

good morning to all of you we continue with our discussion on electrostatics in the last lecture we had introduced gauss's law

so let us recall that if you have a set of charges  $q_1, q_2, q_3$  etcetera and if you have an imaginary surface as we call it the gaussian surface then the flux electric flux through this imaginary gaussian surface is given by  $q_1 + q_2$  by epsilon zero the electric flux through this closed surface is equal to the charge enclosed by the closed surface divided by epsilon zero please remember that the electric field at all points on the surface is the electric field generated by all the charges in the system including  $q_3$  here so electric field at this point is the sum of electric field because of  $q_1, q_2$  and  $q_3$  while in the flux equation the total flux crossing this gaussian surface is equal to the total charge enclosed by the gaussian surface divided by epsilon zero

so the charges could be positive or negative

so you have to keep track of the sign of the charges here

so if  $q_2$  happens to be equal to minus  $q_1$  then net flux becomes zero

so please remember that a net flux is equal to zero does not imply no charges present in the system we can have zero flux because of cancellation of positive and negative charges or because of no charge being inside the surface we can generalize this to say that the total flux to any gaussian surface is the sum of all the charges present within the surface divided by epsilon zero and in fact if you have

so the flux was defined as the flux was defined as if you have a surface the flux we had defined as  $\mathbf{E} \cdot d\mathbf{s}$

so if i had a surface  $d\mathbf{s}$  here  $d\mathbf{s}$  vector and electric field was like this then the flux through this  $d\mathbf{s}$  is actually  $\mathbf{E} \cdot d\mathbf{s}$

so this is for a set of point charges i can actually generalize this into an integral form the total flux is actually integral  $\mathbf{E} \cdot d\mathbf{a}$  which is equal to charge enclosed by epsilon zero this is an integral over area of the gaussian surface and this circle on the integral sign implies its a closed integral

so it means the entire surface the surface is supposed to be closed

so the net flux emanating from the close surface is charged and closed by epsilon zero

so if you have an arbitrary surface you take the  $\mathbf{E} \cdot d\mathbf{a}$  you take the element of area at every point calculate the electric field at that point integrate and finally you get the total flux coming out of the surface must be equal to charge and close by epsilon zero

so this equation is true for any surface any close surface its the charge enclosed includes all positive negative charges within the surface and i must emphasize again that the electric field which is existing in this equation here is the total electric field produced by all charges the gaussian surface is an imaginary surface

so i can choose any arbitrary surface as a gaussian surface in problems i will choose a gaussian surface which will help me to resolve or calculate the electric field etcetera and

so the choice of the gaussian surface depends on the symmetry in the problem

so we will discuss some examples where it will be obvious the kind of gaussian surfaces i will choose

so gaussian gauss law is very useful when there are symmetries present in the system gauss law is always valid it becomes useful for me to calculate electric field for a given charge distribution or the charge distribution for a given electric field when there is symmetry in the system and as i mentioned in the last lecture the gauss law is based on the inverse square law of the electric field

so all fields which which behave like an inverse square law will satisfy this so for example gravitational field also will satisfy an equation like a gauss's law and this law is valid for any distribution of charge and any gaussian surface

so in the last class what we did was we used this law to consider where the charges are there in a conductor

so if i had a conductor we considered an arbitrary conductor solid conductor here and we put an excess charge  $q$  and we were trying to find out where the charges are sitting are they within the meat of the conductor or are they on the surface or are they in both places

so we used gauss's law and because the electric field inside the conductor has to be zero at all points because if there was electric field then the charges would move and in a static situation there cannot be any electric field within the conductor

so we use this fact and we took gaussian surfaces inside the conductor and because the electric field at all points on the surface is zero the net flux is zero and then you can actually shrink the surface the sphere to smaller and smaller values the charge continues to be zero and finally you reach a point and that means there can be no charge within the conductor

so you can take gaussian surfaces at different points and you can show that there are no charges within the conductor all the charges are distributed on the surface the charge excess charge is at the surface

so the charge distribution at the surface is such that the net electric field within the conductor becomes zero

so if you have an arbitrary charged conductor the charges are not necessarily uniformly distributed on the surface you may have less charge here more charge here etcetera etcetera

so charge distribution actually adjust itself in such a fashion that the electric field within the conductor is zero if you had a cavity within the conductor suppose i had a conductor here and i had a cavity

