a very good morning to all of you we will continue with our discussion in the in electrostatics ah let us recall in the last lecture we had discussed about capacitors and capacitance

so capacitors are made up of two conductors separated by air or by an insulator and they carry equivalent opposite charges

so plus q and minus q and this particular device is called the capacitance capacitor and its it stores charges and it actually stores electrostatic energy we had calculated the capacitance of a parallel plate capacitor a cylindrical capacitor and a spherical capacitor

so today what i want to do is to calculate how much energy is stored in an in a capacitor

so the topic today is electrostatic energy stored in a capacitor

so ah as before i have two conductors one carrying charge plus q the other carrying charge minus q equal to opposite charges there are electric field lines between these charges and the we start with two conductors which have no excess charge and we slowly move charges between the two conductors

so that one of them gets positively charged the other gets negatively charged

so we are moving electrons from here into this conductor to leave positive charge here and an excess negative charge here and we this process is called charging the capacitor

so the capacitor is connected to a battery and that battery charges the capacitor

so the question is when i charge the capacitor how much energy stored in the capacitor

so to calculate this let us follow the following procedure

so suppose at the end we have a charge plus q and minus q and charge is q and potential difference b and we know that k q is equal to c times v but c is the capacitance

so now i start with a pair of conductors which have which are neutral and no excess charge now as i start to move charge electrons from this conductor to this conductor i start doing work because the electrons are being pulled by the conductor here and i have to move the electrons in a away from its star its force

so by well for moving the electrons from here to here i must do charge and that is the cha energy which is stored in the capacitor

so let me assume that at some instant of time the charge is q and the potential is given by v is equal to q by c but c is a capacitance

so at some instant of time there is a charge plus small q and minus small q on the two conductors they have a potential difference given by v is equal to by c but c is the capacitance now at this point to further increase the charge i move a small infinite decimal charge dq from one capaci one conductor to the other conductor

so i move an infinite decimal charge d q and because the potential is v the work done in moving charge d q will be v times d q which is equal to q d q by c

so i replaced the potential difference between the two two conductors by q by c and i get this

so i start with two conductors with zero charges and keep on moving charge from one to the other to finally charge the capacitor to a plus q and a minus q and

so the total work done in charging from zero to capital q will be w is equal to zero to q q by c d q which is equal to one by c integral zero to q q d q which is nothing but q square by two c

so the external agent has to do a work amounting to q square by two c in charging the capacitor and it is this work done that is stored as electrostatic energy within the capacitor

so energy stored is equal to u is equal to q square by two c

so this is why

so while charging the capacitor i am doing work and that work which i am doing is stored as electrostatic energy in the capacitor this formula is very similar to the energy potential energy stored in a spring mass system

so if i was if i have a mass connected to spring with spring constant k remember that the energy stored in pulling the string by an extension x is half k x square

so the displacement plays the role of the charge here and k is something like one by c in this equation

so just like a pulled spring stores energy a charge capacitor stores energy and that is the energy stored in the capacitor now using the relationship q is equal to cv i can write the energy stored in another form

so q square by two c which is equal to one by two c q is equal to c v

so this is c square v square which is equal to half c v square thats another form of electrostatic energy i can also write it in a in a different form i replace only one of the q by c c b

so which is equal to half q b

so there are three forms of energy i can have energy is equal to q square by two c or energy is equal to half c v square or energy equal to half q b all of them are equivalent and we can use any of them at any given time depending on the problem i will use one of these equations to calculate the energy stored now i can actually calculate i can put this energy in a slightly different form by taking the example of a parallel plate capacitor

so remember in a parallel plate capacitor having an area a and a separation d suppose this is positive charge here and there is negative charge here there are electric field lines coming downwards and

so the energy stored is q square by two c or equal to half c b square now c is equal to epsilon zero a by d we have already calculated and b is equal to e times d

so i can put u is equal to half epsilon zero a by d into e square into d square is the electric field between the capacitor plates and ah

so this is equal to half epsilon zero e square into a times d

so i write it in this form separate the factors into two parts half epsilon zero e square into one d cancels off and i get a times d now what is a times d this is volume enclosed

