallel combination of two 6 ohm resistances

so therefore the effect of these two is equivalent to a 3 ohm resistance

so therefore this circuit that i have written down i could have first simplified it now i will first solve this equation

so let us look at that supposing i assume supposing i assume that my current that is coming in from here is i_1 there is a 3 ohm here and let us suppose an i_2 is going out from here now this junction suppose i have a current i_2' which is going through now remember i_2' is not really current in any one of these two it is a current through an equivalent resistance which i found out

so what i will do here is this that i have one equation which is i_1 one junction rule i_1 is equal to i_2 plus i_2' this is the first junction now then i have the following now notice that once i have done that i don't need any more junction rule the reason is i have two loops i have three unknowns here i_1 i_2' and i_2 one is taken care of by the junction rule and the second one and the third one will be taken care of by choosing two loops

so look at this now

so this was a 12 volt

so minus 3 i_1 going like this minus 3 i_2' plus 12 is equal to 0.

so i in other words i_1 plus i_2' is equal to 4 this is one of the equations in the second loop what i have got is

so this was my right hand loop let me look at the second loop in that loop what i have got is 3 i_2

so remember that whatever is coming in

so i write down 3 i_2 minus 3 i_2' equal to 6 okay what i have done actually is this actually i should have written minus 3 i_2 this is i_2 then i am going up the current

so therefore plus 3 i_2' and here i get a plus 6 but this is the same

equation as that

so using these you will be able to solve for the things like what is $i_{\text{double prime}}$ and what is i_1 i_2 these are the 3 things and i've got the equations corresponding to that now having done that what you will find will be the following that your solutions will turn out to be i am not solving this because they are trivial equations and

so this is equation number one this is equation number two this is equation number three what you get is i_1 equal to $10/3$ amperes i_2 equal to $8/3$ amperes and $i_{\text{double prime}}$ equal to $2/3$ amperes but you recall that i told you $i_{\text{double prime}}$ is not a current through any of the branch of my original circuit but i can see what happened there because this $i_{\text{double prime}}$ came from this circuit and these are two equals resistances

so therefore whatever is coming in there must have been equally distributed

so therefore if you call this let us say i_3 and that as i_4 that must have arisen due to the $i_{\text{double prime}}$

so therefore i can assume that i_3 equal to i_4 equal to one third half of $i_{\text{double prime}}$ which is equal to one third amperes i would advise you instead of doing this part to start doing directly by assuming i_1 i_2 i_3 whatever the way we have written it and you have got two junctions there and you have got three loops there you can do it another way instead of doing this shortcut couple of lectures back we talked about a an infinite resistance circuit let me give you an example of how it works in this situation at that time we had simply asked for using the concept of parallel and series combination we were asked to calculate what is the effective resistance what i will do is do the same thing but i will now put a battery in one end of the circuit

so let me draw this circuit there is a six volt battery here this is not quite the same circuit as we did earlier let us say this is one ohm one ohm one ohm this is two ohms two ohms two ohms and this continues in a infinite ladder now the question is this what is the current that passes through this resistance

so let me let me say that how much is this current now let us look at that

so we will assume the following we say that suppose i assume my resistance r okay to be the equivalent resistance there then this circuit that i have got i can do like this see imagine forget about this battery look at how this battery this works i have a resistance here and a resistance here now supposing i were to cut it here then what remains is exactly similar because i have said this is infinite actually i should use the word semi-infinite because on one end i have kept it but it is infinite that way

so if the resistance of this whole thing is r then what i am getting is the following i am getting a circuit of this type there's one ohm i've got a two ohms there and i have got a resistance r there

so this now this 2 ohm and this are there in parallel

so therefore this is equivalent to a battery 6 volts there a 1 ohm here and an effective resistance there

so this is 1 ohm and this is the combination of 2 and r

so this effective resistance is $2r$ divided by $2 + r$ now notice what i am saying now this tells me that the current through this circuit will be the series resistance of 1 ohm and $2r$ by $2 + r$ but then if i did not cut it here i consider the whole situation that is nothing but a resistance r

so therefore my r should be equal to $1 + 2r$ by $2 + r$ solve this this is quadratic very simple

so r will turn out to be equal to 2 ohms sorry yeah r will turn out to be equal to ohms just take the quadratic equation and take the positive solution there

so therefore what is the current through the circuit the current through the circuit is what you do here

so current is 6 divided by this is just a series resistance
so $1 + 2r$ divided by $2 + r$ is $4/2 + r$ is also 4.

so this is equal to 6 divided by $1 + 4$ by 4

so that's equal to 3 amperes

so therefore what we are saying is this this 3 amperes that we have got is passing through my 1 ohm resistance because that is the one which came there and it distributes to this 2 and r which is also 2 because this is this this was r
so this current that is going in there will distribute here as well as to this part and since this resistance is the same as that resistance the current through the 2 ohm resistance is that the nearest 2 ohm resistance is 1.

