

morning to all of you we will continue with our discussion on electrostatics in the last lecture we had started to discuss electrostatic potential and potential energy

so we started to discuss electrostatic potential energy and electrostatic potential electrostatic potential energy is the work done by an external agent to move a charge from one point another point

so we had derived that the potential energy of a pair of particles of a pair of point charges q and q is v u is equal to q q by four pi epsilon zero r where this is the charge small capital q the small charge q and this distance is r

so that defines the potential energy of a pair of point charges if you have multiple point charges for example potential energy of a system of of charges u will be equal to

so if your three charges q one q two by four pi epsilon zero r one two plus q one q three by four pi epsilon zero r one three plus q two q three by four pi epsilon zero r two three

so essentially you have one charge q one here another charge q two say another charge q three

so the potential energy is essentially defined by the separation between this this is r one two this is r one three and this is r two three

so we have the potential energy for a system of charges and i must mention as i mentioned last time that this potential energy is independent of the sequence at in which you are assembling the charges

so it does not matter whether you bring q one first and then q two and q three or you bring q two first and then q one and q three its independent of the sequence of uh assembling of the charges distribution and also remember that this is an energy that is contained in the complete system of charges we then defined the electrostatic potential as the work done in bringing a unit charge from infinity to that point

so for example the potential for a point charge q will be v of r is equal to q by four pi epsilon zero r where q is some charge here and r is the distance from here from the charge and that is the potential at this point ah the pointed distance r from the point charge that is the actually the work done in bringing a unit charge from infinity to this point here and remember that v of r is a scalar quantity ah we will show you i will show you later on that v of r and the electric field corresponding electric field are ah related that i must write a vector here the potential at any point and the electric field at that point are related to each other ah sometimes it is easy to calculate the potential and from the potential to calculate the electric field and we will see some examples a little later i also introduce the unit of potential its volt

so one volt is equal to one joule per coulomb that is the work down on energy in moving one charge from infinity to that point at this point i might like to mention that there is a unit of energy which is used in many places called the electron volt abbreviated as e v

so one electron volt is equal to the charge of one electron into one volt which is one point six ten to the minus nineteen joules

so that is a unit of energy it is the energy required bring to move a charge of one electron across a potential difference of one volt ah i also can relate the work done by an external force in moving unit charge from a point r i to r f is w is equal to v at r f minus v at r i

so for a point charge w will be equal to q by four pi epsilon zero one by r f minus one by r i

so the work done in moving a charge from one point another point depends on the potential difference between these two points and that is a typical relationship which tells you the work done for moving a unit charge is the difference of

potentials between these two points potentials follow a superposition principle
so if you have a number of charges q_1, q_2, q_3 etcetera and if you had a point here

so if i call this distance r_1 this distance r_2 this distance r_3

so the total potential at this point at a point p is actually equal to q_1 by $4\pi\epsilon_0 r_1$ plus q_2 by $4\pi\epsilon_0 r_2$ plus q_3 by $4\pi\epsilon_0 r_3$

so in general this is actually q_i by $4\pi\epsilon_0 r_i$ and if you have a distributional charges

so if i have a volume with some distribution of charges i can take a infinitesimal volume here with a charge dq and i want to calculate the potential at this point r at a distance r from the origin this is the origin and if i call this r' v at r is actually $\frac{1}{4\pi\epsilon_0} \int \frac{dq}{r'}$

so r' is the distance of this point from the elementary charge dq and i integrate over the entire volume or the surface or the line to get the total potential at that point

so we can use the superposition principle to obtain the total potential at any point in the presence of multiple charges

so now i want to discuss some examples to show you how i can calculate the potential

so first lets start with the first example potential of a charged conducting sphere

so i have a sphere which is a conducting sphere and it has an extra excess charge q put on it let r be the radius of the conductor

so we have actually shown before that the electric field produced by a conducting sphere charge conducting sphere is the same as the as if the entire charge was situated on the at the center of the sphere as far as outside regions are concerned inside the conductor the electric field is zero

so actually we can get the electric field here as $\frac{q}{4\pi\epsilon_0 r^2}$ into r^2 this is for r greater than r which is equal to zero for r less than r no electric field inside the conductor and an electric field of $\frac{q}{4\pi\epsilon_0 r^2}$

