good day to all of you this is the second lecture in the subject of electrostatics let me recall what we did in the last lecture i started with some demonstration experiments where i showed that if you take two pieces of straw and rub them with wool the two straws repel each other ah the ripple repelling force comes even when i put one of the straws close to the other straw do not even touch it one straw is pushing the other straw with certain force we also saw that if i take a glass rod which is rubbed with silk close to the plastic straw then the glass rod attracts the plastic straw the plastic straw which was charged gets repelled by another plastic straw but the glass rod attracts the plastic straw

 so there seem to be two kinds of forces one which is a repulsive force and one which is a attractive force to explain this we introduce the concept of charge as i mentioned last time charge is an attribute of a particle much like mass

 so if you have different particles just like they have different masses they can have different charges and we explain the experiments of repulsion and attraction by saying that there are two types of charges there are two types of charges one is called positive the other is called negative and the positive charge repels the positive charge negative charge repels the negative charge while positive charge attracts the negative charge and negative charge attracts a positive charge

 so with these two charges we are able to explain the phenomenon of repulsion and attraction that was seen in those experiments we also discussed different properties of charges for example first ones conservation of charge if you take an isolated system with certain charges then the total charge in that isolated system does not change by isolated i mean you do not allow any charge to come from outside into the system or any charge to leave the system this says that the total charge within the isolated system is constant it does not imply that the number of negative charges and number of positive charges is constant as i mentioned in the last lecture sometimes it is possible to generate new charges within the isolated system but whenever you generate a negative charge you will also in parallel generate a positive charge

 so the total charge of the system will remain constant and this is a property of conservation of charge we also saw that charge is quantized that is the charge comes in quanta of approximately a charge which is 1.
6 10 to the minus 19 coulomb

 so all charges that you will ever find positive or negative will all be integral multiples of this number

 so you can have any integral multiple of this quantum of charge you cannot have a charge which is say 2.
9 times this number

 so this quantization of charge is another very important property and third thing is additive additivity of charges

 so if you have n one positive charges and n two negative charges then the total charge in the system will be n1 minus n2 into e

 so you just add charges just like you add numbers keeping the sign of the charge in mind

 so when you have equal number of negative and positive charges the net charge becomes zero we then introduce the concept of conductors and insulators conductors are those substances in which there are free electrons that can move within the material

 so if you push some charges on this conductors they will distribute themselves all around the surface of the conductor insulators are materials in which this free movement of charges does not take place

 so if you charge an insulator at some point the charge sticks to that point and

is unable to move around the insulator i also talked about semiconductors which have conductivity between conductors and insulators and semiconductors form a very very important part of the electronics revolution we then introduced coulomb's law as the force between two charges

so let me recall if i have a charge q one another charge q two if this is my origin i have one vector here which is called r one vector another vector here which is called r two vector here this vector is r two one vector and r two one vector is equal to r two minus r one

so we said that according to coulomb's law the force on charge two because of charge one is one by four pi epsilon zero q one q two by r two one square into r two one unit vector

so the force is proportional to the product of the two charges inversely proportional to the square of the distance between the two charges and is along the direction along a line connecting the two charges as i mentioned this formula is valid whether the charges are positive or negative

so if both charges are positive then the direction of the force f two one is the same as the direction of r two one unit vector r two one vector goes from q one to q two and

so if the charges are both positive the force on q 2 is in this direction which is repulsive

so this charge is repelling this charge similarly if q 1 was negative and q two was also negative then this force again becomes positive two product of two negative numbers here

so the force has the same direction as the direction of r two one unit vector

so the force on q two because of q one is again in this direction which is again repulsive if the sign of q one and q two were opposite if one was positive the other was negative then the direction of the force f to one is opposite to the direction of the unit vector r two one and the force becomes attractive

so if for example this is positive and this is negative then the force acting on this charge is towards q one which is attractive now please remember that this force on one because of two is equal to one by four pi epsilon zero q one q two by r one two square into r one two unit vector now what is r one two unit vector r one two vector is equal to r one minus r two vector which is actually minus of r two minus r one vector which is equal to minus r two one vector