so if i look at this energy i can interpret this equation by saying that the energy stored in the form of an electrostatic field and the energy density or the energy per unit volume stored in the electrostatic field is given by half epsilon zero e square this is the volume of the capacitor and that is the

so if i multiply this quantity by the volume i get the total energy

so this must be the energy per unit volume

so if i have an electric field inside the between the capacitor plates i have an electric field e and i find that i can interpret this equation as if half epsilon zero e square is the energy per unit volume stored in an electrostatic field although i have derived this equation for a parallel plate capacitor this is a very general equation and

so if you have an electric field e at any point the electrostatic energy that is contained in that electric field is half epsilon epsilon zero e square if it is free space

so that is energy density and this is a nice way of interpreting this electrostatic energy now although i have derived that for a parallel plate capacitor i want to take an example another example and to show that this equation will also be working correctly

so i take a spherical capacitor

so remember i had a spherical capacitor there was a conductor here and there is another conductor outside

so r_a is the radius of this conductor and r_b is the radius of that conductor

so let me draw charges here

so i had plus charges here and i have minus charges in the outside

so the outside conductor is a finite thickness contour

so we have already calculated the capacitance of this capacitance of the spherical capacitor is four pi epsilon zero r_a r_b by r_b minus r_a in an earlier class we had calculated the capacitance of a spherical capacitor ah c is equal to four pi epsilon zero r_a r_b by r_b minus r_a

so the energy stored u is equal to one by two q square by c which is equal to q square by eight pi epsilon zero r_a r_b into r_b minus r_a

so using the formula q square by two c i get an expression for the energy stored in a spherical capacitor as q square by eight pi epsilon zero r_b minus r_a by r_a r_b

so thats one way of calculating now let me calculate the energy stored in the electric field which lies between the two conductors and show you that if i assume the electrostatic energy density is as half epsilon zero e square i will get the same expression for the total energy stored

so let me redraw the capacitor again

so i have the inner conductor here and the outer conductor

so inner conductor is positively charged

so i have positive charges here and the outer conductor has negative charges now

so to calculate

so energy density i know is equal to half epsilon zero e square now first

so for this to use this i must calculate the electric field at different points

so first thing remember that this is the conductor here that is a conductor

so the entire electric field in this configuration lies between the distance r_a and r_b there is no electric field within this conductor there is no electric field within this conductor there is no electric field anywhere else except in the region between r_a and r_b and to calculate this i must know the electric field

so what we have done before is exactly like this

so i take a gaussian surface of radius r and calculate the flux crossing this

so flux crossing is four pi r square into e which because e is radial and

so the flux crossing the spherical surface is four pi r square into e which is equal to charge enclosed by epsilon zero

so e is equal to q by four pi epsilon zero r square

so we have seen this before this particular spherical charge distribution actually is equivalent to a point charge situated at the center of the sphere that's the electric field and please now note that the electric field depends on the position in a planar in a parallel plate capacitor the electric field was uniform here the electric field depends on position

so i just cannot multiply this number by the volume i must integrate

so what i do is i take

so this is my inner conductor here

so i take a surface lying between ah a volume lying between r and r plus d r

so this is r plus d r

so in this volume i want to calculate the energy and then i will integrate over the entire distance from r to r_b

so what is the energy contained in the volume between r and r plus d r the energy density of epsilon zero e square which is q square by four pi epsilon

zero whole square r s power four e square into the volume of this which is four pi r square into d r the area of the sphere into the thickness which is four pi r square d r

so this gives me equal to q square by number one four pi sine zero cancels off and i get eight pi epsilon zero into d r by r square

so one r square cancels off and i get h q square by eight pi epsilon zero d r by r square

so as you can see the energy stored changes with position because the electric field is strong here

so there is more energy density the electric field is weaker gets weaker as you go away from the center and the energy density keeps on decreasing

so that is energy density energy lying between r and r plus d r

so the total energy total stored energy u is equal to integral r a to r b q square by eight pi epsilon zero d r by r square which is equal to q square by eight pi epsilon zero into one by r a minus one by rb which is equal to q square into rb minus r a by eight pi epsilon zero r a r b and if you compare this with the expression that we had just now obtained from the other equation you see this equation here they are the same equation q square by eight pi epsilon zero into r b minus r a by r a r b