so r is the distance of any point from here

so i can calculate the potential at this point $v(r)$ is equal to $\int_{\infty}^r \frac{1}{r^2} dr$ which is equal to $-\frac{q}{4\pi\epsilon_0 r}$ $\int_{\infty}^r \frac{1}{r^2} dr$ which is actually equal to $\frac{q}{4\pi\epsilon_0 r}$ this is for r greater than r because the electric field which i am using here in this integration is the electric field for a point lying outside the sphere

so that is the potential as far as outside points are concerned and the potential is exactly the same as if the entire charge was concentrated at the center

so i can calculate what is the potential at the surface of the sphere which is equal to $\frac{q}{4\pi\epsilon_0 r}$ this is at r is equal to r

so the potential keeps on changing until i reach the surface of the conductor and this the variation is given by $\frac{q}{4\pi\epsilon_0 r}$ now inside the conductor there is no electric field

so i do not have to do any work in moving a charge inside the conductor from the surface to any other point inside the conductor which means that the potential inside the conductor must be the same as at the surface of the conductor remember potential is related to the work done in moving a charge

so because there is no electric field within the conductor i do not have to do any work in moving the charge anywhere within the conductor which means that the

potential inside the conductor must be the same as this

so what i see is first of all the entire conductor is at the same potential
so conductor forms an equipotential surface it is a surface where the potential remains constant and

so if i were to draw the potential as a function of position

so if this is my sphere carrying a charge

so let me try to draw a figure here which shows V as a function of r
so this is the radius as a position as a function of position if i plot you see that as i come suppose the charge is positive as i come closer you look at the potential distribution here one by r

so as i come closer to the sphere sphere small r in decreases and

so the potential increases

so the potential increases one by r from on ah here and here then inside the conductor there is no change in potential

so as i move away from the sphere the potential drops as 1 by r and inside the conductor the potential remains constant

so this is actually q by $4\pi\epsilon_0 r$

so inside the conductor potential is constant but i already have calculated before the electric field as a function of position electric field

so let me look at the same boundary here you know that electric field goes as one by r square here here the electric field goes as one by r square

so it goes faster than one by r but and it

so it rises faster like this and then the electric field becomes zero inside the conductor and then again it drops first

so you can see the electric field decreases faster as a function of r inside the conductor the electric field is zero inside the conductor the potential remains constant

so please note here that i can have regions where the electric field is zero but the potential is not zero the potential will remain constant in that region

so conductor is an equipotential surface

so let me calculate some numbers here let me put some actual values and calculate

so let me take a sphere of radius r is equal to ten centimeter which is point one meters

so this is a conducting sphere ok let me assume that we have a charge of one nano coulomb ten to the minus 9 coulomb placed on the sphere

so what is the potential on the sphere which is q by $4\pi\epsilon_0 r$ which is equal to 10^{-9} into 1 by $4\pi\epsilon_0$ is 9×10^9 divided by point one which is equal to ninety volts

so if you take a sphere of radius point one meters and put a charge of one nano coulomb on the sphere this sphere gets a potential of ninety volts which essentially means that you need to spend energy to bring a charge from infinity to this point if the charge is positive

so what is the electric field electric field on the surface is actually as you can see here electric field varies as one by r square potential varies as one by r

so the electric field on the sphere surface of the sphere must be q by four $\pi\epsilon_0 r^2$ and radially directed

so this is equal to V by r which is equal to ninety by point one which is equal to nine hundred volts per meter

so on the surface of the electric field on the surface of this spherical conductor you have an electric field which is pointing like this of nine hundred volts per meter its pointing like this here pointing if the the surface charges if the charge is positive positive q if the charge is positive electric field is

pointing away and the inside inside the conductor the potential remains constant now i must mention here a certain aspect that happens ah in in nature and that is if you have if you look at electric fields in air if the electric field becomes stronger and stronger the electric field can knock off electrons from the atoms and

so it creates a breakdown you you can see a spark taking place in air and the the in at the typical conditions the maximum electric field in air

so that there is no breakdown E_{max} is equal to three to ten to the power six volts per meter three million volts per meter is the maximum electric field you can have if you try to generate an electric field beyond this point then the there will be a breakdown and the the electric field will be

