

so in the last lecture we had discussed with several examples the series and the parallel connection of resistances towards the end we started talking about the series and the parallel combination of cells

so what we said is that when the cells are combined like resistances we can combine them in series or in parallel we discussed the series combination last time but let me quickly review this

so in series combination as is common the i remains the same and the Δv that is the voltage across differs

so basically the circuit is very similar to the way you define a resistance series combination

so this is battery number one i already told you that we need to represent a battery by a combination like this

so this is an emf source and the resistance here is the internal resistance

so let us call it r_1 and this is e_1 and then i have a second one and as is customary i am combining or connecting the negative terminal of one with the positive terminal of the other uh though if you were to connect it differently then all that it will mean is you will have to subtract the voltages

so this is the second battery

so this is e_2 and r_2 and we said that we want to find out what is the equivalent combination in other words supposing you wanted to replace this combination with a single source of potential difference and

so this is let us say the point a and this let us call it point b and point c so between the points a and c what should you combine

so this is a and this is the combination i am looking for and of course there is a internal resistance

so i will again combine this and say this is e and let us say r and this is the point c

so what we said is the principle is the same as we do in case of resistances that is start from any point let us suppose i start from the point c and go in the direction of current supposing this is the direction of current later on today when we discussed the general rule for finding out current we will realize that it is a material what is your assumption regarding the direction of current because if you have made a mistake in the beginning and your answer turns out to be with a minus sign it simply means that the direction of current is opposite to what you have assumed to be true

so let us look at that i start from the point c go in the direction of current this is the direction of the current now whenever you go in the direction of current there is a potential drop when you cross a resistance

so drop i will put a minus sign

so therefore what i have is v_c minus i times r_2 and here i increase the potential because it's going from negative to the positive

so therefore my emf has to be added here

so e_2

so with this i have come to the point b then since this is a series connection the current still continues to be the same and the same current goes through r_1 giving me a drop of minus ir_1 and then again i add up another e_1 and that by that i have come to the point a

so this tells me v_a minus v_c that is the potential difference between the two ends of this section is e_1 plus e_2 minus i times r_1 plus r_2

so if you look at this expression it tells you that the equivalent emf is given by e_1 plus e_2

so this is e equivalent and the equivalent internal resistance is simply given by r_1 plus r_2

so that is req

so this is what actually you substitute this

so let's write down that eq is equal to e_1 plus e_2 and req is equal to r_1 plus r_2 now supposing i had instead of this combination i had connected the polarities differently and let us suppose i had this combination here

so what we have is a situation like this this is a seat of emf e_1 with internal resistance r_1 and we connect this to another battery but this time instead of connecting the negative terminal of one to the positive terminal of the other we connect the two ends having the same polarity and this is e_2 with an internal resistance r_2

so let's give the same type of notation as we have been given for the battery

so this is seat of one ear and this is directed like this and this is the seat of the other unit like that

so let's look at what happens in this case

so let me take this point as a and let me take this point as c now we can assume the current to go in any direction we like we have pointed this out several times

so let us assume that the current is flowing from c and flowing out from the end a

so in which case if i start at c and come to the point a what i have is v_c which is the potential at the point c minus because i am traveling in the direction of the current i times r_2 then i have a further drop because inside the battery i am going from positive terminal to the negative terminal

so minus e_2 once again when i enter the next seat of emf i have a minus i times r_1 but this time i have a plus c_1 because i am going from the negative terminal to the positive terminal and i reach the point a

so this is

so this tells me that $v_a - v_c$ is equal to $e_1 - e_2 - i(r_1 + r_2)$ if you compare this with our previous expression what we find is there is no difference other than the fact that there is a minus sign between u_1 and e_2

so basically what we are trying to say is this that the emfs can be also treated as algebraic quantities depending upon the direction in which the current is moving in if the current goes out from the negative terminal to the positive terminal then of course uh this is uh emf is positive but on the other hand if the reverse happens then it is negative now the question is this this combination that we have shown here is not really a preferred combination you will find that in all appliances where you use serial combination of batteries the terminals that would be connected would be negative terminal with the positive terminal the next question that we have is why use a serial combination of cells at all why not simply use a cell which has a which supplies a higher voltage now the reason why i cannot do that is the following that in addition to the voltage there is another important quantity which characterizes a battery and that is what is known as the capacitor

so the capacity rating is essentially a measure of what is the life of a cell and as we are all aware that batteries work by the principle of chemical reaction

so whatever is the chemical reaction taking place there supposing you have an electrolytic cell

so you have an electrolyte now what happens is that these iron suture if you want to call them active ingredients the positive ions go towards the negative terminal and the negative ions go towards the positive terminal and they pick up or give up give away electrons themselves now becoming uncharged now once they become uncharged they become inactive and do not take part in further chemical

reaction and that would mean that the battery's life has come to an end in the market there is usually an assorted voltages and ratings available and typical for instance is lead acid batteries which supplies you two volts the more common double a or triple a batteries they supply you 1.

