

foreign morning to all of you we continue with our discussion on electrostatics in the last lecture we had introduced concepts of electrostatic potential energy and electrostatic potential

so let us recall if you have a collection of charges then there is potential energy stored in the collection of charges

so if you have all the charges of at infinitely separate separate distances and from there if you bring one charge at a time and assemble the entire charge distribution you need to do work on the charges to assemble them and this work that you do is actually stored in the form of potential energy of the entire charge distribution as i mentioned before the energy is contained in the distribution of charges it is not present in one charge or the other it is in the entire distribution of charges and it does not matter in what arrangement you bring the charges and assemble the entire distribution in whatever method you employ finally when you assemble a charge distribution it has a certain amount of potential energy built into the system we then defined the potential electrostatic potential as the work done in bringing a unit positive charge from infinity to that point

so if you if you have a positive charge point charge at infinity you bring it from infinity to that point where you want to calculate the potential the amount of work you do in bringing the charge defines the potential at that point and potential is a scalar quantity and as i mentioned last time it is in many problems much easier to calculate potential and from potential as i will tell you electric fields can be calculated

so ah as an example what we did was last time was we calculated the potential of a point charge suppose you have a point charge q the potential at any distance r from here is equal to q by four pi epsilon zero r

so thats a scalar quantity and potential of this point charge depends only on the distance of the point from the point charge and the potential follows the principle of superposition

so if you have multiple charges then the total potential at any point is the sum of the potentials created by each and every individual charge with this principle we had calculated the potential ah of a dipole

so if you have a minus q and a plus q charge here separated by distance two a then this is a dipole with a type of moment like this then we calculated what is the potential at a distance r from here and some angle θ

so θ is the angle made between the line joining the point where you are calculating the potential and the center of the dipole and the dipole axis and the potential at this point depends both on the position as well as the angle subtended by this line

so i ah then we introduced the concept of equipotential surfaces these are surfaces where the potential electrostatic potentials remain constant

so these could be surfaces of arbitrary shapes the shape depends on the kind of electric fields that you have if you have a uniform electric field pointing in this direction the equipotential surfaces are plane surfaces perpendicular to the electric field lines if you have a point charge then we saw that the electric field the equipotentials are spheres

so if you have a point charge here q then the equal potentials are spheres like this these are all equipotential this is one equipotential is another equal potential they are all spheres surrounding the charge point charge and the points the spheres have center at the point charge itself and as you know the electric field lines are like this away from the point charge if it is the positive charge it is towards the point charge if it is a negative charge

so these are the electric field lines

so as i have drawn here the electric field lines are perpendicular to the equal

potential surfaces we had discussed this again last time that if you have an equipotential surface suppose i have some equipotential surface say like this this is an equipotential surface then all it implies is the potential at every point on the on the surface is the same i am drawing a section of that surface on this plane here

so there is a particular curve

so there is a particular surface over which the potential remains constant

so that means that to move a charge from this point to this point i do not need to do any work in fact i do not need to do any work in taking a unit positive charge from any point on that surface to any other point on that same surface because they are equipotentials the potential is the same all along the surface which implies that there cannot be a component of the electric field which is along the equipotential surface

so it implies that the electric field lines have to be perpendicular to the equipotential surfaces here will be like this here will be like this

so these are the electric electric field lines which are always perpendicular to the equal potentials this we saw in the example of a sphere that on a point charge that the equipotentials are spheres and the electric field lines are radial lines from the point charge away

so using this what i want to do which is which we started doing last time is to relate equipotentials and electric fields potentially potential and electric fields now before then i want to show you a picture of the equipotential surfaces of a dipole and the corresponding electric field lines just like electric field lines equal potentials are another way of representing the electric field distribution or potential distribution which is helpful in understanding and in picturizing the potentials and electric field

so what we would like to do is to relate electric fields and potentials

so we started to do this in the last lecture

so again let me recall i want to relate a potential v to the electric field of the distribution