so please note that either of the formulations gives me the same total energy contained in this case i had to be little careful because the electric field is not uniform

so the energy density stored in the electric field electrostatic field changes with position

so when i calculate in such a case i must calculate the electric field at different points and then i will get the energy density at different points and then i have to integrate over the entire volume where there are electrostatic field exists

so these are two ways of calculating and this tells me that energy is stored in the ah in a in an electrostatic field and

so when a capacitor is charged i store electrostatic energy and that energy can be discharged at any later time from the capacitor and

so the capacitor does work and releases that energy now i want to discuss something about dielectrics and polarization remember we had discussed dielectrics are those materials in which there are no free electrons

so the unlike conductors in conductors there are free electrons the outermost electrons of the atoms are freed from the atom and they are free to move anywhere within the conductor

so when you put a conductor in an electric field the electric field then applies the force on these charges on the electrons which then move because of the electric field and they continue to move until the electric field within the conductor becomes zero

so if we have seen that in the static case there cannot be any electric field within the conductor now in a dielectric there are no free electrons but there are atoms which contain charges

so usually as we have discussed before the center of the negative charge of the electron cloud surrounding the nucleus and the center of the positive charge of the nucleus are coincident at the same point

so you do not see any electric field from the atom but when you put an atom in an electric field the atom gets polarized

so what happens is you may start with an atom with a positive and negative charge which are coincident at the center

so when you apply an electric field like this then what happens is you have a small separation of negative and positive charges and this form the dipole we

have seen this is a dipole and it is characterized by dipole moment
so when you place a dielectric within a an electric field the atoms get
polarized and the we say that the dielectric gets polarized in this process
so we will call such a dielectric to be polarized dielectric
so placing a dielectric within an electric field immediately polarizes the
dielectric
so now first let me see what happens if i had a uniform electric field and i
had a block of conductor
so i had a conducting block here and i had electric field in this direction a
uniform electric field for example i place this conductor between the plates of
a parallel plate capacitor
so i have this conductor now the moment i apply an electric field on the
conductor inside the conductor there is instantaneously some electric field that
electric field now will move the char electrons and the electrons will get
accumulated on one side leaving a net positive charge on the other side
so you have electrons attracted here to this side and there will be net
positive charge on the other side of the conductor we have seen this before
so this leaves a surface charge density on the conductor and the charges will
continue to move until the net electric field within the conductor becomes zero
so if the electric field which i have applied is e naught and if sigma is the
ah surface charge density here the electric field because of these two surface
charge densities is sigma by epsilon zero and that must be equal to e naught
so e naught is like this and the electric field because of the the surface
charges is like this and they must be equal to cancel the electric field here
so i generate a surface charge in c which is epsilon zero e zero
so this surface charge density creates gets created which creates its own
electric field
so the electric field within the conductor becomes zero that is the story of a
conductor now what happens if i place a dielectric inside an electric field
so let me take a dielectric now
so i have a dielectric and i again have a uniform electric field applied in
this upward direction now as we have seen in the absence of electric field there
are atoms within the dielectric whose positive and negative charges coincide the
moment an electric field is applied the negative charges get attracted and i
will schematically draw each of these atoms as a over shaped object here this is
essentially denoting each of the dipoles which is formed because of the
application of the electric field and i know that the charges will be plus on
the upper side and minus on the lower side
so the electrons get attracted in the downward direction because of the
electric field leaving a net positive charge
so there are each atom becomes a dipole with the dipoles pointing upwards
so remember the dipole moment of a dipole is a vector joining the minus to the
plus charge
so these are all dipole small small dipoles which have the dipole moments
pointing upwards now
so i have just drawn some atom there are billions of billions of atoms in the
dielectric
so this dielectric gets polarized this is supposed to be a polarized dielectric
now you can see if you take any small volume within the dielectric volume is
small compared to the size of the dielectric this the the measure the transfer
size of the dielectric but small large compared to the atomic spacing you can
see that there will be an equal number of positive and negative charges within
that volume
so effectively there is no volume charge density created within the electric

within the dielectric but look at the surface at the surface here there are negative charges left which are not compensated by positive charges similarly there are positive charges left on the upper surface which are not compensated by the negative charges