so r one two is oppositely directed

so if this is q one this is q two this is r one this is r two this is r one two

so the force as you can see here f one two becomes one by four pi epsilon zero q one q two by r now because r one two vector is minus of r two one vector the magnitude of r one two vector is equal to magnitude of r two one vector

so i can write here r two one square instead of r or r one two and r one two unit vector is minus r two one unit vector

so i put a minus sign here and i put r two one

so as you can see here the force on charge one because of two is exactly equal and opposite to the force of q one and q two

so if force if q one repels q two with a certain force q two repels q one with the same force in the reverse direction and this is nothing but a ah statement is a conformation of newton's third law this force as you can see here is very similar to the law of gravitation except that in gravitational there is only attractive force because there is only one kind of mars ok now we go to let me try to recall what we did in the experiment remember i had two straws in which we had ah rubbed the toss two straws with wool and we found that it was repelling each other

so what is actually happening is when you rub straw on a another surface the straw picks up some electrons from the bull and becomes charged excess charge of

negative and wool loses the charge

so please remember there is no generation of new charge all that has happened is some charge from the wool has moved to the plastic straw similarly when i dropped the other straw some charge moved from wool to the other straw and this was the same charge as moved to the first straw

so the two straws after rubbing have identical charges similar charges both are negative and hence they repel each other as we saw in the experiment on the other hand when i rubbed glass with silk it

so happens that glass loses electrons into the silk

so if you lose electrons from glass glass will have excess positive charge compared to negative charge and it gets positively charged

so when you bring glass close to the plastic straw the positive charge in the glass attracts the negative charge in the straw and as we saw in the experiment they stick to each other

so this is essentially the explanation of what we observed the other day ah in the demonstration

so now let us start with some concepts that please remember i told you that the unit of charge is coulomb but one coulomb is a huge charge because you remember one electron electron charge in magnitude is one point six ten to the minus nineteen coulombs

so in one coulomb the number of electrons is one by one point six ten to the minus nineteen which is approximately six into ten to the power eighteen
so six times ten to the power eighteen electrons are there in one coulomb and that is a very very huge charge

so we normally in problems we deal with much smaller charges things like for example 1 micro coulomb written like this this is 1 micro coulomb this is ten raised to minus six coulomb one nano coulomb which is ten to the minus nine coulomb i thought it may be interesting for you to have a knowledge of the following quantities which represent for example there is milli which is m which corresponds to 10 to the minus 3 millimeter 10 to the minus 3 meters milligrams 10 to the minus 3 grams then you have micro which is written as mu and that is ten to the minus six then you have nano which is written as small n which is ten to the minus nine then you have pico which is written as p which is actually ten to the minus twelve

so you see we are decreasing by a factor of thousand every time then we have femto written as f this is ten to the minus fifteen then we have ato ten to the minus eighteen

so one coulomb would mean ten to the minus eighteen coulombs then you have zepto which is small z is ten to the minus twenty one and yocto small y which is ten to the minus twenty four

so these are all quantities which are which can be used normally we tend to use milli micro nano pico femto but today these days there are also physical quantities at the scale of ato and zepto

so i thought it may be interesting for you to ah to learn that what are the what are these ah descriptions of these kinds of numbers on the other side we also have kilo which is k ten to the power three then you have mega which is capital m ten to the power six then you have giga g which is ten to the power nine then you have tera capital t which is ten to the power twelve then you have peta which is ten to the power fifteen then you have exa e which is ten to the power eighteen then you have zeta capital z which is ten to the power twenty 21 and finally yota capital y which is 10 to the power 24 i thought i will just mention about this because many times in physics and engineering you will tend to use quantities which are quite small in numbers or large numbers and then you will have to use some of these powers to to explain those quantities ok

so now i would like to introduce another very very important concept in electrostatics and that is the principle of superposition
 so earlier what we have discussed is the force acting on a charge because of the presence of another charge it is not necessary i just have two charges there may be many many charges
 so what happens if i have instead of two charges three charges
 so let me look at the following problem
 so i have a charge q two i have a charge q one and i have a charge q three
 so this is my some origin here this is my origin here o this is r two vector this is r one vector and this is r three vector you have these vectors r one two vector here and here you have r one three vector
 so the question is the following these are the three charges here three point charges the question is what is the force on q one in the presence of both charges q two and q three
 so what i do is the following i keep q one here and q two here and move q three to infinity very far away
 so you know that if i move q three to a very large distance it will hardly have any force on q one because remember the force on q one because of q three will decrease as one by this distance square
 so if this distance becomes very very large then the force of q three on q one will almost become zero
 so then i will have a force on q one because only because of q two
 so let me call this f one two
 so f one two is the force on the charge q one because of charge q two only in the absence of any other charge then what i do is i bring back q three put it back in this position move q two to infinity very large distances and again if q two moves to very large distances because the force of this charge q two and q one is inversely dependent on square of this distance as this distance keeps on increasing the force of q 2 on q 1 will become almost 0.