5 volts nickel cadmium battery which is another common battery available gives you 1.

2 volts lithium-ion battery gives you 3.

6 volts and it's more convenient that instead of designing a battery with the specific voltage depending upon what your work requires you pick up from the standard combinations that are available and design by using parallel and series combination the voltages and the ratings that you want

so let us look at how to add cells in parallel

so the combination is very simple that just the way you do resistances

so i have situation like this

so let us look at this combination this is once again our same notation e_1 r_1 e_2 r_2 and in this case let us now notice here the most common combination in this is join the similar polarities end to the common point this is the way the parallel combinations work if you look up your remotes where parallel combinations of batteries are used you will find that both of them say that put the positive terminal here

so so the way it works is this supposing the current here is i_1 and the current that is coming through that is i_2

so this is your point a let's call this point b_1 let us call this point b_2 and this point to c and obviously since this is the negative terminal current is coming like this and current is coming like this and of course this is the way it is now suppose v_{b1} and v_{b2} are the voltages here and there

so my v is v_{b1} minus v_{b2} and look at what each one of them is

so this difference v_{b1} minus v_{b2} since it's a parallel combination you can compute it either through this branch or through this branch whichever you like so that tells me that i_1 minus $i_1 r_1$ just i just go from the point b to this point since there is a i am going in the direction of current drop is $i_1 r_1$ i pick up emf e_1 and then come back to the point b

so that by that i get this difference and this is also equal to if i proceed like this then this is also equal to e_2 minus $i_2 r_2$ now notice that the net current that i have got is this i here now this is clearly i_1 plus i_2 now i will be talking more about this junction what happens in a junction later today but look at this that i_1 coming in here i_2 coming in there the current is nothing but rate of change of charge

so therefore if all the charges did not go out in the same time frame there would be accumulation of charges now this is what is known as continuity

so by the continuity condition i must have my current i is equal to i_1 plus i_2 and that i have seen is how much i_1 you can see if v is the potential difference there

so this tells me i_1 is e_1 minus v divided by r_1 and i_2 is e_2 minus v divided by r_2 i can rewrite it e_1 by r_1 plus e_2 by r_2 minus v into 1 over r_1 plus 1 over r_2

so look at this expression supposing i now find out what is my v just rewrite this accordingly

so i get firstly i get a minus i into $r_1 r_2$ by r_1 plus r_2 this is simply multiplying both sides with the inverse of this plus r_2 plus e_2 r_1 divided by r_1 now look at this supposing i wanted to replace this by a single battery as before

so i have this

so this current is i that is what is coming in

so notice that what i need to do is to have an equivalent is it notice that what would be my v would be supposing this is some r equivalent and this is some equivalent

so i will have i times r equivalent as a drop

so minus i times r equivalent

so this term must stand for r equivalent and this term here

so i must have plus e eq

so i'll have this is my this term is my e eq

so therefore now look at what i am getting

so my r equivalent is the same expression as what we got for the parallel combination of resistors

so therefore let us write it down we say the equivalent resistance that i have are equivalent is $\frac{1}{\frac{1}{r_1} + \frac{1}{r_2}}$ which is the equivalent resistance of the two internal resistances for r_1 parallel to earth okay let us look at the expression for e eq a little more carefully now this is $\frac{e_1 r_2 + e_2 r_1}{r_1 + r_2}$ let us do something supposing i write it like this $\frac{e_1 r_2 + e_2 r_1}{r_1 + r_2}$ divided by $r_1 + r_2$ let us do something supposing i write it like this $\frac{e_1 r_2 + e_2 r_1}{r_1 + r_2}$ divided by $r_1 + r_2$ and write it as $\frac{e_1 r_2 + e_2 r_1}{r_1 + r_2}$ this is the same expression i have simply put in the denominator $r_1 r_2$ there and r_1 after 1 now this expression here i can rewrite as $\frac{e_1}{r_1} + \frac{e_2}{r_2}$ and this is as we have seen is the equivalent resistance r_{eq}

so you notice that there is a very symmetric relation that we find for e equivalent which is given by e equivalent by r equivalent is $\frac{e_1}{r_1} + \frac{e_2}{r_2}$ by n and you if you had more than two batteries of course you could still use this formula that whatever is the equivalent resistance of all of them and here the same type of formula we can we can do that

so basically what we do is this that in case of a parallel combination the arrangement is something like this you see typically a battery looks like this this is plus i am using the picture which is very commonly seen in normal 1.5 cell torchlight batteries

so this is the way we normally do connect these and

so this is my plus and once again connect these and this is my minus

so why parallel combination

so one of the things that you notice is this the voltage that i get is the same because of the parallel combination but it provides a higher capacity rating for the same voltage

so this is this is the way

so for instance if you added two 1.