so this is the conductor the question are there charges on the inner surface of the conductor someone can show through arguments that even the inner surface there are no charges i can take a gaussian surface like this enclosing the enclosing the cavity and because the electric field at all these points is zero the net charge enclosed by this sphere must be zero but i could have equal amount of positive and negative charges on the surface of the conductor in the surface of the conductor of this of the hole

so we will have to use an ah discussion a little later but i will show you that because of that argument there can be no charges within the cavity of the conductor on the surface of the cavity of the conductor

so there can be no electric field inside the conductor and all the stuff all the charges are sitting on the outer surface of the conductor

so effectively what is happening is the inner volume of the conductor within the cavity is completely isolated from electric fields and this is used to shield components electronic components by cover covering them with a conductor and you can actually shield the inner area of inner volume from external electric fields in fact i can make on make the cavity bigger and bigger and finally i will be left with just a surface charge with no conductor no nothing everywhere anywhere

so the charge is distributed distributed in such a fashion that the electric field inside is zero

so this charge distribution actually adjust itself to create a zero electric field within the conductor within this volume and i can actually increase the volume of the cavity to almost touch this outer surface and all the charges are

sitting there

so a surface charging distribution like this will produce zero electric field within this within the volume of this charge distribution now i would like to use gauss's law to look at the following problem that is field produced by a charged conducting sphere

so my problem is i am given a spherical conductor of radius  $r$  and i have thrown in some excess charge into this

so the charge which have excess charge which i have thrown is capital  $q$  and now my problem is what is the electric field produced by this charge distribution

so first thing i know is that the charges are all on the surface there is no charge inside the conductor

so all the charges are sitting on the surface

so first question is in what way are they distributed on the surface are they equally distributed are they more on the upper half lower less on the lower half are they more on the right less on the right etcetera etcetera these questions can arise but i can use as i mentioned sometime back i can use symmetry present in the problem to get some solutions

so first thing i notice here is because i am taking a spherical conductor all points on the sphere are equivalent to each other this point is equal to this point is equal to this point is equal to this point all points on the sphere are equivalent to each other which essentially implies that the charge must be equally distributed across the surface of the conductor because if there is extra charge here why there should be excess charge here not here

so if you ask this question because of symmetry the charge gets equally distributed across the surface of the conductor and generates the surface charge density  $\sigma$  is equal to  $q$  by  $4\pi r^2$

so effectively we are looking at a problem in which i have a surface charge density  $q$  by  $4\pi r^2$  on a spherical surface and i want to find the field produced by this charge distribution

so if you recall our discussions with coulomb's law what i would have to do in principle is the following suppose i were to calculate the electric field at this point i have to take a small area here find out the electric field produced by this i take another small area here find out the electric field produced by this point these different areas how they are producing electric fields add up all the electric fields produced by all the surface charges present in the sphere and get the total electric field now that is not a simple problem and here i will show you the power of gauss's law

so using gauss's law we will be able to immediately calculate the electric field produced by this surface charge distribution

so how do i use gauss's law

so in applying gauss's law i must judiciously choose a gaussian surface now in integral form remember i wrote an equation like this for the gauss's law in this integral form if i choose an appropriate gaussian surface if i can take the electric field out of the integral which means if i choose a gaussian surface where the electric field is the same at all points on the gaussian surface i can take out the electric field and i will be able to resolve the problem

immediately to get the electric field

so that is what we are going to do here we will have to choose an appropriate gaussian surface such that i can integrate that and get the electric field now again i must use again some symmetry arguments how do i know what is the direction of electric field here what is the magnitude of electric field here vis-a-vis this point

so i can use gauss's law or symmetry now symmetry considerations now first thing you notice is that because of the spherical symmetry of the problem the

electric field at all points at a given distance from the sphere must be the same the magnitude of electric field at a given distance  $r$  from the center of the sphere must be the same at all points because of spherical symmetry again again as before if the electric field here is different from here if I can spin the rotate the spherical charge distribution then obviously this point will move here this point will move here and but the original spherical distribution and the new circle distribution are exactly the same

so there can be no difference between electric field here and electrical magnitude here