 so then what will i i have is q 1 will have c a force only because of q three
 so let me call that as f one three
 so f one two is the force on q one because of q two in the absence of q three f one three is a force on q one because of q three in the absence of q one q two now i put both the charges in this position and i want to calculate what is the force on q one
 so what is found is the force on q one which i call f one is actually f one two plus f one three that means this force on this charge q one is the sum of the force on the charge q one because of q two in the absence of q three and the force on q one because of q three in the absence of q two
 so the force on q one because of q two which is this quantity does not change even if q three is sitting here that is remains at f one two similarly the force applied by q three on q one is the same as if q two did not exist the total force on q one is the vector sum of the force of q two on q one and the force of q three on q one this is called the principle of superposition that means to calculate the total force on a charge i add up the forces of each individual charge on this charge vectorially and that gives me the total force now this is a very very significant result that cannot be predicted by any logical argument it just happens that electrostatics follows this principle of superposition
 so the force of interaction between any two charges is not affected by the presence of any other charge in the system and that is very very important for our understanding now i must mention that there may be domains of applications where this principle may fail this may happen in extremely small distances or in the presence of very very intense forces in all our discussion we will use the

principle of superposition and

so no matter how many charges are there in the system the force on any charge will be simply the vector sum of the force by each charge on this particular charge which can be obtained from coulomb's law

so in this problem in this example here as we see the total force on q one is the force on q one because of q two plus the force on q one because of q three

so i can write the following expression f one is equal to one upon four pi epsilon zero q one q two by r one two square r one two cap plus one by four pi epsilon zero q one q three by r one three square r one three cap

so this is f one two and this is f one three

so this force is the force on q two because of on q one because of q two irrespective of whether q three was there or not similarly this is the force on q one because of q three independent of the presence of q q one a q two

so this is the principle of super position

so if you have a large number of charges some large number of point charges i can write the total force on a charge q one as sigma j is equal to two to n

so suppose n n charges one upon four pi epsilon zero q 1 q j divided by r 1 j square r 1 j cap note that the summation does not include the charge on which i am calculating the force in this figure q one is not applying any force on itself the force on q one is determined only by the charges q two and q three

so similarly here in the summation the force on the charge q one depends on all other charges corresponding to q two q three etcetera up to q n and this is the corresponding distance of the jth charge from q1 and this is the unit vector along the line joining the jth charge to q1

so this helps me to calculate the total force on any charge

so for example if i wish to calculate the force on charge q two in the system i would have to write like this f two vector will be sigma j is equal to one and j is equal to two to n three two one sorry one by four pi epsilon zero q two q j by r two j square r two j unit vector

so this does not include charge q two it includes charge one q one and q three two q n all the other charges except the charge q two on which i am trying to calculate the force please remember that force is a vector quantity and

so i must add all these forces vectorially now to make things clear i want to take one or two examples to calculate the total force and to understand how does one estimate the force on one particular charge in the presence of other charges

so let me look at example one

so i am given the following problem i have a charge which i call q one here another charge q two and a third charge q three now as an example i am taking all the three charges along one line let me write this as minus 20 nano coulomb

so this is actually minus 20 into

so this is minus 20 10 to the power minus 9 coulomb this is for example let me take plus 5 nano coulomb this is 5 into 10 to the power minus 9 coulomb and let me assume this is plus 8 nano coulomb

so this is eight into ten to the power minus nine cool and i am given that this distance is one meter and this distance is half a meter

so i have three charges kept along a line q one q two q three q one is minus twenty nano coulomb q two is plus five nano coulomb q three is plus eight nano coulomb