so high that you will see a spark coming out of that conductor

so you can see here that if you take a radius of 0.1 meters

the maximum potential of this conducting sphere V_{max} is equal to E_{max} into the radius of the sphere which is three ten power six into point one which is equal to three into ten to the power five volts which is written as 300 kilo volts if you in decrease the radius to r is equal to one centimeter which is point zero one meters V_{max} reduces by a factor of ten and you get thirty kilo volts

so you cannot have a a conducting sphere of radius of one centimeter and raise it to a potential greater than thirty kilo hertz because if you try to redu increase the potential by charging more the electric field becomes

so intense that there will be spark in air and the charges will get out from this from the spherical conductor

so there is an upper limit to how much of charge you can put on the conductor you can from here calculate for this radius what is the maximum charge that you can put on the spherical conductor i would like to discuss another important example and that is potential due to a dipole remember in an earlier class we had discussed the electric field produced by dipole we had calculated the electric field along the axis and on the equatorial plane and right now i want to calculate what is the potential of a dipole

so let me draw the dipole here

so this is minus q this is plus q remember the dipole moment is from minus q to plus q

so i want to calculate the potential at this point

so let me just let this point be the center of the dipole this distance let me call r let me call this distance r_1 let me call this distance r_2 this is the point p this distance is r from the center o to this point p is r this distance from minus q to p is r_2 from plus q to p is r_1

so remember potential satisfy superposition principle

so V at p must be equal to potential at p due to plus q charge plus potential at p due to minus q charge now because this distance is r_1 the plus q charge produces a potential $\frac{4\pi\epsilon_0 q}{r_1}$ and minus q produces minus q by $\frac{4\pi\epsilon_0 q}{r_2}$

so this is actually q by $4\pi\epsilon_0$ one by r_1 minus one by r_2 now let me call this angle as θ now you have all done geometry ah and calculated the relationship between the various lengths of a triangle

so let me write out the equation here r_1^2 is actually equal to $r^2 + a^2 - 2ra\cos\theta$

so this distance was two a remember we had marked a dipole as two equal and negative equivalent of the charges separated by distance

so $2a$ is the separation between the two charges

so r_2^2 plus a^2 minus $2a$ $r_2\cos\theta$ and r_2^2 is equal to $r^2 + a^2 + 2a$ $r_2\cos\theta$

so actually substituting this r_1 and r_2 in this equation I can calculate the potential at any value any point if I know the distance of that point from the center and if I know the angle made by that line joining the center of the dipole to the point with the dipole axis

so this formula can be used to calculate potential at any point and please remember potential is a scalar quantity

so I am just adding the ah the quantity is potential because of plus q plus potential because of minus q now we had also introduced a point dipole where the dipole size is very small compared to distances

so let me try to calculate an approximate expression for the potential when the distance r becomes very large compared to the size of the dipole

so if r is much greater than a I can make an expansion of this and obtain approximate expressions for r_1 and r_2

so if you look at r_1 square let me write down r_1 square again

so r_1 square is equal to r square plus a square minus $2ar \cos \theta$ which is equal to r square into $1 + \frac{a^2}{r^2} - 2 \frac{a}{r} \cos \theta$

so r_1 is approximately equal to r into $1 + \frac{a^2}{r^2} - 2 \frac{a}{r} \cos \theta$ is per half square root

so $1/r_1$ is approximately equal to $1/r$ into $1 + \frac{a^2}{r^2} - 2 \frac{a}{r} \cos \theta$ is power minus half I just inverted now if r is much greater than a then I can approximate actually these are these are exact relation they are they are not approximate they are exact now I approximate

so $1/r$ into ah you know the binomial expansion here

so I get $1 + \frac{a}{r} \cos \theta$ approximately

so I have neglected I have neglected terms of order $\frac{a^2}{r^2}$ and greater

so $\frac{a^2}{r^2}$ $\frac{a^3}{r^3}$ etcetera all these things have been neglected in writing this approximation

so $1/r_1$ is approximately $1/r$ into $1 + \frac{a}{r} \cos \theta$ similarly I can make an approximation for r_2 square

so r_2 square was equal to r square plus a square plus $2ar \cos \theta$

so I leave the exercise to you

so you can show that $1/r_2$ is approximately $1/r$ into $1 - \frac{a}{r} \cos \theta$