so first thing I notice is that the electric field magnitude depends only on  $r$  it cannot depend on the position around the sphere it it it will not change as I change from here to here to here as I change my position but keeping the distance from the center constant the electric field magnitude will remain the same

so that is first information I have got second one what is the direction of electric field now electric field here here can have a direction in this direction I can have a tangential direction here or it can have a direction which is perpendicular to the page now you see again because of spherical symmetry I cannot have an electric field like this because there is no difference but in this direction in this direction

so if because this the spherical distribution is absolutely symmetric about the center it is a spherical symmetry

so electric field cannot have this component similarly the electric field cannot have a component perpendicular to the page either coming out or going in because of complete symmetry in the problem

so the only possibility is electric vector is like this here electric vector will be like this here electric vector will be like this

so electric vector will have to be pointing in a direction which is radial which is along the line joining the center of the sphere to the point

so electric field is now becoming radial its pointing away from the center of the sphere and it has the same magnitude all across the surface of a sphere of radius small  $r$

so now having this information

so I can write here  $\mathbf{E}$  is along  $\hat{r}$

so  $\hat{r}$  is the direction the radial vector direction

so this is  $\hat{r}$  the vector unit vector joining the center of the sphere to any point here that is  $\hat{r}$  remember we had introduced this in coulomb's law

so from the symmetry of the problem I have been able to say that electric field magnitude depends only on the distance from the center and the electric factor has to be along the radial direction

so now let me try to use gauss's law

so let me draw the figure again here this is my spheric sphere with the spherical charge distribution  $q$  and I am taking a sphere of radius small  $r$

so here the electric vector has to be like this and remember the normal is also like this

so let me go back to this formula  $\oint \mathbf{E} \cdot d\mathbf{A}$  is equal to  $q$  enclosed by epsilon zero now at every point

so  $d\mathbf{A}$  are elements of areas I an element of area here element of area here

so this is these are all  $d\mathbf{A}$  directions  $d\mathbf{A}$  directions here  $d\mathbf{A}$  direction here but  $\mathbf{E}$  is also like this at this points

so anywhere you choose on the surface of the sphere  $\mathbf{E}$  and  $d\mathbf{A}$  are parallel  $\mathbf{E}$  and this surface charge surface element are parallel to each other

so  $\mathbf{E} \cdot d\mathbf{A}$  is actually nothing but  $E dA$

so sorry  $\epsilon \cdot dA$  becomes  $\epsilon dA$  now because i have chosen the gaussian surface as a sphere and because of symmetry the electric field is the same at every point on the sphere i can take the electric field out of the integral and i get an equation which is this  $q_{\text{enclosed}}$  by  $\epsilon_0$  now it is this is possible only because electric field happens to be the same at all points on the sphere because this area this integral is over the sphere

so i move my area element from from one point to another point on the surface of a sphere and as i move electric field does not change electric field here magnitude of electric field here here here everywhere is the same

so i can take the electric field outside and what is this this is the area of the sphere

so this is  $E$  into  $4\pi r^2$  is equal to  $q_{\text{enclosed}}$  sorry this is sorry this area the area of this of the sphere

so this is  $E$  into  $4\pi r^2$  is equal to  $q_{\text{enclosed}}$   $\pi \epsilon_0 z$  and  $q_{\text{enclosed}}$  is nothing but the charge which i have added to this and that is a by  $\epsilon_0$

so i get the electric field of a charged conducting sphere as  $E$  is equal to the magnitude  $q$  by  $4\pi \epsilon_0 r^2$  and because the direction of the electric vector is along  $r$  cap i get electric field  $E$  is equal to  $q$  by  $4\pi \epsilon_0 r^2$   $r$  cap

so this is my chart sphere and electric field at this point is like this along this line the electric field at this point is along this direction electric field here is along this direction

so what is this

so this is the electric field produced by a charged conductor carrying a charge capital  $q$  this is also the electric field produced by a point charge of magnitude capital  $q$  at the center of the sphere because if i had a point charge here the electric field at any distance would be given by the same equation

so what i have seen is please remember i am taking gaussian surface outside the conducting sphere because inside the conducting sphere electric field is zero anyway

so what i am seeing is that the electric field produced by a charged spherical conductor is the same as the electric field produced by a point charge at the center of the sphere