so i want to find out what is the force what is the electrostatic force on q three

so i want to find out what is the force because of these two charges on q three now please note that all the three charges in my example are along one line

so the force of this charge on this charge does not depend on the existence of this charge at all it's on the way but if this this force of this charge on this

charge it depends only on the separation between these two charges and the two charges between similarly the force of discharge on this charge is independent of the presence of the charge human according to our principle of superposition now in physics we would should take the origin and axis to help us solve the problem easily i could in general take an origin at any place axis in any direction but it will help me to take the origin here

so i take a coordinate system like this

so this is y axis and let me take x axis like this

so let me redraw the figure here

so i have ah q one q two and a third q three

so this is y axis x x now this will help me to solve the problem little more easily as i mentioned it is not necessary i could have taken the origin at any point

so the force on q three is actually equal to f three one plus f three two the force on q three because of q one plus force on q three because of q two now what is f three one one by

so that is the force on q three because of q one four pi epsilon zero q one q three by r three one square r three one unit vector now what is r three one this ah

so the distance is given to me are one meter and this is point five meters

so r three one magnitude is equal to one point five meters and r three one unit vector is nothing but unit vector along the x direction because this line is along the x direction and the the line joining q one to q three is along the x axis

so r three one becomes simply one point five meters

so actually r three one vector is one point five x cap similarly what is f three two f three two is equal to one by four pi epsilon zero q two q three by r three two square r three two unit vector r three two vector is actually r three vector minus r two vector which is equal to point five into x cap because again if you can see here the line joining two to three is along the x axis

so r three two is also the same as it has the same unit vector direction x cap and has a magnitude

so magnitude of r three two vector is actually point five meters which is the distance from two to three

so i have got the total force on q three is the sum of the force on three because of one plus the force on q three because of charge two and these are the two numbers

so i can actually calculate immediately the two forces

so f three one is equal to

so i need to calculate this

so let me substitute this number one by four pi epsilon zero is nine ten to the power nine into q one was given as minus twenty nano coulomb q three is eight nano coulomb divided by the distance square which is one point five square which is two point two five and this comes out to be minus six point four into r three one unit vector

so six point four into ten to the minus seven x cap newton that's a force minus six point four ten to the minus seven x cap what does the minus sign indicate the minus sign minus x cap it is the vector direction

so that indicates that the force is in this direction and you can appreciate that because this is negatively charged this is positively charged

so it is a force of attraction

so this charge which is negative is attracting this charge which is positive and

so the force on q three because of q one is along minus x cap direction what

about f three two lets calculate that f three two is equal to one by four pi epsilon zero q two q three by r three two square r three two unit vector which is equal to nine into ten to the power nine q two is five nano coulomb and q three is eight nano coulomb divided by point five square which is point two five and this comes out to be one point four four into ten to the minus six x cap newton this is a positive force this is this forces along x cap direction which means that the charge q two is actually repelling q three because the force on q three because of q two is along the plus x cap direction the force on q three because of q one is along minus x cap direction

so that is a force of attraction

so q one attracts q three irrespective of the presence of q two the force of q one on q three is exactly given by the coulomb's law without any worries about the presence or absence of q two similarly the force on q three because of q two is again obtained by coulomb's law and it happens to be repulsive because they have the same sign charges

so i can calculate the total force on q three which is equal to minus six point four ten to the minus seven x cap plus one point four four into ten to the minus six x cap which happens to be equal to eight into ten to the minus seven x cap

so many newtons

so what does it imply this force this charge which is sitting here will feel a force which is in the plus x cap direction because the forces has a vector direction along x cap

so this force this charge is trying to attract this this charge is trying to repel it and it

so happens in this calculation that because this is closer to these two charges or closer much closer than these two this repulsive force is stronger than the attractive force of this and the net force on this charge becomes repulsive rather than attractive

so if i have this problem here if i fix these two charges this charge will be trying to be pushed out away from this point along this line now one obvious question arises is there any point in this plane at which point there is no force on the charge q three can i have a situation can i have a point on this plane where if i place q three it does not suffer any force now the first thing you can notice is because force is a vector quantity if you take the charge q three on at any point other than the x axis

so let me just put q one here q two and q three

so if i move q three from here to here this force will be like this because it is attractive this is minus twenty nano coulomb this is plus five nano coulomb and this force will be like this repulsive