so for this what we do is we consider two equipotential lines equipotential surfaces one with a potential v naught and another with a potential v naught plus $d v$ two potential equipotential surfaces which are close to each other in potentials one has a potential v naught the other has v naught plus $d v$ now as i just mentioned the electric field line will be perpendicular to the direction to the surface equipotential surface

so this may be the direction of electric field here now what i do is i move from a point a on this equal potential to another point b on the nearby equipotential

so when i move in this direction from this way equal potential to this e potential i need to do some work

so work done in moving a unit charge from a to b is equal to v naught plus $d b$ minus v naught which is equal to $d b$

so you know that potential difference is the work done in moving a unit positive charge from one point to the other

so when i have to move a point charge from a to b the work done is potential at b minus potential at a

so v naught plus $d v$ minus v naught which is $d b$

so let me call this ah vector as $d l$

so work done is also equal to is also equal to minus e dot $d l$ the force i need to apply is opposite of the directional electric field

so the work done by the external agent is minus e dot $d l$ which is equal to if this angle is theta this is equal to minus $e d l \cos \theta$ now if you see here what is $e \cos \theta$ $e \cos \theta$ is the component of the electric field vector

along the length direction ab this is the dl element electric vector points like this

so $e \cos \theta$ is the component of the electric field along the direction in which i am moving

so this can be written as $\int -e \cdot dl$ where e is the component of the electric field along the direction of motion

so i have an equation that $\int -e \cdot dl$ is equal to Δb this implies e is equal to $-\nabla b$

so e is the electric component of the electric field along the direction of in which i am moving the charge

so suppose for example i consider a coordinate system here

so let me say this is x axis this is y axis this is these are the equipotentials here

so if i move from suppose like let me move parallel to the x axis

so this is v naught some potential equal potential v naught v naught plus db

so i move along the direction parallel to the x axis

so my dl vector is actually dx vector

so i am moving along the x axis

so the equation which i just now wrote down dl vector is along x axis

so what i will get is the electric field e_x will be equal to $-\nabla b$ by dx

so the the partial derivative of v with respect to x is nothing but $-e_x$ i am writing partial derivative because the potential depends on in general all three coordinates x , y and z in a similar fashion if i move along the y parallel to the y axis then i can get the following equation e_y is equal to $-\nabla v$ by dy and similarly e_z is equal to $-\nabla b$ by dz

so these are three very important relations which relate the three components of the electric vector with the dx , dy and dz differentials

so in fact from here i can write e vector is equal to $i \hat{a}_x + j \hat{a}_y + k \hat{a}_z$ which is equal to $-\nabla b$ by $dx + dy + dz$

so if i know the potential distribution of a given charge distribution if i know b as a function of x , y and z i can calculate each of the three derivatives partial derivatives and hence calculate the electric field as a function of position

so this is a very powerful method and we started looking at an example

so as an example i want to look at calculation of electric field of a point charge point charge

so i have a point charge q here and i know that v of r is equal to

so r is this distance q by $4\pi\epsilon_0 r$

so if i had a coordinate system here x , y , z if this point at p had a coordinate x , y , z then r is the distance of this point from the or from the origin in which the point charge is sitting

so r is equal to square root of $x^2 + y^2 + z^2$

so v of x , y , z is equal to q by $4\pi\epsilon_0$ square root of $x^2 + y^2 + z^2$

so now this is the potential as a function of x , y , z

so i can calculate the three electrical components

so e_x is equal to $-\nabla b$ by dx which is equal to $-\frac{q}{4\pi\epsilon_0} \frac{1}{(x^2 + y^2 + z^2)^{3/2}} \cdot 2x$

so you must be able to differentiate this quantity which is $\frac{1}{r^2}$

so $\frac{1}{r^2}$ is equal to $\frac{1}{(x^2 + y^2 + z^2)^{3/2}}$ which is actually q by $4\pi\epsilon_0$

epsilon zero into ah i will write this as $x^2 + y^2 + z^2$ into x by square root of $x^2 + y^2 + z^2$

so what i have done is i have split the $x^2 + y^2 + z^2$ raise to three by two into $x^2 + y^2 + z^2$ and square root of $x^2 + y^2 + z^2$ now what are these two quantities these are nothing but