so the electric field produced by a charged spherical conductor behaves similar to a charge at the center of the sphere now remember we did not have to do much integration just because of some symmetry arguments and by an appropriate choice of the gaussian surface i would i was able to take the electric field out of the integral and integrate the area and get the electric field as a function of position

so this is a very interesting

so as you can see here the power of gauss's law here that by using symmetry arguments i can calculate the electric field of a spherically charged distribution ah i can also put this in a slightly different form remember  $q$  is the total charge and i mentioned that it is uniformly distributed on the surface of the char of the sphere

so the surface charge density is the total charge by  $4\pi r^2$  this is the pressure density and

so if i calculate the electric field very close to the surface of the conductor just outside  $r$  is equal to capital  $r$

so electric field will be ah  $q$  by  $4\pi \epsilon_0 r^2$  into  $r$  which is equal to  $\sigma$  by  $\epsilon_0$  into  $r$  cap

so  $r$  cap is nothing but the unit is a normal vector

so this is actually also equal to  $\sigma$  by  $\epsilon_0$  into  $n$  cap

so what it implies is at every point at this point there is an electric field  $\sigma/\epsilon_0$  pointing in this direction  $\sigma/\epsilon_0$   $\sigma/\epsilon_0$

so actually if you see this is more general result we will see later on if you have a charge surface charge here it produces an electric field and we will calculate the electric field produced by surface charge density and in this conducting case in the conductor case where the electric field is zero inside and outside it is  $\sigma/\epsilon_0$

so this is a very very interesting example where we have seen the power of gauss's law in being able to calculate the electric field produced by a charged conductor

so inside the conductor the electric field is zero and outside the conductor the electric field produced is exactly the same as if the charge was concentrated at the center of the sphere you may have come across a similar situation in studying gravitation the gravitational attraction of a spherical mass distribution is exactly the same as if the entire mass was concentrated at the center of the spherical distribution because the two forces gravitational force and electrostatic forces follow similar laws the results are very similar now i want to look at another example and in that example i will be able to calculate by using coulomb's law and by using gauss's law and then again you will see how gauss's law simplifies the calculation

so this is the field due to a line charge density and i am assuming infinite infinitely long

so i have a line a straight line which carries a charge per unit length  $\lambda$

so  $\lambda$  is the charge per unit length of this charge line charge distribution and my objective is to calculate the electric field produced by this line charge now i am considering the infinitely long line charge obviously infinite line charge do not exist but ah if you have a very long line charge very close to the line charge distribution the line charge distribution will behave as if it was infinitely long

so my objective is to find what is the electric field at this point at some point

so what do i do i travel first let me use coulomb's law to try to calculate what is the electric field here i will have to do some integration and then later on i will show you the gauss's law the power of gauss's law

so i drop a perpendicular here and

so let me call this as z axis this is some point here let me call this distance r from here please note that this point is the same at this point because of infinitely long line charge all points at any value of z if you choose it is the same

so i am calculating at some point and as you will see because of again symmetry the electric field here and here will be the same because it is the z invariant system the charge does not as you move along the z axis nothing happens okay

so now let me take a small element of charge here of length  $dz$  and the electric field produced by this charge will be along the direction let me assume a positive charge you can do the same calculation with negative charges electric field will be pointing towards the charge here i am just for simplicity or for this specific problem i am considering a positive charge density please note that ah

so this is addressing z from the axis a distance z on the other side also i can have an element now this element will produce an electric field like this this distance equal to this distance this charge is equal to  $dz$

so this electric field and this electric field are exactly equal in magnitude so you can see it will have a horizontal component and a vertical component this will have a horizontal component and a vertical component the vertical component this angles are equal all these angles are equal

so this vertical component of this and vertical component of this are exactly equal and in opposite direction the horizontal component of this and horizontal component of this again are equal but in the same direction

so the electric field at this point will be oriented in this direction because these two components cancel of each other

so thats one thing which i have seen already from this distribution but let me now calculate what is the electric field produced here by this

so what i will have to do is essentially get only the horizontal component finally because that is what is going to add and the vertical components at any point the vertical component of electric field will become zero because for every element of charge that creates a upward component there will be another identical charge element which will create a downward component of equal magnitude

so it will cancel off

so let me calculate the magnitude of the electric field

so let me call this  $dE$  magnitude of electric field is charge here which is  $\lambda dz$   $\lambda$  is the charge per unit length multiplied by the length which i am taking  $\lambda dz$  is the charge contained in this divided by  $4\pi\epsilon_0$  into this distance square