so obviously these two forces cannot cancel each other even if they have similar magnitudes

so if you take any point anywhere on this plane except along the x axis you will find that these two vectors cannot cancel each other at all

so we expect the point if at all to lie on this line on the x axis of this problem

so to solve the problem let me assume that i have q one here q two here and some point here which i where i put q three and let me assume this distance is x this is one meter

so i i put a i assume the charge q three to be present at some point x such that the net force becomes zero

so two things i must satisfy one is that the force directions must be equal the force by this charge and by the force by this charge on the the charge q3 must be equal and opposite to each other

so first thing is by moving it on the x axis i have ensured that the force of q

one and q three and q two and q three are along the same line i still do not know whether they will be in the same direction or opposite directions

so first thing is by ensuring that my charge is on this plane on this line of x axis the force of repulsion of q one on q three and the force of attraction of q two and q three will be along the same line and not only they must be opposite in direction but also equal in magnitude

so let me try to find out where is the where is varies or where are the points on the x axis where the forces between q one and q three and q two and q three become equal in magnitude

so i want to have i want to look at a point where f one three magnitude is equal to magnitude f two three

so what is the force ah because of one on three

so this distance this distance is x

so i will have f one three magnitude is equal to one by four pi epsilon zero q one q three by x square

so there is no x cap this is the magnitude similarly f two three magnitude is equal to one by four pi epsilon zero q two q three

so this is all magnitude i'm putting magnitude here divided by now this distance

so that is x minus one whole square

so magnitude of this is the magnitude of force of q one on q three this is the magnitude of force of q two and q three and let me put them equal to each other to find out the value of x

so i have one by four pi epsilon zero q one q three by x square is equal to one upon four pi epsilon zero q two q three upon x minus one whole square

so i cancel this off i cancel of q three

so i get x minus one square by x square is equal to mod q two by mod q one and this q two is five nano coulomb and magnitude of q one is twenty nano coulomb

so that is one by four

so this implies x minus one by x is equal to plus minus half

so this there are two solutions one is if x minus one by x is equal to plus half this implies if i cross multiply two times x minus one is equal to x which implies x is equal to two

so this is one point where the forces of q one and q two on the q three charts are equal the second solution if i look x minus one by x is equal to minus half this implies two times x minus one is equal to minus x which implies x is equal to two by three two x plus x is equal to two

so thats x equal two by three now if i look at these two points the two solutions they seem to be like this

so i have q one which is minus twenty nano coulomb q three q two which is plus five nano coulomb and there is q three here which is plus eight nano coulomb and this is my distance x

so there are two solutions the first solution is to the right of of q two because x is two meters and this distance i know is one meter

so this first solution is two meters that means one meter from q two if i keep the charge the net force will become zero then the magnitude of the force because of q one on q three and magnitude of the force of q two and q three become equal now this point is such that remember at this point q one has a force like this and q two has a force like this because this is q one has a attractive force and q two has a repulsive force these two forces are opposite in direction equal in magnitude

so this point x is equal to two meters is a point where the force on q three becomes zero because the force of q one and q three is equal to force on q two and q three and the force because of q one and q three and because of q two and

q three are in opposite directions

so q one tries to attract q three q two tries to repel q three but with the same magnitude along opposite directions

so the result is q three has no force acting on it now i would like you to think about the second solution which is between q one and q two because that solution is x is equal to two by three meter at this point what is the solution which i have got in my problem

so please give a thought and find out what is happening at this point where the two forces become equal i leave it as a small exercise to you just give some thought and try to find out what is the significance of this point now i would like to take another example which is ah

so this example was two three charges on a same line now i want to take another example where the charges are not along the same line but in a different in a plane

so let me take as an example

so this is q one q two and q three

so let me assume nano ten nano coulomb here plus the nano coulomb plus 10 nano coulomb and plus 5 nano coulomb and just to give you some numbers let me take this as 20 centimeter ah this is 10 centimeter and this is divides equally

so this is 10 and this is 10.

