

welcome back

so let me begin by giving a summary of what we did in the last lecture

so first thing is we talked about series and parallel combination of resistances

so one of the things that i wanted you to understand is that these nomenclatures have specific meaning in fact i gave you an example of a situation where a resistance combination looks arranged parallelly but they are actually a series combination

so the way the it works is this that we defined parallel combination as a collection of resistances in which the arrangement is such that the voltage across any member of the combination is the same

so the way it works is this supposing this is a point a

so i have one resistance here another resistance here i could have any number of them actually third resistance here

so let us just call them r_1 r_2 r_3

so what we are saying is this that the drop in the potential if you go through the resistances whether you go through this this or that is the same Δv

so this would mean that since the values of resistances are different the current in the branches would be different

so we will say that voltage across each member is the same but current carried is different

so voltage across is Δv supposing here i have i_1 this is i_2 this is i_3

so expressions for Δv are either $i_1 r_1$ or $i_2 r_2$ or $i_3 r_3$ they are the same

so when you look at a combination the way to recognize whether it is a parallel combination or not is to check whether the voltage drop across various members are the same

so that's one the we next defined what is called a series combination

so that's of course a much simpler one

so there what happens is this that there are various resistances in that combination let's again call them r_1 r_2 r_3 and this is the point a to b now this is defined as a combination in which the same current passes through same current passes as a result since i is the same and here ours are different again

so therefore this is ir_1 this is ir_2 this is ir_3 this is the drop across them

so therefore they drop Δv_1 across this Δv_2 across this Δv_3 across this

so the net drop across a b

so let us call this v_{ab} is i times r_1 plus r_2 plus r_3

so let us return to the example that we had started working out towards the end of our last lecture but couldn't complete it

so supposing i have a combination of resistances like this let's this was a 21 volt battery and what i had there was a switch which was initially open

so let us number them let us call this point a this is b this is c d and e and f

so what we said is this that when this switch is open

so this section doesn't actually do anything to the circuit but look at this pair of points the voltage drop across ce because c is connected to this side is connected to this side of the battery is 21 volts voltage drop across df is also 21 volts we are given some numbers uh to represent the

so this was 4 ohms this was 12 ohms

so i call this r_1 call this r_2 and this is r_3 and this is r_4

so r_4 was 8 ohms and r_2 was also it was let us look at what happened there

so if you look at it the current comes out from this positive terminal and part of it goes here part of it goes there

so let us call this is getting cluttered up

so let me use a slightly different ink let us call this i_1 and since this is passing through r_3 let us call this i_3

so look at this if a current i is coming through this then clearly your i gets divided into i_1 and i_3 and because current is nothing but flow of charge and there is no accumulation of charge anywhere

so rate of flow of charge here plus the rate of flow of charge there must be equal to the rate of flow of charge which came through the circuit

so since we have said that the drop across this is the same as 21 volts

so what we notice is the following that here since this part of the circuit doesn't matter

so r_1 is in series with r_2

so in branch c r_1 is in series with r_2

so therefore this circuit has a net resistance of 4 plus 8 equal to 12

so let's call this r_{12} that's equal to 12 volts and likewise r_3 is in series with r_4

so let's call this r_{34} that's equal to 8 plus 12 which is equal to 20 ohms now having done that you notice that this circuit is now reduced to a circuit like this

so this is 21 volts this is 12 ohms and this is 20 ohms but this 20 and 12 are now parallel

so we call this r_{12} and we call this r_{34}

so we say r_{12} and r_{34} are parallel remember the formula for the parallel combination that i gave equivalent resistance

so the equivalent resistance of this combination is given by $\frac{1}{r_{eq}} = \frac{1}{r_{12}} + \frac{1}{r_{34}}$ and that's equal to $\frac{1}{12} + \frac{1}{20}$ and if you invert this r_{eq} will turn out to be equal to 15 by 2 that is 7.5

5 ohms

so what i now do is this i draw a further equivalent circuit which is at 21 volts here and a 7.5

5 ohms there and now i can easily find out how much current i was there because that is what is coming out

so which tells me that the current i which the battery supplied to the whole combination is given by 21 divided by 15 by 2 which if you work out it will become 42 by 15 which is 2.8

8 amperes

so therefore if you look at the original circuit now

so what we said is just go one step back now if you go one step back what you realize this that this i which came in which was 2.8