2 volts batteries in parallel and let us suppose each one of them has let us say rating of 1000 milliamp hours because that the typical small batteries they don't uh consume a lot

so what we get as a result is 1000 plus 1000 i get 2000 milliamp hours for the same voltage and as a result of course you can calculate how much is the power difference

so let me now use these to work out a few example problems the which will sort of tell you in what situation you can use this combination to your advantage

so let me start with this remember i told you that normally parallel combinations are connected similarly

so let me give some voltages this is three volt this is two volts and this is one volt and let's say each one of them is one ohm simply and this point is a let me take also the load to be one for simplicity trying to illustrate a point rather than actually work it out

so look at what i have said what i have said is this that this parallel combination i can replace by a single battery and how much would that be

so the equivalent resistance here is simply the parallel combination of three

one ohms which of course will give you one by three ohms and the equivalent voltage emf is given by this formula $e_{eq} = e_1 + e_2 + e_3$ by $r_{eq} = r_1 + r_2 + r_3$ now $u_1 = r_1 = r_2 = r_3$ are the same and each equal to 1

so this is nothing but just sum of the 3 voltages which is 6 volts

so that tells me e_{eq} is 6 into 1 by 3 which is equal to 2 hertz

so what we are saying is this that this combination you can replace with a single battery of 2 volts having an internal resistance which is one third of what each one of them had

so ah but you know if this is all that is there in a circuit then of course the combination of batteries is not supplying any current this is the circuit is open

so if it is not supplying any current supposing this is my i_1 this is my i_2 and this is i_3 and this is of course as we have seen is i and why what we are saying is this is 0 no current now we can actually check how does it work see the way to do it is the following that suppose i come from here to there

so we have already worked out the potential difference between a and b is 2 volts and

so therefore if i start from here go along this path then i notice that i start from 3 volts minus 1 into i_1

so my equation is 3 minus 1 ohm into i_1 that must be equal to the potential difference between these 2 which is because there is no current it is the same as 2 volts which tells me that i_1 is simply 1 ampere let us look at supposing i went like this because this is a parallel combination i can go any way i like

so i pick up 2 volts minus i_2 into 1

so 2 minus 1 into i_2 that's equal to 2

so that gives me i_2 equal to 0 if you did the same thing by the third branch you will find i_3 equal to minus 1 ampere that is i am the direction i have shown is opposite to what actually take place now let me ask a slightly different question suppose in this somehow i shorted the central resistance that is the internal resistance if somehow other i could sort

so what is meant by shorting shorting simply means that the two ends of the resistance that you have got you are going to connect them by a wire of no resistance at all

so let me draw that picture again okay

so basically what we have said is the following that supposing you did this so that this resistance is effectively removed from circuit and not only that till now i discussed about a circuit which was open but let me also connect a to b there is a resistance here which is 1 ohm resistance that's your load resistance and this was 3 this was 2 this was 1 and this was each one of them was one this is not really material anymore

so let us look at what does not do for such a problem

so firstly this is almost an introduction to what i will be later on discussing as the loop law but i don't need it right now because that's the way i have been discussing that i can go around any path as long as i remember the following if i climb up a potential i add that amount of potential in a battery if i am traveling in the direction of current there will be a potential drop of i times r if the current through the resistor is i

so let us look at what i can talk about now notice one thing

so let me do the following suppose i decide to travel along this path what i am doing is the following that remember that our currents were i_1 i_2 i_3 and this current here was i earlier it was 0 but now because i have provided a path for the current

so this is i this is equal to $i_1 + i_2 + i_3$ now look at what would happen if i took this path a go to b go to c go on the top line come back to d and come

back to a that's a loop we'll be discussing about loops as you go along but look at what am i doing

so firstly since this is i_1 i pick up an e_1

so $e_1 - i_1 r_1 - r_1 i_1 = 1 - i_1$ which is $i_1 + i_2 + i_3$ times 1 of course that's equal to 0 because i have simply returned back to the same point

so this gives me since my e_1 is 3

so i get 3 is equal to $2 + i_1 + i_2 + i_3$

so that is my equation number 1.