5 amperes let me write down nearest two ovaries let us make things a little more interesting this time since we have already learnt about a capacitor in a circuit the way i would do it this let me draw a circuit this time with a capacitor ok

so this is the thing

so what we want to know is how much is the current which is passing through let us say this one let us give some names the first thing that you should understand regarding direct currents that is passing through a capacitor part now remember once equilibrium is reached there is a direct current

so known current can pass through a capacitor the capacitor plates of course get charged but

so there would be a potential difference across them but current is not passing

so what it tells me is this that there is no current which is flowing through this resistance there

so no current here that doesn't however mean that there is no current through this one and the reason is very simple that if current came through this it will get stuck here

so therefore there is no path but current can be there in this because there is another loop of which it is a part

so first let us write down no current through capacitor after transients have died out there are no current in a dc capacitive situation okay let us now start giving some names supposing this i call it as i then let us call this i_3

so i notice here at this junction i have i_3 going out i have i coming in

so therefore at this junction what is coming in is $i_3 - i$

so clearly $i_3 - i$ comes here since there is no current in this branch

so what i have here is also $i_3 - i$

so effectively

so far as current is concerned i have taken out this part of myself there will be potential difference but this doesn't contribute to my voltage law

so let us look at this here the simplest thing to do in such situation is to make some intelligent observations and the first observation that i make is this that between these two points let us just number them let us call them a b

so between a b i know the potential difference because this is not contributing to my circuit

so the potential difference between a b is the same as the potential difference let's say across a prime b prime but which is 6 volts

so therefore

so let's write down potential difference across a b is equal to potential difference across a prime b prime which is equal to 6 volts

so therefore my current i_3 is simply $6 / 4 = 1.5$ amperes

so therefore it is equal to 1.5

5 ampere

so one unknown thing has been removed now what i do is this i look at this loop

in this loop let's do my kirchhoff's law

so i have got minus i_3 into 4 remember i_3 is already known this current here is i_3 minus i_1

so therefore minus i_3 minus i_1 into 2 plus 2 because i am going like this then there is a plus 3 here minus i_3 minus i_1 i_3 minus i into 3 well i_3 minus i

so you will be able to now you remember that i already know what is i_3

so do that this is a trivial number

so you get i equal to 1.

7 amperes that was the only unknown that we had now supposing i am interested in finding out what is the potential drop across these two

so let me redraw that circuit and what we have shown is that the current i was equal to 1.

7 amperes i_3 is 1.

5 amperes

so therefore the current in this section a to c which was i_3 minus i is actually minus point two

so i have shown it as the direction opposite to what i took

so in this section uh i_3 minus i amount of current namely 0.

2 ampere current is passing now my question is what is the potential difference across the two ends of the capacitor

so let us call it d

so what is the potential difference across cd now this is fairly simple remember that in this section there is no current

so therefore the potential drop across this two plate is the same as the potential drop across c and a

so this is equal to Δv_{cd} is the same as Δv_{ca} and i know the current here

so therefore by the rule that we have been repeatedly talking about when i go from c

so let's talk about vc then i go down the potential hill

so therefore minus this was 2 volts further minus current is 0.

2

so 0.

2 into 2 ohms and with that i come to this end and since there is no current passing through this section and this is the resistanceless wire

so therefore i can come to point d

so this is equal to v_d which tells me that v_c minus v_d is equal to 2.

4 volts and that is the potential drop across the plates of the capacitor and since v_c is at a higher potential than v_d this side of the plate is positively charged and this side of the plate is negatively charged in many problems blind application of kirchhoff's law is very time consuming and becomes clumsy however

often symmetry of a problem helps us in reducing the difficulties associated with application of kirchhoff's law to illustrate this let me consider a cubical network of 12 conductors let me draw this i will not be showing the resistances with wiggly lines but i will assume that each of the 12 arms of the cube have a resistance of r and let us name them

so let's call it $a b c d$ it's called $e f g$ okay let me assume a battery is connected between the diagonally opposite corners a to d which is v and each arm has a resistance which is equal to r now notice one thing these arms for example $a f a h$ or $a b$ are symmetrical with respect to the diagonal $a d$ and likewise the other three here $ed dg$ and dc are symmetrical with respect to

so we are looking at symmetry with respect to diagonal ad now that tells me that the current that is distributed in each such arm must be equal

so let me take these three currents to be equal to i each
 so this is i this is i this is i now likewise these are the currents which
 are going out of the point a distributed as i i i
 so that the battery is actually supplying $3i$ and these currents will be
 entering the point d
 so therefore these which are entering the point d should also be i i i now
 when the current i arrives at a point f because of the fact that the arms f g
 and f e are symmetrical
 so therefore these will have i by 2 each and likewise a reaching h will divide
 into 2 that's also i by 2 and this will also be added and you can check that the
 junction rule is automatically satisfied there and likewise this will be i by 2
 this will also be added now notice that we have only used the symmetry of the
 problem in getting into this situation
 so let's look at now this loop a b c d let's call this e x x v and y and l
 so this is basically this outside loop which contains
 so this is the way
 so look at what do i get out of this
 so i get minus ir minus i by $2r$ another ir
 so therefore total is minus 5 by 2 ir and then of course plus v
 so therefore what i get is 5 by 2 i r is equal to v which tells me current i is
 given by 2 by 5 v divided by r and if i give some numbers for example if r is
 equal to 1 ohm and v is equal to let's say 10 volts then the current i will turn
 out to be 4 amperes the the current that is supplied by the battery is $3i$ which
 is simply 6 v divided by 5 now supposing my question was what is the equivalent
 resistance between the points a and b now this can be easily answered by
 observing that my battery is supplying $3i$ amount of current
 so current from the battery is $3i$ now supposing r equivalent is the equivalent
 resistance of the circuit between the points a and b then by definition it
 follows that v divided by r equivalent must be equal to $3i$ and which is equal to
 3 into 2 by 5 v by r because that was the value of i
 so this is equal to 6 by 5 v by r
 so you can immediately see that the equivalent resistance between the points a
 and b which are on the diagonally opposite corners is 5 by 6 r and if each r is
 1 ohm then of course this is just 5 by 6 ohms now this is an example where if
 you had a priori assume that there are 12 different currents in 12 different
 conductors you would have a mess but because we were able to observe asymmetry
 we were able to do this problem without much of an effort what we'll do next
 time is to take up some problems which are complicated and there are no obvious
 symmetries in the problem and also talk to you about a few applications of what
 we have learnt under the current this chapter you