so the moment i put a dielectric in an electric field the atoms get polarized each atom becomes a small dipole moment and you are left with a surface charge density on the lower side negative surface charge density on this side and a positive surface has density on this side

so the result of polarizing a dielectric is to leave surface charge density on both surfaces in this in this diagram this particular surface and this particular surface

so this dipole now will create its own electric field

so there are charges negative charges on the lower surface positive charges net charges on the upper surface that produces a downward looking electric field which partially cancels this electric field in a conductor the downward directed electric field is equal to the upward directed applied electric field require resulting in a complete cancellation in the dielectric case as you will see the cancellation is partial

so let me assume that in the dielectric i am left with a a positive charge a positive charge on the upper surface

so the upper surface has a net positive charge and the lower surface has a net negative charge and there is no other volume charge density inside

so i have another uniform electric field

so this results in what i would call as bound surface charge density

so sigma b and minus sigma b sigma b is the bound surface charge density it is called bound surface charge density because these electrons are not freed from the atom they are still attract attached to the atom only thing that has happened is they have got a little stretched the center of the negative charge has got displaced with respect to center of the positive charge with the result that is each atom becomes a dipole and when this dielectric gets polarized i am i it results in a bound surface charge density on these two surfaces and we say that the dielectric is polarized and to quantify this quant this this polarization we define a vector called polarization which is denoted by p this is the dipole moment per unit volume

so you take a small unit volume or you take a small volume delta v calculate the total dipole moment of delta v and divide the dipole moment by the delta v to get the polarization of the of the dielectric and ah this polarization is caused by the external electric field

so inside the dielectric there is an electric field e and there is also polarization p

so this polarization vector must be proportional to the electric field applied and

so we have a relationship like this epsilon zero chi e this is epsilon zero is the permittivity of free space and chi is called the electric susceptibility it measures how much susceptible is the dielectric to polarization

so that is a susceptibility

so p is called the polarization and its related pos is proportional to electric field now this relation is true for small electric fields and if your electric fields become very strong then this equation breaks down and we need to modify this equation but we will not discuss that here for small electric fields which are usually found polarization of the dielectric is proportional to the electric field and as this vector relationship shows that p and e are in the same direction now

so let me calculate

so how do i relate polarization to the surface charge density
so we have we have shown that there is a bound surface charge density and this bound surface charge density comes because of polarization
so how is the speed related to the surface charge density
so to calculate this what i do is the following i take a cylinder a small cylinder of length l area a and polarized like this
so let me assume that the two ends of the cylinder are at right angles to the polarization the polarization is along the length of the cylinder and the polarization in the dielectric is equal to p
so let me write a scalar relation to my nodes along this direction
so the polarization is p and volume is a times l
so the dipole moment of this cylinder is p times a times l p is the dipole moment per unit volume a times l is the volume of the dielectric a surface area multiplied by the length and p times a times l is the dipole moment of the cylinder now i can also write this dipole moment in a slightly different form let me assume the charges are cube minus q and plus q here
so if i have two charges plus two and minus two separated by length l i can write the dipole moment as q times l
so this implies that q times l is equal to p times a times l or q is equal to p times a
so q is the charge accumulated on this side plus q minus q and the surface area is a
so i can define i can get the bound surface charge density sigma b is equal to q by a which is equal to p
so when i have this when i look at the cylindrical object which is polarized along the axis of the cylinder i find that the surface charge density here bound surface charge density on both sides as is minus sigma b and plus sigma b
so the surface bound surface charge density is related to polarization like this and in this example i have assumed that the surface end surface is perpendicular to the polarization vector now you may not have always the same situation you may have surfaces which are not perpendicular to the polarization to find out what happens in the situation like this
so let me draw the same cylinder here now now the surface happens to be at an angle
so let me assume that this angle is theta the polarization is still like this theta is the angle theta is angle made by made between this inclined surface inclined area and this normal
so i can define as we have defined before a unit normal along the area and polarization vector is like this and this is theta now please remember just like before the charges which are accumulating here is minus q and on the surface is also plus q the same charge gets accumulated on the other surface now the area instead of being a is a by cos theta this area is larger than this area this area is perpendicular area thats an inclined area
so if a if an inclined area that area is larger than this area a and its a by cos theta
so the bound charge density now becomes sigma b is equal to q by a by cos theta which is equal to p cos theta because q by a is sigma is p into cos theta which is equal to p dot n p vector is like this n vector is the normal to the surface is output normal the surface the volume of the of this dielectric is here n cap is the outward normal and p cos theta is nothing but p dot n
so this is a specialized relation when p and n are parallel but in general if you have a surface in which which is ah on the one side of which the dielectric with the polarization p they it creates a surface charge density bound surface charge density of p dot n