so within this loop for example

so here again what we will do is this i have i_1 into 1 and then i travel through this red section there is no resistance and pick up the voltage 2 but this time it is subtracted because i am going from positive to negative and then return back there

so that will give me a an equation like this that i_1 because that immediately solves for i_1

so i get i_1 say there is an i_2 there but it is not coming into the picture in this loop when i go along because the resistance is not there

so i get i_1 equal to $e_1 - e_2$ which is equal to $3 - 2$ which is equal to 1

so that immediately gives me the solution i_1 is equal to 1 ampere now you could do the same thing for for instance for any of these loops you can do for this loop and show that i_3 is equal to minus 1 ampere and i_2 is equal to 2 ampere let me take another example supposing i have a simpler battery system r_1 equal to two ohms and let's take them to be the usual 1. 2 volt batteries this is 0.

15 ohms this is 0.

15 ohms now i first find out what is equivalent resistance

so that is simply 0.

15 parallel to 0.

15

so it is 0.

0.75 ohms and e equivalent divided by this r_{eq} which is 0.

0.75 is equal to 1.

2 divided by 0.

15 twice

so it is 2 into 1.

2 divided by 0.

15

so this is equivalent to a battery eq which has the same voltage namely 1.

2 volts now this is purely because of symmetry that is the two internal resistances are the same now you could verify that this indeed is true by realizing that supposing this is i_1 and this is i_2 now by symmetry i_1 and i_2 must be the same because this battery combination and this battery combination is identical

so therefore what i get here is a current which is $2 i_1$ because i_1 is equal to i_2 .

so let me look at the any one of the loops come like this go along any branch so if you look at the kirchhoff's law for this situation you find that minus $2 i_1$ times 2 ohms minus i_1 times 0.

15 plus 1.

2 equal to 0.

now which gives i_1 into i will not be doing the algebraic addition for a particular reason

so let's write it as $4 + 0$.

15 equal to 1 point which gives the current i_1 to be 1 .

2 divided by $4 + 0$.

15 amperes of course now the actual current in the circuit is $2 i_1$ that's equal to 2 .

4 divided by $4 + 0$.

15 which you can rewrite as 1 .

2 divided by $2 + 0$.

0.75 amperes this $2 i_1$ is the current that is passing through the 2 ohm resistance so notice that this 1 .

2 ampere that 1 .

2 volts that we have written down that is the equivalent emf of the parallel combination of the cells as you know that two equal valued emfs give the same emf when they are put in parallel and this denominator is the serial combination of two plus the equivalent resistance of the internal resistances with this let me give an introduction to the most celebrated law of current electricity which is known as kirchhoff's law we have seen lots of examples of situations where i can simplify the systems or circuits by looking at the series or parallel combination in today's and the next lecture we will be giving several examples of circuits which are too complicated to be broken into such simple combinations which are either series or parallel

so what we will do is this we will try to get a method of how to solve for such circuits and

so these are done in a very systematic way by a set of two laws

so these are known as the ketchup's law but before i discuss what is kirchhoff's law let me define but let me first put one circuit just as an illustration of what i am talking about this is a circuit which you can saw by solve by other methods but i am just using it for the purpose of illustration

so that you can see what is actually happening there

so let us call this number these points one two three four five six.

now first i define a branch point or a junction what is meant by a junction

so junction is a point in the circuit wherever three or more branches or three or more conductors join whether there is resistance in it or just a resistance less wire it does not matter

so a junction is a point in the circuit where three or more conductors are joined together in this example we have several junctions for example this is a junction a you can see it this is one conductor this is one conductor this one conductor as i told you it is totally material whether the conductors that are coming in have resistances or not

so a is a junction this point b is a junction this this and that c is a junction d is a junction

so these are what are called as junction then i define what is called a loop loop is much simpler as the name suggests any closed path in the circuit is called a loop

so in this circuit which let me redraw our numbers were one two three four and let's call these numbers five six.