so i will get an expression for e_x

so e_x is equal to q by $4\pi\epsilon_0 r^2$ now what is this this is as you can see here this quantity is r^2 and this quantity is r

so i get r^2 into x by r

so that is the electric field component along the x axis i will leave it to you as an exercise to show that e_y will be equal to q by $4\pi\epsilon_0 r^2$ into y by r and e_z will be equal to q by $4\pi\epsilon_0 r^2$ into z by r you look here this equation for potential is symmetric in x , y and z

so when you calculate differential with respect to y all you need to do is to replace x by y and you will get the expression for e_y similarly expression for e_z

so the electric field total electric field is nothing but $i\hat{c}ap e_x + j\hat{c}ap e_y + k\hat{c}ap e_z$ which is nothing but q by $4\pi\epsilon_0 r^2$ into $i\hat{c}ap x + j\hat{c}ap y + k\hat{c}ap z$ by r and now we can recognize this quantity this numerator in this this numerator is nothing but r vector x , y , z are the coordinates of this point x , y , z is the coordinate of this point and

so r vector this is our vector join the vector joining the point charge q to the point p is nothing but r vector

so i get the following expression for electric field of a point charge e is equal to q by $4\pi\epsilon_0 r^2$ into r vector by r and what is r vector by r it is nothing but unit vector along the r direction $4\pi\epsilon_0 r^2$ into $r\hat{c}ap$ and this is exactly the electric field that of a point charge from coulomb's law

so what i have shown you through this simple example that knowing the potential of a point charge which is given by this i can actually calculate the electric field of the point charge through this calculation and

so this relationship which i have written here is a very very useful relationship for very ah different charge distributions and

so given any charge distribution i can first calculate the potential distribution of the charge distribution once i know v as a function of x , y and z i can use these three relationships to calculate e_x , e_y and e_z and from there the total electric field e vector

so this was a very simple example which i wanted to show you as an example which can be used to calculate the electric field of a point charge now i want to use the discussion that we are having to look at if the following problem with conductors with cavities

so let me start with a following situation i have some conductor arbitrarily subconductor

so this is a conductor i put an excess charge q on the conductor

so what happens as we have discussed earlier that all this excess charge will sit on the surface of the conductor because you cannot have any electric field inside the conductor because and the state static situation because if there is any charge inside the conductor in the presence of electric field the charge will move and it will never be a static situation

so finally when you are reached a static situation that is there are no more changes the electric field inside the conductor must be zero and we use gauss's law to show that this implies that there are no charges within the conductor no excess charge within the conductor

so all the excess charge that you have put on the conductor is on the outer surface on the surface of the conductor now i must mention here that the charge distribution on the conductor is not uniform for an arbitrary shaped conductor the charges will adjust themselves on the surface such that the electric field produced at any point within the conductor becomes zero

so for example the charge distribution here will be such that the electric field at this point which is created by various charges from all points here so they are all different directions

so all these total electric field vectorial sum of all the electric fields so if i take a conductor like this and if i take a point here this charge here is producing electric field like this this charge from here is producing electric field like this a charge from here is producing electric field like this a charge from here is putting the electric field like this this charge is producing electric field like this this charge is producing like this

so i must add all the electric field contributions of all the charges present on the surface and i should find it zero here

so the charges adjust themselves on the surface in such a fashion that the net electric field within the conductor at every point within the conductor is actually zero now if you have a spherical conductor by symmetry the entire charge sits uniformly distributed across the surface of the conductor

so if you have a charge q and if the radius is r you get a surface charge density q by $4\pi r^2$ because then here in this situation because of symmetry the charge distributes equally on the entire surface of the conductor

so we have seen this before now the question arises that suppose i have a cavity within this conductor

so cavity within the conductor

so i have a conductor of arbitrary shape and i have a cavity

so this is the conductor here and i have cavity and now i put a charge q on the on the conductor

so the question is where are these charges now sitting are they sitting on the outer surface only or are they sitting on the inner surface of the conductor or are they sitting on both the inner surface and the outer surface of the conductor