so you can see in this example on the on the left side this side n vector is like this n cap and p vector on this is like this
 so p dot n is minus
 so you have a minus minus charge density here on this surface cylindrical surface n cap is like this p is like this and p dot n is zero
 so there are no surface charge densities on the cylindrical surface because that is parallel to the p vector and the normal to the surface is perpendicular to vector and p dot n becomes zero on this surface which is inclined at an angle theta this surface bound surface chance density is p dot n which is sigma v
 so that is a very general relationship whenever you have a polarization p in a dielectric the it creates a bound surface charge density of p dot n
 so this this relationship is useful for us to analyze ah dielectrics electrostatics in dialect with dielectrics
 so now i want to calculate the capacitor with a capacitor with a dielectric all of our earlier discussions we assumed that the capacitor plates are have no they have just placed and there is air or vacuum in between we do not consider any medium to be present within the place of the capacitor now i want to have a capacitor in which i am going to assume that there is a dielectric kept filling the entire space
 so i have a dielectric here within the entire space of the between the parallel blade capacitor
 so again let me draw positive charges here and there will be negative charges on the plate here
 so there is an electric field in the downward direction
 so this will result in a negative accumulation of negative bound charge here and an accumulation of positive bond charge here
 so let me write here
 so this is plus sigma f this is minus sigma f this is minus sigma b this is plus sigma b on the surface of the dielectric we have bound surface charge densities which i am calling minus sigma b and plus sigma b on the surface of the conductors we have free charges which are plus sigma f and minus sigma f now i want to calculate what is the electric field within the dielectric
 so we follow the same procedure as before we use gauss's law
 so i take a gaussian surface like this i take a gaussian cylindrical surface with an area a and in the vertical direction now you can see here because of the symmetry of the problem the electric field will be downwards there is an electric field created by the charges plus and minus on the conducting plates which is acting downwards there is a electric field created by the dielectric polarized dielectric which is upwards but as we have seen before this we will see that this cancellation is not perfect
 so there is some some electric field still left within the within the dielectric unlike a conductor where the electric field should be zero there is no such condition for dielectrics
 so the electric field lines are like this
 so this is the gaussian surface there is no electric field within the conductor
 so the flux on the surface is zero this cylindrical surface of the con of the gaussian surface is parallel to the electric field
 so there is no flux on that there is only a flux from here
 so if e is electric field e into a which is the flux must be equal to the charge enclosed now the charge enclosed has two components free charges here and bound charges here
 so the total charge is sigma f minus sigma b into area because that is the surface surface charge density and c is the surface charge density multiplied by area the charge divided by epsilon c

so thats gauss's law the electric flux through any closed surface is equal to charge enclosed by epsilon zero that's what i am using here

so a cancels off and i get epsilon zero a is equal to epsilon zero e is equal to sigma f minus sigma b now sigma b we have just now shown sigma b in this case will be ah p which is equal to epsilon zero chi e

so i have epsilon zero e plus sigma b which is epsilon zero chi e is equal to sigma f

so this implies epsilon zero into one plus chi into e is equal to sigma f

so chi is a

so we define new quantities now i define dielectric constant k as one plus chi and then i have permittivity of the dielectric epsilon is equal to epsilon zero into one plus chi which is equal to epsilon zero into k epsilon zero is the permittivity of free space epsilon is the permittivity of the medium of the dielectric and

so epsilon n is equal to epsilon zero times k and k is usually greater than equal to one k is equal to one for free space or vacuum and k is always more than one

so if i go back to this equation i get the electric field in the dielectric as e is equal to sigma f by epsilon zero k which is equal to sigma f by epsilon and you can see here that because k is more than one this electric field is smaller than the electric field when there was no dielectric placed within the plates of the conductor between the plates