now there are many many loops here

so for instance if you look at this left hand side small square like loop 1 4 c b 1 is a loop

so is d23cd i i'm showing loops which we are traversing in different directions sometimes clockwise sometimes anti-clockwise then for example one two three four one is also this one two three four one there are many which are not immediately

visible as a loop but there also start for example from one

so one to a traveling on the outside five six d 2 b 1

so 1 a 5 6 d 2 b 1 there are there are a 5 6 d value sorry a 5 6 b then you can come down 3 c 4 a w

so these are various examples of loops that are there in this there are two laws first law is called the junction

so this is kirchhoff's law the first law is called the junction rule the junction rule says that the algebraic sum of currents flowing towards a junction is zero

so let me write it down then i will illustrate what it means algebraic sum of currents arriving at a junction is zero

so this is also you could have stated it in a different way you could have said algebraic sum of currents leaving a junction is zero i will explain what it means

so let me first write down this sum over i i i equal to 0

so supposing there is a circ there is a junction where my currents arrive like this

so let me just give some name i_1 i_2 i_3 i_4 i_5 now i use the word algebraic it's not just i_1 plus i_2 plus i_3 plus i_4 plus i_5 equal to zero i have to check is it going towards the junction or it is going away from the junction supposing i decide that the current arriving at a junction is positive then my i_1 is positive i_3 is positive i_4 is positive but this is negative because it is going leaving a junction this is negative this is a leaving assumption

so this is what is meant by algebraic shock if i_1 i_2 i_3 i_4 and i_5 are just the magnitudes of the current with the direction as shown here then you know that for if i_1 i_2 i_3 etc are just magnitudes with reaction directions as shown then it tells me that i_1 plus i_3 plus i_4 which are coming in

so hence positive minus i_2 minus i_5 equal to 0 this is what is meant by algebraic sum being equal to 0 remember right in the first lecture we pointed out a current is not a vector this is not the way the vectors are vectors add by specific rules parallelogram rules between two

so this illustrates why we have made a statement that currents are not vectors we have seen current densities are vectors ok

so that is junction rule simple rule just take care of the direction of the current then you can write down the first rule next rule is called loop rule this says in any loop any closed loop the sum of voltage difference is 0 that is sum over i v i equal to 0 remember we have been talking about this for a very long time

so what is junction rule junction rule is a near statement of continuity of charge flow because at any junction whatever charge is coming out because there is no accumulation they have to go out and hence the word algebraic sum is zero now the loop rule arises because for static fields we have seen that integral e dot dl is 0 and we have been talking about this that suppose i have a resistance

so this end is positive this end is negative now this is uh in the sense that if the current is flowing like this from

so the direction in which the current is flowing is such that the positive end is connected to the positive terminal of the battery

so therefore this is the way the current is going

so higher potential of the resistance is the point where the current enters and of course we normally talk about a ohmic conductor

so Δv that the potential drop this is equal to i times r and that drop has to be taken care of as you go around the loop now in the seat of emf Δv is greater than 0 if we are going from the negative terminal to the positive you recall that you have been having this type of notation for our battery and this

is the mark that we have

so what we are saying is this that in using kirchhoff's law if you are traveling like this then your Δv turns out to be positive and the reverse would be true that is if you are traveling in the reverse direction if from positive

so let me put it in the same picture

so this is greater than zero if on the other hand we are describing it like this then this Δd is less than now you do not have to know a priori what is the direction in which the current is flowing you are allowed to use any s make any assumption regarding its direction and at the end of the calculation if the numbers turn out to be negative then you know that your original assumption was wrong and the direction of current should have been opposite to what you have assumed

so let me return back to this example that i gave you earlier

so let me look at a picture like this i will do numerical problems next time but at this moment let me illustrate how it is going consider a circuit like this

so let me number them this is r_1 this is r_2 let us call this r_3 let's call this r_4 now this says r_4 let us just call this load resistance is r this is r_5

so how do i look at this how many unknowns are there let us look at this first so there is this general current which is going out let us call this i now this i gets divided at the junction remember i said that the algebraic sum at this point is 0 .

so i is coming in but supposing you assume that i_1 and i_2 are going out this immediately tells me that if i_1 i_2 and i_3 are assumed magnitudes of the current then i_1 plus i_2 must be equal to i and keeping that in mind supposing we say this is i_5 this is let us say i_3 this is i_4 now look at how many unknowns are there i got i i_1 i_2 i_3 i_4 and i_5 6 unknowns are there i need to get equations corresponding to that generally i should be looking for three branch equations and three loop equations now if in principle if you blindly do things you will get many more equations because as i have told you there are many ways you can count the loops junctions are fixed because i need to simply see whether there are three points or not for example here i have a junction here i have a junction here i have junction here already four junctions are there this is a loop this is a loop this is a loop but loops there are too many

so when i try to solve this problem if i blindly write down the number of equations these equations will be many more than the number of unknowns that i have that makes the many of these equations are not independent

so one has to be careful in choosing which equations you write down what i will do next is to take several examples of circuits and solve them using what these two sets of two rules which are known as the keychops laws you