so that's the problem we would like to see the first thing is that let me take as before we have done i take a gaussian surface which is completely lying within the conductor and encloses this cavity

so this is the gaussian surface this is the gaussian surface enclosing the cavity and that gaussian surface lies entirely within the conductor now because the electric field inside the conductor is zero the net flux crossing this gaussian surface must be zero because electric field at every point on the surface is zero

so if i integrate $\mathbf{E} \cdot d\mathbf{A}$ then i will get zero

so that means that this gaussian surface must be enclosing zero net charge now as i mentioned before zero net charge implies that there must be either no charge within the gaussian surface or an equal amount of positive and negative charges please remember zero net flux does not imply no charge it is possible that there may be no charge or there may be an equal amount of positive and negative charges if i have equal amount of positive and negative charges within the cavity then the net flux crossing the surface of the gaussian surface will still be zero

so let me assume that this particular inner cavity surface also contains some charges

so but i must because the net flux through the gaussian surface is zero there must be equal amount of positive and negative charges within the on the surface

so let me write let me draw some charges here

so i have plus plus plus some plus charges here and maybe there are some negative charges on some other point on the surface

so there are positive and negative charges sitting on the in the cavity

so now what is going to happen is remember there cannot be any electric field within the conductor

so there must be electric field lines like this from the positive charge to the negative charge within the conductor cavity these electric field lines cannot enter the conductor because inside the conductor the electric field must be zero now let me take the following ah path

so i take a point charge from here move along this along the line and continue to move like this into the conductor and come back to this to this point sorry to this point

so i start from here go along this and take a path and come back

so i want to calculate $\int \mathbf{e} \cdot d\mathbf{l}$ along this path now remember we had discussed this before electrostatic fields are conservative fields and $\int \mathbf{e} \cdot d\mathbf{l}$ must be zero which implies that if you have any electric field distribution if you start from a point and come back to that same point the net work done in taking a charge from a point p to a through any circuit and coming back to the same point p the net work done must be zero now look at this path which i have taken

so i move from here and then i move along this now you see because the electric field on at all points on this path in this region of the path is zero this integral has no contribution from this path now in this in this path here there is an electric field and there is a finite length which i am travelling

so what i will find for this path if i call this path c i will find that $\int \mathbf{e} \cdot d\mathbf{l}$ along the path c is not equal to zero in this path now this is inconsistent with the fact that $\int \mathbf{e} \cdot d\mathbf{l}$ must be zero and

so what i conclude is there can be no x no excess charge excess charge within the on the surface on the inner surface

so this inside cavity cavity this inner surface of the cavity can cannot have any excess charge because if it had charges there must be equal amount of positive and negative charges which then will result in an electric field within the cavity of the conductor and then if i do this integral of $\int \mathbf{e} \cdot d\mathbf{l}$ along a circuitous path which partly passes through the ah cavity and partly passes through the conductor i will find that $\int \mathbf{e} \cdot d\mathbf{l}$ is not equal to zero which is inconsistent with the fact that $\int \mathbf{e} \cdot d\mathbf{l}$ must be zero and hence there cannot be any excess charge within the inner cavity of the conductor

so if i have a conductor like this in any arbitrary conductor if i have a cavity and if i put charges q on the cavity on the on this conductor all these charges must be sitting on the outer surface of the conductor there cannot be any charge on the inner surface of the cavity all charges are sitting excess charge all the excess charge which i put i am assuming positive extra charge here all those charges are sitting on the outer surface of the cabinet of the conductor and there are no charges within the conductor cavity

so ah if you if you touch any of these points this is there is no charge at all on this inner surface of the cavity now if it

so happens let suppose the conductor is a spherical conductor and i have a cavity here wherever i have a cavity whatever charge if i put here this charge will be equally distributed all across the surface of the spherical cavity spherical conductor and there is no charge within the surface within the inner surface of this conductor which is the cavity surface there is no charge there at all now i want to see what happens if i place a charge within the cavity of the conductor