so the electric field within the dielectric actually smaller than the electric field which was created in free space and this the reduction is by a factor k which is the dielectric constant of the dielectric

so let me take an example here

so before that i will give you some values of dielectric constants of standard materials which are used in various capacitors

so i have a table here make area and k

so pyrex glass is a type of gas 4.
7 polystyrene which is 2.
6 paper is 3.
5 porcelain which is six point five titanium ceramic one thirty

so there are dielectrics with very strong strontium titanate which is even larger 310 and its interesting to know the dielectric constant of water which is eighty point four

so these are some ah dielectric constants of some dielectrics which are used either in capacitors or otherwise we would we should know some of the numbers and we can see here there is a wide variety of dielectrics with dielectric constant varying right from almost close to one in fact air at the dilatory constant which is very very close to one slightly more than one very close to one right up to a couple of hundred

so its a very wide range of materials and depending on the kind of capacitance i need i can use different dielectrics for for the capacitance

so let me take an example here ok

so we have calculated this electric field as e is equal to sigma f epsilon

so let me go back and look at what is the capacitance of this capacitor law

so i have the dielectric filling this space between the

so electric field becomes sigma f by epsilon now sigma f is equal to q by a is the area of the plates and e is equal to potential difference divided by the separation

so b by d is equal to q by a epsilon this implies v is equal to q into d by a epsilon

so we we know the capacitance is given by q is equal to c v q is equal to c

times v
so the capacitance of this capacitor with the dielectric filling c is equal to
so c is q by v which is epsilon a by d remember when the space between the parallel plates was filled with free space the capacitance was epsilon zero a by d now the capacitance is epsilon a by d which is actually epsilon zero k a by d because epsilon the permittivity of the dielectric is epsilon zero times the directory constant
so the capacitance as increased by a factor k
so as an example let me take ah
so here is an example
so area of plates 100 centimeter square and spacing between the plates is one centimeter
so with air first air separating the plates epsilon is equal to epsilon zero approximately and you can calculate the capacitance c air is equal to epsilon zero a by d which comes out to be eight point eight five eco farads if you put a dielectric and dielectric constant is two point six then the capacitance with the dielectric becomes about 23.
01 picofarad
so there is an increase in capacitance by the factor the dielectric constant k and the capacitance increases by filling the capacitor capacitor with dielectrics
so actually if you want to have a higher capacitance we can use dielectrics to fill and increase the capacitance ah let me try to clarify further here
so let me take for example the same capacitor but instead of filling the entire space with the dielectric suppose i had dielectric only partially filling and i had positive charges here negative charges on the lower plate of the conductor
so see that these are the two conducting plates
so this induces a negative bound charge here and that induces a sorry positive bound charge here a negative bound charge on this side
so you have an electric field pointing downward here pointing downward here and also pointing downward here but slightly weaker
so you can actually calculate the electric field in this in this space e a
so if i call this sigma f and this is sigma b e a is equal to sigma f by epsilon zero and if the dielectric constant of this is k the sigma dilat e dielectric is equal to sigma f by epsilon zero times k which is equal to sigma f by epsilon and you can see that e a is greater than e of the dielectric
so what the dielectric does is gets polarized a polarized dielectric creates an electric field in the opposite direction as the applied electric field and there is a partial cancellation in a conductor this cancellation is complete while in a dielectric the cancellation is only partial ok
so we can look at various examples but before i do that i would like to discuss a very important issue which is gauss's law in dielectrics till now we have discussed gauss's law ah with no medium in between
so we had a charge surface charge density for example or a conductor which is charged or a point charge set of point charges all in free space and we never took in account of any medium now i want to find out what happens to gauss's law in the presence of a dielectric
so to understand what happens let me consider the following situation i have a this this is the conductor and this is the dielectrics is a conductor and this is dielectric
so let me assume there are positive charges on the on the surface of the conductor this will lead to a bound surface charge density on the surface of the conductor
so this is sigma f and this is sigma b pressure free charges on the surface of