8 amps amperes got divided into this part and got divided into that part

so therefore the current in this section the current in this section

so this was 21 volts and we showed that this is 2.8

8 amperes this is 12 ohms this is 20 ohms we call it i_1 we call this i_3

so this point is c d e f this was the situation where the a b were not connected

so in this case the drop across c ce is the same as the drop across df is equal to 21 volts

so $\Delta v_{ce} = \Delta v_{df}$ is equal to 21 volts

so therefore my current i_1 is 21 divided by 12 as r_1 plus r_2

so that's equal to 1.75

75 amperes i_2 is equal to 21 by 20 that is equal to 1.05

05 amperes which of course agrees with our original current which is 2.8

8 amperes now you can use this to find out the potential drop between the points c and a remember that a was here this was the point a

so now you know i_1 the r_1 was 4 ohms

so i_1 times r_1 is the drop across c_2 c was at 21 volts

so the point a will be 21 minus i_1 times 4 you can easily calculate and find out that this v_a was equal to 21 minus $i_1 r_1$ that's equal to 21 minus 7 by 4 into 4 that's equal to 14 volts and v_b equal to 21 minus $i_3 r_3$ that's equal to 21 minus 21 by 20 into 12.

that's equal to 8.

4 volts now let us close that switch now when you close that switch the entire character of that circuit changes

so let's look at what is happening when i close that switch

so let us redraw that picture again

so this was a 21 volts battery i had a 4 ohms here r_1 just indicate without this is r_2 which was 8 on this side there was a r_3 which was 12 and an r_4 which is 8 again and i had this switch connected

so this was c_d this is a this is b this is e this is f now let us look at what type of a combination is this first thing you realize is this that this when you close the switch point a and point b because this is a short this is a short because there is no resistance across this

so therefore v_a must be equal to v_b likewise v_c is equal to v_d and v_e is equal to v_f since c and d have the same potential a and b have the same potential this tells me that this 4 and 12 are parallel if you recall i had said that the way to recognize whether some resistances are in parallel or not you have to find out whether the potential drop across them is the same we can redraw the circuit in the following way

so remember that i had the 21 volt drop here and the circuit is essentially equivalent to the following situation this is r_1 and this is in parallel with r_3 and here i have an r_2 in parallel with r_4 the points with reference to original labeling is this is a point c or d as the case may be this the point is a this point is b this is a' this is b' they are the same points really a or a'

so let us say a equivalent to a' b equivalent to b' and the last point is e or f as you desire

so since r_1 is parallel to r_3 my resultant is r_{13} equal to $r_1 r_3$ divided by r_1 plus r_3 and that's equal to 48 divided by 16 that's equal to 3 ohms likewise r_2 is parallel to r_4 and that leads to r_{24} equal to $r_2 r_4$ divided by r_2 plus r_4 they were all the same resistance

so this is 8 into 8 by 8 plus 8 which is equal to 4 ohms

so at that stage my circuit is this i have a three ohms here which we called as r_{13} and i have four ohms there which he called as r_{24} and this was 21 volts since r_{13} and r_{24} are in series my net resistance here is 7 ohms

so therefore my current is 3 amps now notice this thing the this point this was my point c this is my common point a or a' and this was my common point e or f

so since 3 ampere current is reaching c then my drop across ca is 3 into 3 that is 9 volts and drop across $a e c a$ and this is $a e$ is 3 into 4 equal to 12 volts now you return to the original circuit let me redraw the original circuit again

so my current in this section ca is not equal to the current in this section cv in fact if we remember we called this one i_1 this one i_3 and this was i

so my i_1 plus i_3 that must be equal to the current supplied which is 3 amperes and we have seen that the drop across ca is equal to 9 volts

so i can easily calculate how much is the current i_1

so current i_1 is obviously 9 divided by 4 this was 4 ohms which is equal to 2.25 amperes is equal to 12 volts

so if we call this i_2 now

so that would be equal to 12 divided by 8 ohms that was the resistance of this so that's equal to 1.

5

so you notice that 2.

25 ampere is coming in at c but 1.

5 ampere is going out through this section

so therefore there must have been a current in this section

so $i_a - i_b$ which is the difference between the two because there cannot be any accumulation of charges there

so therefore i_{ab} must be this minus this that's equal to 0.