so i have now an example in which i have ah

so let me take for example a spherical conductor and let me have some cavity here

so this is my conductor and i place a charge here say plus q now i want to know what happens to the situation now you see that there cannot be any electric field within the cavity within the conductor

so what this plus q will do is to attract negative charges onto the surface

so there will be an accumulation of negative charge on the surface of this cavity now if i assume the stuff the cavity to be a spherical cavity and this point charge to be placed at the center then you can see from symmetry that this negative charge must be equally distributed across the cavity surface because if you now take a gaussian surface like this which is lying within the conductor the net flux must be zero and the net charge enclosed also must be zero

so because you have put a plus q charge here there must be a minus q charge of accumulated charge on the inner surface of the conductor now these charge obviously are coming from the conductor and hence they will leave an equal amount of positive charge on the outer surface of the conductor and if the conductor is a spherical conductor that q positive charge will get equally distributed all across the surface of the conductor

so what i am now seeing is if i did not have a charge within the cavity of the conductor all the excess charge that you put on the conductor are all sitting on the outer surface without the cavity if you have a cavity and if you put a charge within the cavity then this charge will attract if this charge is positive it will attract an equal amount of negative charge onto the inner surface of the cavity such that this gaussian surface encloses zero net charge

so if you have a plus q charge here there will be a minus q charge accumulated on the inner surface of this cavity and

so that the net flux will be zero on this gaussian surface the net charge enclosed by this gaussian surface is zero and these negative charges will leave an equal amount of positive charge on the outer surface of the conductor and if this conductor is a spherical conductor then this positive charge will be equally distributed on the outer surface and

so these two charges together are not producing any electric field outside the conductor outside because the sum of these two electric fields must be zero everywhere

so from outside for the outside point it looks as if there are positive charges on a spherical conductor and we know that the electric field here because of this positive charge on a spherical conductor is exactly the same as if the entire charge was concentrated at the center of the spherical conductor

so look here that there is no information as to the existence of this cavity or the accession of the charge all that you see from outside is a conductor with equally charged distribution all across the surface of the conductor now think what will happen if i move this charge from the center to a side point if i move it here what will happen to the electric field what will happen to the charge distribution on the inner surface what will happen to the charge distribution outer surface what will be the outside electric field distribution

so i leave this problem to you please give some thoughts to find out what will happen

so i want to leave a problem here for you

so assume a spherical conductor of radius r and a spherical cavity of radius r_s and assume that i put a charge minus q here

so consider a spherical conductor with a spherical cavity centered in the conductor

so this sphere and these two spheres are concentric
so their centers meet and ah a charge minus q is placed at the center of the cavity

so calculate the surface charge density on the inner and outer surfaces and we calculate the electric field everywhere

so from the discussion that we have had

so this is the conductor here and of an out the radius is r_0 here and the spherical cavity has a radius r_s both the spheres have the same center and at the center of the cavity i have placed a charge minus q

so i want you to calculate the surface charge density on the inner surface and outer surface of the conductors and calculate the electric field at every point in this problem now i want to take this discussion little forward and look at the following problem

so let me assume that i have a pair of spherical conductors

so one conductor like this and another smaller conductor and joined by a conducting wire

so this is a conducting wire

so this is radius a this is radius b both are conductors and this is conducting and this is again radius b now what i am going to do is i throw some excess charge on the system i throw a charge on the system

so the charge will because these are conductors joined by another conductor here the charge will distribute itself and let me assume the charge on this conductor is q_a and the charge on this conductor is q_b please remember the two surf the two spherical conductors are two different spherical conductors of different radii and the charge will distribute in such a fashion that you have some charge q_a on the sphere of radius a and the charge q_b on the sphere of radius b now we have discussed before that conductors form an equal potential cir equipotentials

so the potential at both the sphere and the sphere and the wire must all be the same because if there was a potential difference then that will lead to an electric field and that electric field will ensure the charges then move and until the potential becomes equal all along the conductor

so the con this conductor as well as this conductor will have the same potential now i want to approximate ah we have calculated the potential of a spherical conductor and