the conductor here and bound charges on the surface of the dielectric
so let me take a gaussian surface like this and assuming this to be flat plate plane surfaces the electric field lines are perpendicular to this interface there is no electric field on the on this surface because it is inside the conductor there is no electric field crossing these surfaces because electric field lines are parallel to this curved surface the only electric field crossing is ah on the surface
so if the surface area is a i will write e times a is equal to just like before sigma f minus sigma b the net charge multiplied by a by epsilon zero
so electric field crossing this entire surface gaussian surface is e times a because there is no electric field here this line there is no crossing there is no flux on the surface the only flux is from the surface within the dielectric
so i will write this as epsilon zero e plus sigma b is equal to sigma f and sigma b i know is nothing but p
so i write this as epsilon zero e plus b is equal to sigma f now there is a name given to this vector its called the displacement vector d vector is equal to epsilon zero e plus b
so this is scalar relationship but if i look at in a vector form this is a displacement vector
so this equation becomes nothing but d is equal to sigma f now i can multiply both sides by area and write this as sigma f into a now what is i can interpret d times a as the flux of the displacement vector through the same gaussian surface what this was the flux of the electric field through the gaussian surface d times a i can interpret as the flux of the displacement vector on the same gaussian surface
so i get that the flux of the displacement vector is equal to total free charge enclosed by the surface this was free charge the total charge including free and bound charges here i get displacement flux of the displacement vector is equal to the free charge enclosed
so i get i get this equation i can write this in a integral form at the following integral d dot d a is equal to free charge enclosed the gaussian the gauss's law for electric fields was integral e dot e a is equal to total charge enclosed by epsilon zero here for the displacement vector this is the gauss's law i can also write this as epsilon e dot d a is equal to v charge n close please remember that on this in this equation in this gauss's law form the charge on the right hand side is only the free charge and that is all that i needed to know to find out what is the displacement vector and from the displacement vector i can use this equation to calculate the electric field vector now let me take one example of gauss's law in a dielectric
so let me assume that i have a conductor of radius a surrounded by a dielectric of radius b this is conductor and let me assume that the charge is the charge q on this q and this is the dielectric now because of this spherical symmetry the electric field lines will be radial the displacement vector lines will be radial
so i take a gaussian surface of radius r
so i use this equation d dot d a is equal to three charge enclosed because the displacement vector is perpendicular to the surface this does be simply d times four pi r square is equal to q q is the surface charge enclosed the charge enclosed please remember there is a dielectric here
so if i were to draw the charges if the dielectric has positive charge there will be negative bound charges on this side but i am not bothered about the bound charges at all in using this form of gauss's law because it only requires knowledge of free charges
so displacement vector is actually q by four pi r square in fact no matter what value of r you take whether within the dielectric or outside the dielectric this

is a displacement vector

so for r ah less than b greater than a but less than b the displacement is equal to q by four pi r square and this is equal to epsilon e because there is a dielectric with permittivity epsilon

so e will be equal to q by four pi epsilon r square for r greater than a for for r greater than b t is again given by q by four pi r square but now in this case d is epsilon zero e

so the electric field in this case will be q by four pi epsilon zero r square

so you see what i have got is using this form of gauss's law i have been able to calculate what is electric field in the situation like this

so once i know the electric field in all the regions i can use this electric field in the expression for polarization to calculate the polarization once i know the polarization i can calculate the surface charge density

so this is a very very powerful form of gauss's law ah in with using dielectric with reducing displacement vector and is applicable whether you have dielectrics or not and this is very useful especially in situations where there is symmetry

so i would like to summarize what we have been doing in electrostatics we started with coulomb's law then we introduced the principle of superposition where we calculated the total electric field by a number of charges and then with that we also introduce the concept of electric field lines and then we specifically calculated the electric field to q to a dipole and also calculated what are the forces and talks on dipoles and then we introduced the very important principle of gauss's law and used that gauss's law to calculate electric fields in different symmetric situations we introduced the concept of gauss flux electric flux and then we also discussed conductors equal potential surfaces electrostatic potential energy and electrostatic potential and finally we discuss some capacitors and capacitance and also dielectrics insert capacitors and how the electric fields get modified and finally we just introduced gauss's law in a dielectric and these are very general principles of great application in the field of electromagnetics thank you you