75 amperes now what you can do is you can do a consistency check by repeating this calculation here

so this drop was 9 volts you can find out how much is the current here and then similarly you can find out how much is the current there okay and you will find that the result would be the same that this minus this would be the amount of current that is flowing into the section at a b

so this gives you an example of how to calculate things in which we have parallel and series circuits

so let me show you some little complicated problems suppose i have a situation like this now what i will do is that instead of drawing wiggly line lines for resistances let me for the moment just draw them like this but i will tell you where the resistances are

so this is a this is b and what i am saying is these are all resistances and the upper part each is one ohm and the lower part of each is two ohms the connecting wires are of course not are resistances

so let us look at what can i say about this circuit

so look at this situation

so notice that here i have these two because there is no connecting remember that the connection to the circuit which means connection to external um battery or whatever is the source of emf is between a and b

so therefore if i look at this section and this section this is in series because whatever current comes through this that must come back to this

so one and o 1 are in series similarly 2 and 2 are in series

so therefore this section that i have got here these are connected to the next one

so this is one this is one this is two this is two

so this is the same as a two one resistance being parallel to a 4 ohm resistance

so i can redraw this circuit like this and then of course each of these sections will be connected to the next section and four of them will be there

so this is two ohms and this is four rows since two ohms is in parallel with four ohms i get the equivalent resistance to be equal to 2 into 4 divided by 2 plus 4

so that is equal to 8 by 6

so which is equal to 4 by 3 ohms

so this is nothing but a single 4 by 3 ohm resistance and four of them are connected in series

so therefore net resistance of this combination is 4 into 4 by 3 which is equal to 16 by 3 ohms let me take another example i will consider an infinite resistance network

so the network is like this you have resistances all of them have the value r and they are connected also my resistances are what is the effective resistance of such a circuit now in order to do that you have to make an observation remember i've said it's an infinite sector

so supposing there were an end to it now what do you mean by end of an infinite register section imagine the number to be n which is very large

so look at the last section

so this is let us suppose the last section see if i were to cut this like that then whatever remains there has the same structure as the other one the longer one and since i have said it is infinite it won't make any difference at all

so therefore if my equivalent resistance for the whole network is r_{eq} then after they cut it out what remains is also r_{eq} you take out an identical section from the end of an infinite network you are still left with an infinite network so therefore the r_{eq} which is the equivalent resistance what i am looking for is equal to the equivalent resistance of the infinite section to the left of this red line

so this is equal to r_{eq} plus this sector which i have marked with red now let us look at what this actually means now notice what has happened as a result is this this r_{eq} section has become parallel with this r

so r_{eq} is parallel to r

so therefore the effective resistance that is coming from this section and that section this r

so that is simply equal to let's call this r_{eq} the effective resistance r_{eq} into r divided by r_{eq} plus r

so this amount

so i will draw this again

so we are saying now that i have a resistance here which is this quantity that is let's call this r_{eq}' and of course these two are but this circuit is a simple circuit which is r and r_{eq}' are in series

so therefore this is equal to r_{eq}' plus $2r$ and r_{eq}' i have already obtained a relationship which is r_{eq}' divided by r_{eq}' plus r plus $2r$ now if you solve this this leads to a quadratic equation

so let me write down r_{eq}' divided by r_{eq}' plus r in the denominator i have to add this to $2r$ and that's equal to r_{eq}' let's simplify them which is r_{eq}' and $2r$ equals

so it is $2r$ square plus $3r_{eq}'$ in the numerator and denominator i cross multiply

so i get r_{eq}' square plus r into r_{eq}'

so therefore r_{eq}' square minus $2r_{eq}'$ minus $2r$ square is equal to 0

so the solution for r_{eq}' is $2r$ plus or minus square root of $2r$ whole square that is $4r$ square plus 4 into 2 $8r$ square divided by 2 and that's equal to $2r$ plus or minus take away $\sqrt{4}$

so i get 2 times square root of 3 times r divided by 2 which is equal to 1 plus or minus root 3 r obviously i pick up the positive sign

so equivalent resistance of this infinite network is 1 plus root 3 times r once you know the value of r you can calculate now this idea that series resistances and parallel resistances i can find effective formula can be used to solve problems which do not look like the standard resistance network problem