so the ah if with the charge q_a

so potential on the surface of this conductor v_a is q_a by four pi epsilon zero into r_a remember the potential of a spherical charge distribution

so if i have a sphere and if i have a charge q on this conducting sphere and so as far as outside region is concerned this sphere charge sphere acts like a point charge at this point

so the potential at any point r from here is q by four pi epsilon zero r and on the surface at r is equal to r which is the surface of the conductor potential at r is equal to q by four pi epsilon zero r

so the potential on the surface of the conductor is equal to the charge carried by the conductor divided by four pi epsilon zero times radius of the conductor

so that is the equation which i am using here

so what i am saying is as an approximation i am assuming that the potential of this spherical charge and this spherical conductor are the same in approximately equal to v_a is equal to q_a by four pi epsilon zero r_a ah sorry a the radius of this sphere and v_b is equal to the charge on that conductor divided by four pi epsilon zero into ah b b is the radius of that conductor a radius of this conductor and i know that v_a is equal to b b because both conductors are at the same potential this implies that q_a by a is equal to q_b by b ok

so now suppose σ_a and σ_b are the charge densities
 so if this charge surface charge densities σ_a and σ_b
 so here it is σ_a and here it is σ_b then q_a must be equal to σ_a into four pi a^2 and q_b must be equal to σ_b into four pi b^2
 so i had this equation i had this relationship q_a by a^2 is equal to q_b by b^2
 which means that σ_a into four pi a^2 by a^2 is equal to σ_b into four pi b^2 by b^2
 so i get σ_a into a^2 is equal to σ_b into b^2
 so i also know the electric field at the surface of this conductors if you have a surface charge density σ electric field is equal to σ by epsilon zero
 so this for a surface charge density σ the electric field is σ by epsilon zero
 so what i get is electric field on the surface of this conductor radius a is σ_a by epsilon zero and electric field on the surface of the conductor b is σ_b by epsilon zero
 so and i have this relation σ_a a^2 is equal to σ_b b^2
 so it implies that e_a times a^2 is equal to e_b times b^2
 so e_b by e_a is equal to a^2 by b^2
 so two electric fields are related to this
 so let me redraw the figure here you have one sphere of radius a connected to another sphere of radius b
 so all it implies is the electric field at this point on the surface of this is e_a and the electric field here is e_b
 so the ratio of these two electric fields is
 so e_b by e_a is a^2 by b^2
 so you see that a smaller sphere which means if b is less than a e_b is much bigger than e_a
 so the smaller the sphere the stronger the electric field
 so what happens is if you have two spheres if you have two spheres joint like this the two spheres are form an equal potential and the electric field just surrounding the smaller sphere will be much much higher than that surrounding the larger sphere
 so actually i can generalize this and to say that if you have a conductor with which is not spherical but which has some sharp edges like this then the charges will distribute in such a fashion that this is a smaller radius here compared to this radius here
 so here the σ will be suppose i call this σ_1 here and σ_2 here the σ_2 will be much bigger than σ_1 and
 so the electric field at this point will be much stronger
 so in fact i can draw electric field lines like this
 so suppose i have a conductor like this and if i put positive charges
 so there will be some positive charges and they will there be more positive charge accumulated here
 so the positive charge density will increase
 so the electric field lines here there will be some electric field line like this the electric field lines will be much stronger here they are closer electric field lines than here
 so the electric field lines will crowd around the corner point of the conductor
 so this is a very very important aspect in a spherical conductor there are all the points have the same radii of curvature
 so the charge is distributed equally along the surface of the conductor but here if you have sharp edges on the conductor then you have extremely large charge densities generated there and as we have seen before this electric field at this point if it exceeds the breakdown of air then you will have a spark

created at that point in fact this is a very interesting concept and this is this concept is used in ah where you must have seen lightning rods that are used to pick up the lightning

so you have sharp edges conducted with the sharp edge on the top of the ah residence and this conductor is joined by conducting wire to the ground