so $1/r_1 - 1/r_2$ is approximately equal to $2a/r^2 \cos \theta$

so $1/r_1$ was $1/r + \frac{a}{r^2} \cos \theta$ and $1/r_2$ was $1/r - \frac{a}{r^2} \cos \theta$

so when I subtract $1/r_2$ from $1/r_1$ you I get this

so I get a potential V at P is equal to q by $4\pi \epsilon_0$ into $2a$ by $r^2 \cos \theta$ remember we had defined the dipole moment magnitude of dipole moment as q times $2a$

so V of P is equal to $p \cos \theta$ by $4\pi \epsilon_0 r^2$ now let me look at the figure here

so remember here

so this is the figure let me draw here again

so I had the ah dipole like this

so this was \vec{p} vector and θ is this vector this angle

so at this point I am calculating the ah potential

so this is r cap and this is \vec{p} vector and this is θ

so what is $p \cos \theta$ $p \cos \theta$ is nothing but $\vec{p} \cdot \hat{r}$

so this is equal to $\vec{p} \cdot \hat{r}$ by $4\pi \epsilon_0 r^2$

so let me write again here

so if i had a dipole like this p and if i take a point p at a distance r from the dipole and this if this angle is θ then v at r is equal to q by four pi epsilon zero sorry $p \cdot r$ cap by four five seven zero r and this is valid for r much much greater than a which is what we have assumed in writing in deriving this equation

so two things you notice that unlike a point charge where the potential varied as one by r for a dipole the potential varies as one by r square remember we have seen this same thing in the electric field case the electric field of a point charge varies at one by r square while the electric field of a dipole varied at one by r cube

so the potential is decreasing as one by r square from the dipole and also it depends on the angle θ

so as you change θ and keep the distance of the point p constant if i move along the along the point with r is the constant then θ changes r remains constant but $p \cdot r$ will change and

so the potential will change

so the potential not only depends on the distance of the point from the dipole but also the angle made by this line with the dipole axis

so for example

so if i write in terms of θ this is equal to $p \cos \theta$ by four pi epsilon zero r square

so if you take ah θ is equal to zero v of r along this line θ is equal to zero is p by four pi epsilon zero r square θ is equal to zero is this is p this line this is θ equal to zero and for θ is equal to π b of r minus p by and

so this is please remember this is minus q this is plus q the dipole moment is a vector from minus q to plus q

so the dipole moment is pointing like this and

so the potential on this side is positive the potential on this side is negative and for θ is equal to ah π by two v of r is equal to zero π by two is this line

so along the equatorial plane the potential is zero ah you can immediately understand this because this point any point on the equilateral plane is equally distant from the plus charge and the minus charge and because the potential is the sum of potential produced by plus charge and the potential produced by minus charge and the charges have equal magnitudes the total potential on the axis is zero

so the potential of a dipole goes as one by r square and the potential also depends on the angle between the the p vector and the position where you are calculating d

so just for a summary let me look at a third example and that is i want to calculate potential of an infinite linear charge density

so i have a line charge here ah

so λ is the line charge start per unit length and i want to calculate the potential at some point here ok

so this distance is r now remember we had calculated the electric field of a infinite line charge

so let me recall to calculate the electric field i take a gaussian surface which is a cylinder of an ah of radius r the electric field by three symmetry arguments we said electric field must be pointing away from the line charge

so the electric field must be in this direction here in this direction here if the line charge is positive and

so we calculated the total flux total flux was ah $2 \pi r$ into l if the length

of this is $\oint \mathbf{E} \cdot d\mathbf{l}$ into electric field must be equal to the charge contained by ϵ_0

so we get electric field as λ by $2\pi \epsilon_0 r$ this we have already seen and electric field vector is \hat{r} where \hat{r} in this direction now the \hat{r} is a vector along a direction \hat{a}_h which is along the line perpendicular to the drawn perpendicular to the line charge

so at this point the \hat{r} will be like this at this point the \hat{r} will be like this

so that is electric field

so i can actually calculate the work done in bringing a charge from some point r_a to r_b

so let me take a point here

so this is a point a distance r_a this is a point at distance r_b

so this distance is r

so i want to calculate what is the work done

so work done is equal to minus r_a to r_b λ by $2\pi \epsilon_0 r$ \hat{r} into dot product with $\hat{r} dr$