so let me give you another example let us look at a resistor which has this shape this is a conical shaped resistor this section has a radius a and this section has a radius b and let us suppose that the length here is l now how do i find out the red supposing the material has a resistivity ρ how do i find out what is the resistance of such a sample of course we assume that the same current is passing through this each section which is circular image

so let us look at this now i can first assume or looking at what is the radius of the circle at a distance x from one of the ends

so this is what i am looking for this distance is x now this is linear

so we write down the radius of that section at a distance x is given by a plus

b minus a over l multiplied by x

so supposing at that distance i consider a small cylinder of width dx

so my resistance of that section dR at x will be given by ρ times the length which is of course dx divided by cross sectional area which is π into this radius square which is $a + b$ minus a by $l \times h$ square

so what i do is this to find out how much is R ρ over π is of course a constant number i integrate this quantity dx divided by $a + b$ minus a over l times x whole square and the x is from 0 to l this is a straightforward integration i don't go through every step of that integration and you can find out immediately that this quantity is given by ρ by π into b minus a over $a + b$ and that's equal to ρ by π l times or l over $a + b$ is the length you notice that supposing a were equal to b then i will get the resistance to be given by ρl by πa square which is what i expected for a normal cylindrical conductor another example i will give you where again i use the same principle of adding up resistances but this time with a slightly different application suppose i have a stack of aluminum and graphite

so let me draw that

so this is a stack of aluminum followed by a stack of graphite having the same cross section

so let us say this is the length of aluminum section this is the length of carbon or the graphite section and the data that is given to us are the following that the resistivity for aluminum at 0 degrees is 2.75×10^{-8} ohm meter and its alpha value alpha aluminum that

is the temperature coefficient of resistivity is 0.004 per degree centigrade the corresponding data for graphite is 5×10^{-6} ohm meter and alpha for carbon is negative 0.005 per degree centigrade now see this is essentially a series circuit because any current flowing into lets say aluminum will also pass through the carbon if there is a complete circuit now what we are looking for is what should be the ratio of the lengths of aluminum section to the carbon section

so that the temperature coefficient of the combination will be zero now what it means is this that supposing i am looking at the resistance of the aluminum section at a temperature t plus the resistance of the carbon section at a temperature t

so i get resistance of aluminum at 0 degrees into $1 + \alpha_a l$ into Δt plus resistance of carbon at 0 temperature $1 + \alpha_c l$ into Δt now what we are essentially looking for is this that what should be the length of the ratio of the length of l aluminum to l carbon such that the this quantity is independent of temperature what it means is i am looking for $r_a l + r_c l$ that should be identical to $r_a l_0 + r_c l_0$ now this were

so what i require is that the contribution from the temperature dependent part of these two should cancel

so what it means is $r_{aluminum} \alpha_{aluminum} \Delta t$ is i should have written as Δt equal to minus $r_{carbon} \alpha_{carbon} \Delta t$ now remember my resistance is proportional to the length and inversely proportional to the cross section but in this case my cross section is the same

so what i require is that $\alpha_{aluminum}$ times well i can put a row $r_{aluminum}$

0 is $r_{aluminum} \alpha_{aluminum} \Delta t$ would be cancelling out from both sides equal to minus $\alpha_c \rho_c \Delta t$ the corresponding data are given that enables you to find out what is the ratio of l aluminum to l carbon and that works out to $l_{aluminum}$ divided by l_{carbon} is approximately 227

so these were examples where the series and parallel resistances were used let me give you a final example in series and parallel combinations let's look at a circuit like this okay there are too many of them well let me number them this

is r_1 this is r_2 this is r_3 this is r_4 this is r_5 let's call this r_6 r_7 r_8 r_9 and r_2 doesn't look like a series or parallel combination but let us try to look at this a little more carefully

so all that i need to do is whether to observe whether the same current is passing through or is the same voltage between two ends if it is former it is a series resistance if it is later its a parallel resistance

so let us look at whats happening now notice here this point and this point these two points are at the same potential okay and this is a common point

so which tells me r_4 and r_5 are parallel combination

so let us write down r_4 parallel to r_5

so let us denote that equivalent resistance to be r_{45}

so i will remove this section and put a r_{45} there now the moment i have removed this section and put r_{45} there i notice r_2 r_3 and r_{45} are in series