so when you have clouds with charged clouds which are on top of this region here then the very strong electric field is generated between the cloud and the ground and the electric field lines crowd towards the tip of the conductor here and

so the charge which gets discharged from the clouds comes and goes through this conductor to the ground and thus protects the other instruments or houses from getting shock

so this is a very interesting application of this fact that electric field lines crowd along the corner of around sharp edges in fact sharp edges have to be avoided if you want to avoid any strong electric fields in the in the in your problem ok

so now having seen this conductors are equipotential surfaces charges put on conductors reside on the outer surface of the conductor now i would like to bring in another concept which is the concept of capacitors and capacitance if you have any two conductors carrying equal and opposite charges

so what i do is i have two conductors i move i move some electrons from one conductor to the other conductor

so i will leave some ah positive charge

so i move some electrons from this conductor to this conductor

so i will re leave a positive net positive charge on this conductor this conductor will have negative charge

so i have two conductors which are oppositely charged and kept this particular configuration forms what is called as a capacitor

so you can see electric field lines will be generated between these because of these charges the electric field lines will be generated and

so this particular configuration forms what is called as a capacitor

so if you have two conductors which have one conductor has a positive charge and the other conductor has an equal negative charge this pair of conductors forms what is called as a capacitor and these capacitors are usually drawn by a symbol like this this is essentially the conductor which will discuss which in terms of parallel bed capacitor

so let us look at the simplest of these capacitors which is the parallel plate capacitor

so i have ah here i have two capacitor conductors

so this this is actually two plates like this there are one plate here another plate here and separated by a distance d ah let me put ah positive charge on one and an equal negative charge on the other

so this process of putting charges on these conductors is called charging of the conductors charging of the capacitance

so if i connect these two pairs of conductors to a battery i will be able to transfer electrons from one conductor to the other conductor and in that process i charge these two and i disconnect the battery and what i will have is two conductors like this two parallel plates facing each other with one with a positive charge the other with a negative charge now this forms a what is called the parallel plate capacitor these are two plates which are facing each other and forming what is called as a capacitor

so capacitor is a device where you can store charges and you can store energy we will calculate that this particular configuration stores energy in the form of electrostatic energy and which can be used later on for very many many

applications now as we have been discussing these two conductors form equipotential surfaces

so the negative charge here attracts the positive charge on this surface here and these two the inner surfaces of these two conductors get charged with positive charge here and negative charge here

so let me assume the surface charge densities of σ and $-\sigma$
so we have discussed this problem before that if you have a charge intensity σ then it creates an electric field

so electric field produced by this surface charge density in this direction here is $\frac{\sigma}{2\epsilon_0}$ everywhere on this side and it is on this side is $\frac{\sigma}{2\epsilon_0}$ this negative charge distribution creates $\frac{\sigma}{2\epsilon_0}$ here and here it creates $\frac{\sigma}{2\epsilon_0}$ here

so we have discussed this problem before and we have shown that between the two surfaces of this conductors we have a net electric field given by

so i have ah this conductor here another conductor here

so ah i am assuming in this calculation that this plates are much bigger compared to the spacing the area of the plates is large very large compared to the size of the plates very large compared to the distance separating them

so i have positive charges here and i have negative charges on this side and i have an electric field with in between which is equal to $\frac{\sigma}{\epsilon_0}$ and the electric field lines are coming like this if the plates are very large in size the linear dimension of the plate is large compared to the separation then i can neglect what are called as end effects that means at towards the ends of these of these conductors the charges will not be uniformly distributed because of end effects but i am neglecting the end effects and i know that towards the center of the parallel plates system i will have uniform electric field now i want to calculate the relationship between the charges which are contained in these two plates and the potential difference between these two

so what is the potential difference between these two

so potential difference

so potential difference V is equal to the work done in moving a charge from one plate to the other and that must be electric field times the distance which is equal to $\frac{\sigma d}{\epsilon_0}$ $\frac{\sigma}{\epsilon_0}$ is electric field d is the distance between the two conductors