so three charges are kept this is one charge here there is another charge here and a third charge

so i want to find out what is the force on q three

so now these three charges are not along the same line they are at three different points on this plane

so again just like i did in the earlier example its good for me to choose an appropriate coordinate system

so i will choose the following coordinate system

so these are my three charges one charge here another charge here another charge here

so let me take this to be y axis and this to be x axis

so this is q one q two and q three

so this is origin

so this is r one this is r two and this is r three

so what are these quantities r one vector is equal to

so please remember this is given as ten centimeters

so that is point one meters into y cap this is r one vector has a magnitude of point one meters and is along the y direction r two vector has a magnitude point one again and is along the minus y direction

so there is a minus sign here and y cap and r three vector is actually this from here to here it has the same magnitude of point one meters and is along x cap

so r one vector is point one y cap point one is this distance into y cap is along y direction r two vector is minus point one into y cap because this distance is point one meters and in the minus y direction r three vector is plus point one into x cap

so r ah three one is equal to r three minus r one which is equal to point one into x cap minus y cap x cap is unit vector along the x direction y cap is unit vector along the y direction similarly r three two will be r three minus r two which is equal to point one into x cap plus y cap

so these are the two vectors because to calculate the force on q three i have to calculate the force on q three because of q one by coulomb's law plus the force on q three by q two from coulomb's law and add to two vector forces

so i would need r three one unit vector and r two unit vector

so this is equal to r three one vector by magnitude of r three one

so r three one vector is point one x cap minus y cap divided by magnitude of r theorem vector is square root of point one x cap minus y cap into point one dot product with point one x cap minus y cap let me write it clearly here r three one unit vector is r three one divided by magnitude of r three one which is equal to point one x cap minus y cap divided by magnitude of r three one vector

so let me calculate magnitude of r three one vector this is ah square root of point one x cap minus y cap dot product with point one x cap minus y cap square root

so that is equal to point zero one into one plus one which is equal to square root of zero point zero two which is equal to square root of two times point one

so r three one unit vector is equal to one by square root of two x cap minus i cap

so r three one unit vector is actually along this direction and r three two will be along this direction

so it has both x and y components

so i leave it for you to calculate and show that r three two unit vector is actually equal to one by root two into x cap plus pi cap same magnitude it is this is x cap minus y cap this is x square plus y cap

so let me calculate the forces now

so f three is equal to f three one plus f three two which is equal to one by four pi epsilon zero q one q three by r three one square r three one cap plus one by four pi epsilon zero q two q three by r three two square r q two cap

so this is equal to now you see here that all the three charges in my problem are equal i have taken q one and q two are equal as ten nano coulomb and q three is different

so this is actually one by four pi epsilon zero q three q one by r three one square r three one cap plus q two by r three two square r three two cap now it

so happens in my problem that q one and q three are equal

so this is nothing but and not only that the magnitude of r three one is the same as the magnitude of r three two because as you can see in the problem this distance and this distance are equal because of the problem which i have taken this is an isosceles triangle and these two distances are equal

so what i get is essentially 1 by 4 pi epsilon 0 q one q three by r three one square into r three one unit vector plus r two unit vector where i have replaced i have assumed that r three two is equal to r three one and q two is equal to q one now what is r three one cap plus r three two cap we have already calculated the two quantities

so r r three one cap is one by root two x cap minus y cap r three two cap is equal to one by row two x cap plus y cap

so r three one cap plus r three two cap is two by root two x cap

so total force becomes 1 by 4 pi epsilon 0 q 1 q 3 divided by r c one square into square root of two x cap

so the force has only an x component it has no y component and you let me substitute this

so 9 into 10 to the power 9 into 10 nano coulomb into 5 nano coulomb divided by r three one magnitude square

so that was point ah r three one magnitude was point point one square root of two

so that is two into point zero two into square root of two x cap

so we can calculate this and find out what is the net force acting on the ah charge but as you can see from here the problem has nice symmetry because this charges are equal in magnitude and

so and the distances are equal from here

so this particular charge both are positive and this is also positive

so this charge will be repelling this chart in this direction this charge will be repelling in this direction with the same force magnitude and their y components will cancel each other and their x components will add and this was expected from symmetry of this problem that the net force on q 3 should have been along x axis direction

so you can substitute this numbers and calculate the total force and i will leave the discussion here for the moment and we will continue with discussion on more on superposition principle and its various applications in the next class you