so which is equal to minus λ by $2\pi \epsilon_0 \hat{a}_h$ r_a to r_b dr by r which is equal to λ by $2\pi \epsilon_0$ \log of r_a by r integral of one by r dr is actually \log and i have taken care of the sign by reversing the \hat{a}_h the inside the \log

so the work done in bringing a charge from r_a to r_b is essentially λ by $2\pi \epsilon_0$ $r_b \log$ now you already see a problem here and the problem is that if your reference point happens to be infinity which means if i take r_a to be infinity

so this is work done in bringing a charge from r_a to r_b

so if i start from infinity i say i see there is an infinity within the \log and there is a problem and that problem is appearing because the line charge density itself is extending over in finite length

so in such situation in problems in which the charge distributions extends to infinity which is of course not practical because usually in practice all charge distributions are finite but in mathematics we tend to use certain distributions in which the charge density is extending over infinity for example in finite line charge or infinite plane sheet etcetera and these are useful to calculate electric fields and potentials but in such situations you will find an infinity of potential at ∞ at infinite distances from the charge distribution

so in these cases what we do is we change the reference point and we say that instead of using reference point to be infinity we will say that we will use the zero potential at some at some r value

so if i say

so let v is equal to zero at r is equal to r_a is equal to capital r and we let the final point to be r

so we will get v of r is equal to λ by $2\pi \epsilon_0$ \log capital r by small r because potential is a relative quantity it is like in cavitation potential a potential at a certain height is measured with respect to potential zero potential on the earth surface

so you can measure the potential differences between points they will not depend on the origin of the reference

so here what we have done is because the potential tends to infinity at infinity we have restricted and said that we will choose the zero potential to be at a finite distance from the line charge distribution and which have which i have chosen as capital r

so you can see if you put small original capital r \log one is zero and you get potential as zero

so that is the original potential

so i brought this example just to indicate you that there may be situations where the potential may be tending to infinity at fault of distances and

so i may have to choose a different reference point for zero potential now i want to bring in some very interesting aspects which is the equipotential surfaces now we have earlier introduced the concept of electric field lines

so we represent electric field distribution by electric field lines

so these are lines of line curved lines in which which are such that the electric field at any point is the is directed along the tangent to that line and the closer the lines are the stronger the electric field is the further they are less the electric field is we can similarly represent potential through what are called as equipotential surfaces

so this is a graphical representation

so what we do is we draw surfaces on which the potential remains constant

so i take all those points for which the potential happens to be say v is equal to v one look at all the points and join them and get a surface similarly i take a surface corresponding to v is equal to v two v is equal to v three and

so on

so i draw surfaces which are such that all points on that surface are at a constant potential

so these are all three dimensional surfaces unlike electric field lines electric field lines are lines and these are complete surfaces

so also notice that since its an equipotential surface suppose the equipotential surface happens to be like this

so the potential on all these points are exactly the same

so i would have to do no work in moving along the potential equipotential because the potential is the same

so potential at this point and at this point are the same

so i need not do any work in moving a charge from here to here

so it implies that there can be no electric field component along the equation surface

so electric field at every point must be perpendicular to the equal potential surface please look at this argument that if i had an equipotential surface then all points on the surface have the same potential

so the work done in moving a charge from one point of the surface to another point on the same surface must be zero because they are at the same potential and potential difference gives me the work not necessary to move a charge from one point another point

so because the electric because the potential is the same there must be no electric field along the direction of motion whatever direction i choose i move like this or like this or like this in any direction if i move on the surface i would have to do no work in moving the charge which means the electric field must be perpendicular like this

so at this point it must be like this if the surface is like this it must be like this

so electric field lines are always perpendicular to the potential equipotential surfaces and this is very very important

so equipotential surfaces and electric field lines form to ah are always perpendicular to each other

so let me take an example of a point charge

so suppose i take a point charge here is a point charge q

so remember for a point charge potential is q by four pi epsilon zero r

so if you take

so r is the distance from the point charge

so if you take points which are at the same distance from the point charge they will have the same potential

so for example at r is equal to r_1 V must be V_1 is equal to $V_1 = \frac{q}{4\pi\epsilon_0 r_1}$