so to move a distance d

so these electric field lines are vertical lines here

so to move a charge from one plate to the other plate i need to do a work the electric field times the distance separating them and $\frac{\sigma d}{\epsilon_0}$ and σ is equal to the charge on the capacitor plates divided by the area of the plates a

so i am assuming plate area a and plate separation d

so σ is $\frac{q}{a}$

so i get V is equal to $\frac{q d}{\epsilon_0 a}$

so what we see is the potential difference between these two conductors is proportional to the charge carried by the conductors now this is for this parallel plate i have shown you but one can show that in general if you have two conductors which have plus q and minus q charges then the potential difference between these two conductors is proportional to the charge carried by the conductors

so we can define a quantity here this is a constant here which we define and we define what is called as the capacitance C

so we had this equation V is equal to $\frac{q}{C}$

so we define C is equal to $\frac{q}{V}$

so C is equal to $\frac{q}{V}$

so that is a relationship

so this one by c is the proportionality constant which relates v and q and this is called the capacitance

so as i mentioned capacitance is a quantity which relates the potential difference between two conductors which are carrying a charge q

so and i have also i have although i have derived this relationship for a parallel plate system this relationship is true in general that means if you have two arbitrarily shaped conductors carrying a charge q plus q and minus q they will develop a potential difference v and the potential difference v between these two conductors will be proportional to the charge carried by the conductors and that proportionality constant is actually the capacitance of the conductor

so we have v equal q by c or q is equal to c times v and this capacitance is a quantity which is a geometrical quantity it only depends on the geometrical parameters such as the area of the conductors the distance between the conductor etcetera it is not dependent on the charges or potential that you are calculating

so c is a proportionality constant now in this we have calculated c as $\epsilon_0 \frac{A}{d}$ that is an approximate relationship for a parallel plate capacitor because we have effectively in this calculation neglected the effects of ends but this is a reasonably good approximation ah if you want to if you were to calculate more precisely you will get a slightly different value of c compared to this number but otherwise this relationship will be still valid v is equal to q by c where c is the capacitance of this conductor pair

so let me calculate let me take an example

so let me take a parallel plate capacitor let me assume the separation is one millimeter

so d is equal to one millimeter and let me assume an area of ten centimeter square

so capacitance of this $\epsilon_0 \frac{A}{d}$ which is equal to $8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} \times \frac{10 \times 10^{-4} \text{m}^2}{10^{-3} \text{m}}$ which is approximately which is equal to $8.85 \times 10^{-12} \text{ farad}$ is actually $8.85 \times 10^{-12} \text{ farad}$

so farad is a unit of capacitance this is named after michael faraday and it is given by the the if you take in volts if you look at this equation here if you take v in volts and q in coulombs c comes out to be an unit called farads and

so this amount of capacitance of this parallel plate capacitor with two plates separated by one millimeter separation and each can each having an area of ten centimeter square is $8.85 \times 10^{-12} \text{ farad}$

so farad is a unit of capacitance and ah this particular is a very very large quantity

so as you can see here this parallel plate capacitor has an capacitance of $8.85 \times 10^{-12} \text{ farad}$

so suppose i were to take this parallel plate capacitor and

so i take the same parallel plate capacitor and if i

so the capacitance is $8.85 \times 10^{-12} \text{ farads}$ and if i apply potential difference v is equal to one volt then the corresponding charge will be c times v which is equal to $8.85 \times 10^{-12} \text{ farads} \times 1 \text{ volt}$ which is equal to $8.85 \times 10^{-12} \text{ coulombs}$

$8.85 \times 10^{-12} \text{ coulombs}$ which is equal to $8.85 \times 10^{-12} \text{ coulombs}$

$8.85 \times 10^{-12} \text{ coulomb}$ that is the charge carried by the conductors

so what we will do is in today we will stop at this point and in the next lecture we will calculate capacitance of other configurations like cylindrical

capacitors and spherical capacitors and we will see in every case that the potential difference and the charge carried by the capacitance are related to each other and the proportionality constant gives me the capacitance of the pair and capacitance are very very important components in electronic circuits and we will understand little bit more about capacitance later in later lectures thank you very much you

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