so r_2 r_3 and r_{45} are in series

so this section now this this and r_{45} here

so our equivalent will be r well this was already 4 5 and i add 2 3

so let's call this r_{2345}

so this r_{2345} is obviously parallel with ourselves

so i write down that that this is parallel there now once we have done that let us call that since the numbers are going on increasing let us call that r_7 prime now there what i do is this that i replace this whole thing with the following circuit is becoming little clumsy

so let me redraw the equivalent circuit at this stage this is v i had r_1 there this we call it r_{10} this was a r_8 and an r_6

so this is r_{10} r_8 this is r_6 and this we had called it as r_7 prime and there was a r_9 hanging in here

so what do we have here clearly r_8 and r_6 are in series r_6 and r_8 in series let's call it a single resistance r_{68} not only that notice that r_{10} and r_9 have common points

so therefore r_9 is parallel to r_{10} this will call it as $r_{9\prime}$

so my circuit at this stage looks like this this was r_7 prime this was r_1 this is $r_{9\prime}$ and this is r_{68} now look at this circuit now

so what we find is that $r_{9\prime}$ and r_{68} are in series and this combination whatever you want to call it let's call it r_{689} this is parallel to r_7 prime

so therefore i can replace this with this resistance an equivalent resistance whatever you want to call it

so let's say it leads to sum r prime and then my r prime and r_1 are in series

so therefore you notice a very complicated looking circuit has been reduced to series and parallel combination it is not true that always we can reduce any circuit to series and parallel combination there are more complicated circuits for which we will find out the rules of how to do it in the next lecture but let us proceed with whatever we have been doing now

so far i have talked about resistances in series and parallel the only other element of the combination of circuits that we have been using are batteries

so batteries are cells as they're called

so the question is this that can we think of putting more than one battery in a circle in other words are the combinations like cells in series and parallel possible i will of course give you more examples of this later but let me try to define what is meant by cells in series and parallel

so first let's talk about cells in series now this has great practical applications if you look up for example a laptop now you will find that the laptop batteries are not single batteries in fact what you have in the laptop is a collection of cells some in series combination and some in parallel

combination in fact in many household applications you use more than for example a normal torch light cell you don't use one battery you put two or three cells end to end with the positive of one being connected to the negative of the next one

so that is an example of something being in series this is routinely done in all appliances at home in in your remote for example in your remote you will find there will be two aaa batteries which are put in parallel

so let me first talk about cells in series now the cells in series are like this supposing i have an emf e_1 with an internal resistance r_1

so this is your first battery i will give you the way the combinations are used normally that is positive end of 1 being attached to the negative end of the next one

so therefore this is the way we would have it

so let's call this one e_2 and internal resistance

so let us find out in nomenclature

so this is a this point is b and this point is c now what i am actually going to do is this i am asking the question is it possible to replace this combination with a an equivalent cell in that is is it equivalent to between a and c having a equivalent combination like this this is ac and if you like i will call them as e equivalent and r equivalent if it is possible to replace them how do i do it now let us look at this let us let us look at here

so what is v_c now if i assume that the current is flowing like this here and is coming in like this then i look at the way the potential develops as i go from c to a

so i start with v_c since this is in the direction of current i have a drop of current $i r_2$

so current is the same

so $i r_2$ then i am crossing a battery from negative to positive

so my potential rises by an amount e_2 i proceed further have a further drop of $i r_1$

so minus $i r_1$ and once again my potential is raised here in going from negative to positive to e_1 until i have reached the point a

so this is equal to v_a

so what is my v_a minus v_c

so v_a minus v_c is e_1 plus e_2 minus $i r_1$ plus r_2

so look at what have i done if instead i had a single cell with an effective emf of e and the total internal resistance of the pair where r_1 plus r_2 then i would have the same potential drop across the points a and c

so therefore i can replace the system by an equivalent emf which is e_1 plus e_2 and effective internal resistance which is simply the two resistances added in series i will in the next lecture talk about how to use cells in parallel i told you that if you look up normal torch light you put cells in series but look up your remote at your home you will find that the two positive ends are going together on the same side and they are in parallel

so i will obtain an equivalent emf and resistances for the parallel combination next time and i will also answer the question which you must be already raising within your mind that why should i use cells in series for instance why not i originally take a battery of a higher emf and of course having a little more internal resistance and

so therefore what is the purpose of using more than one cells instead of using a single cell of a higher emf and different registers we will try to answer that question also as you go along next time

you