so if you take a distance r_1 from here this this all point on the sphere are equipotentials similarly if you take r is equal to r_2 V is equal to V_2 is equal to $V_2 = \frac{q}{4\pi\epsilon_0 r_2}$ that's another sphere that's another sphere

so equipotentials for a point charge are spheres i am drawing a circle in a two dimensional space but you have to imagine this entire thing is rotated around the point charge here

so if i rotate along any axis containing the point charge i this circles will become spheres and all points on the sphere are at equal potential

so for a sphere of radius r_1 the potential is $V_1 = \frac{q}{4\pi\epsilon_0 r_1}$

so this is an equipotential surface that's an equipotential surface and as you know the electric field of a point charge is radial is like this and as you can see here the electric field is always perpendicular to the equipotential surface

so if the charge is positive the arrows are pointing outwards if the charge is negative the arrows are pointing inwards

so i leave it to you to calculate whether in this case

so r_2 is greater than r_1 what about if V_2 is greater than V_1 or V_1 is greater than V_2 please think over it which potential is larger is the potential here larger than here or the potential here is smaller than here

so i leave this problem to you to kind of to find think about it to find out whether the potentials the equipotential surface with a larger radius is at a smaller potential or a higher potential if i have a charge a positive charge here or a negative charge for example

so that is equal potential for a point charge if i had uniform electric field lines

so suppose i had electric field lines like this pointing in the direction a uniform electric field then equipotentials as you can see will be planes perpendicular to this line

so if the electric field happens to be E is equal to $E_0 \hat{k}$

so let me call this as z direction

so the electric field lines are along the z direction \hat{k} direction

so the equal potentials must be parallel to the $x-y$ plane

so this is $x-y$

so equal potential are planes which are perpendicular z axis here because the the electric field is along the z axis

so i have i can show you some figures here two figures which i will show you ah showing the equal potentials for a point charge and

so this is this is equal potential for a point charge

so they are all spheres and that is the center which is the positive which is the charge this black dot is the charge and the equipotential surfaces are all spheres surrounding the charge and as as i have drawn in the earlier case the electric field is radial like this ok from the point charge ah i also plotted equipotentials for a dipole this is calculated from the expression that we had written earlier essentially this equation

so you take different points

so you calculate those points for which this potential remains constant

so as i move the point r_1 and r_2 must vary such that $\frac{1}{r_1} - \frac{1}{r_2}$ remains constant and i can draw

so these are equal potential surfaces and as you can see here the ah these are

actually surfaces

so i can imagine the surface by rotating about this axis

so the electric field lines will be perpendicular

so for example here the electric field will be like this here the electric field will be like this at this point the electric field will be ah like this the electric field here will be like this or every everywhere it is perpendicular its like this

so the the direction of electric field will depend on ah will be perpendicular to all the equipotential surfaces

so you can actually plot equipotential surfaces for different charge distributions by calculating electric field by calculating the potential and from there you can plot equipotential surfaces and you can verify that electric field distribution is always at every point perpendicular to the equipotential surface now a little while ago i mentioned that electric field and the potential are related to each other

so let us try to derive an expression relating electric field and potential

so i want to consider consider two adjacent equipotential surfaces

so let me draw something like this

so this is a potential v naught and there is another surface here p naught plus Δv v naught plus $d b$

so these are two potentials which are very close to each other

so v naught and v naught plus $d v$

so as we know electric field at this point will be perpendicular it will be like this this will be the direction of electric field it has to be perpendicular to the tangent to this line

so this must be like this perpendicular

so now i want to do the following i have a charge here a unit charge which i move like this in some direction let me call this $d l$ vector i move a direction making an angle θ with the electric field direction

so what is the work done by the external force in moving a unit charge from let me call this point a this point b from point a to point b

so remember these are equal potential surfaces here the potential is v v naught here the potential is v naught plus $d v$

so work done must be equal to v naught plus $d v$ the potential at b minus potential at a which is equal to $d v$ work done in moving a charge from a to b is potential at b minus potential at a which is v naught plus $d b$ minus v naught which is d now i also i also know how to calculate work done from electric fields

so work done is also given by $\int \mathbf{e} \cdot d \mathbf{l}$ e is the electric field

so the force on the charge is e vector

so i must apply a force which is opposite to the direction of electric vector which is $-\mathbf{e}$ and i am moving a distance $d l$ from here and what is $\mathbf{e} \cdot d \mathbf{a}$ $-\mathbf{e} \cdot d \mathbf{l}$ which is nothing but $-\mathbf{e} \cdot d \mathbf{l} \cos \theta$ and these two must be equal

so what i get an expression a i see that $\mathbf{e} \cdot d \mathbf{l} \cos \theta$ is equal to $-\mathbf{e} \cdot d \mathbf{l}$

so i can write

so $d b$ the difference in potential between these two points which is also given by $-\mathbf{e} \cdot d \mathbf{l} \cos \theta$

so i get a following expression that $\mathbf{e} \cdot \cos \theta$ is equal to $-\mathbf{e} \cdot d \mathbf{l}$ $d \mathbf{l}$ is the element of length magnitude element of length and i am moving ah $d \mathbf{l}$ vector is the length of the vector $d l$ is the magnitude of the element of length and i am moving in a direction defined by $\cos \theta$ θ is the angle between the electric vector and the direction $d \mathbf{l}$

so for example if i ah

so this is a general relationship

so let me assume i have a figure like this i have an x axis here and a y axis here

so the equipotential happens to be like this

so if i move from here to here along parallel to the x axis

so this is v naught this is v naught plus $d v$ if i move parallel to the x axis error vector is like this then $e \cos \theta$ is not if this is θ $e \cos \theta$ is nothing but x component of electric vector

so my movement $d l$ now is along x axis parallel to the x axis

so $d l$ will be along x axis and θ will be the angle between the e vector and x axis

so $e \cos \theta$ is nothing but the x component of electric field which is equal to minus $\frac{d v}{d x}$ i am writing partial derivative because the potential depends in general on all coordinates x y and z

so the electric field component along the x axis is nothing but minus $\frac{d v}{d x}$ similarly if i move along the parallel to the y axis if i move like this i can relate e_y as minus $\frac{d v}{d y}$ and e_z is minus $\frac{d v}{d z}$ three useful relationships which relate the potential to the electric field

so you can see here the rate of change of potential with respect to x axis with respect to x is the negative of the electric field component along x axis the rate of change of v with respect to y with the negative of that is the electric field along the y axis and $\frac{d v}{d z}$ is minus z

so the three electric field components are related to the potential variation as a function of x y z

so this is what i meant before if i calculate the potential distribution as a function of x y z i can from that expression calculate the electric field distribution

so in many situations it is easy to calculate potential distribution because potential is a scalar quantity and when i integrate its much easier to integrate a scalar quantity in the electric field case i must calculate e_x separately e_y separately e_z separately because electric field is a vector

so let me show you an example of this calculation of electric field relationship

so so potential for point charge

so v of r we have already calculated is q by four pi epsilon zero r where q is here and r is the distance

so if in terms of coordinates i can write that this is four pi epsilon zero $x^2 + y^2 + z^2$ this per half but this point has coordinates x y z and this is the origin

so let me calc let me write here x

so this is x y and z

so you can the distance small r is the distance from the origin which is square root of $x^2 + y^2 + z^2$

so for example i can get e_x is equal to minus $\frac{d v}{d x}$ which is equal to minus q by four pi epsilon zero into you can differentiate this $x^2 + y^2 + z^2$ this power three by two there will be minus half sign and there will be a sign of two x differential of $x^2 + y^2 + z^2$ square is two x with respect to x partial derivative with respect to x and i get this equation which is equal to

so two factor goes off and i get q by four pi epsilon zero ah

so let me write like this ah $x^2 + y^2 + z^2$ into x by square root of $x^2 + y^2 + z^2$

so this is nothing but q by four pi epsilon zero r^2 square and this is this is x by r similarly you can calculate e_y and e_z and from there i will show you that

the total electric field is actually exactly what we had obtained before now i want to close the lecture with a small problem an electric dipole of moment p is equal to $10 \text{ kC}\cdot\text{m}$ is located at the origin in free space calculate the potential at a point P with coordinates x_P is equal to point five meters y_P is equal to zero z_P is equal to point eight seven meters

so i leave this problem to you please calculate the potential at this point P so you have a dipole located at the origin which is oriented along the z axis here and you need to calculate what is the potential at this point you

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