so let me call it  $s$   $s$  square that is the magnitude of electric field produced here and its horizontal component if i call this  $\theta$

so this is only the horizontal component this is not the total magnitude total magnitude is this charge divided by  $4\pi\epsilon_0$  times distance square its horizontal component is multiplied by  $\cos\theta$  because the vertical component which is  $\sin\theta$  is going to cancel off and what is this  $s$  square  $s$  square is nothing but  $r$  square plus  $z$  square

so  $dE$  is equal to  $\lambda dz$  by  $4\pi\epsilon_0$   $r$  square plus  $z$  square into  $\cos\theta$  now  $\cos\theta$  let me calculate this is  $\theta$  this is this  $\theta$

so  $\cos\theta$  is nothing but  $r$  by square root of  $r$  square plus  $z$  square  $r$  by square root of  $s$  square plus  $ah$   $r$  square plus  $z$  square which is this distance and that is  $\cos\theta$

so this is  $\lambda dz$  by  $4\pi\epsilon_0$   $s$  square which is  $r$  square plus  $z$  square into  $\cos\theta$

so this is horizontal component and as i mentioned i do not worry about the vertical component

so let me simplify this

so  $dE$  becomes  $\lambda dz$  by  $4\pi\epsilon_0$  and there is  $r$  here  $r$  square plus  $z$  square raised to the power three by two

so this is the magnitude of the horizontal component of the electric field produced by a small elemental elementary length  $dz$  of the line charge distribution

so how do i calculate the total i integrate over the entire length of the charge distribution and please remember that all the charge distributions i am only integrating the horizontal component

so the horizontal component produced by all the elementary charge distribution are in the same direction

so i just add the magnitudes rather than the direction if i were to calculate the total electric field i must integrate making sure that i am adding the vectors but here because i am calculating the horizontal component the horizontal component of each element will be along the same direction and i am

just adding it up

so the total electric field magnitude will be equal to  $\lambda r$  by four  $\pi$   $\epsilon_0$  integral  $dz$  by  $r^2 + z^2$  raised to the power three by two and  $dz$  goes from minus infinity to plus infinity

so  $z$  position which is the position of this point from the axis drop from this point where I am calculating

so  $z$  goes from minus infinity on the bottom side to plus infinity on the top side now that is a very standard integral

so all I have to do is to do a small change of variables

so I write  $z$  is equal to  $r \tan \phi$

so  $dz$  will be equal to  $r \sec^2 \phi$  and  $r^2 + z^2$  will be equal to  $r^2 + r^2 \tan^2 \phi$  which is equal to  $r^2 \sec^2 \phi$

so the  $e$  becomes I can write an expression for  $e$

so the  $e$  becomes  $\lambda r$  by four  $\pi \epsilon_0$  integral now there was a  $dz$  on the top

so I will have to write  $r \sec^2 \phi d\phi$  there is  $r^2 + z^2$  is the three by two in the denominator

so I will get  $r \sec^3 \phi$  now look at the variable change if  $z$  is minus infinity  $\phi$  is minus  $\pi/2$  if  $z$  is plus infinity  $\phi$  is plus  $\pi/2$  because  $\tan \pi/2$  is infinity  $\tan$  minus  $\pi/2$  is minus infinity

so the integration variable from minus infinity to plus infinity in  $z$  becomes minus  $\pi/2$  to plus  $\pi/2$  in this variable  $\phi$

so minus  $\pi/2$  to plus  $\pi/2$

so some things cancel off here

so I get  $\lambda$  there is a there are  $r^2$

so  $\lambda$  by four  $\pi \epsilon_0 r$  into integral minus  $\pi/2$  to plus  $\pi/2$  this is nothing but  $\cos \phi d\phi$  which is equal to  $\lambda$  by four  $\pi \epsilon_0 r \sin \phi$  minus  $\pi/2$  to plus  $\pi/2$  which is nothing but two

so this  $\lambda$  by two  $\pi \epsilon_0 r$

so please note my integration goes from minus infinity to plus infinity

so I have taken into account all charges present on the line charge and

so this is the total electric field produced and I know the direction of this because I as I showed you in the figure here the direction will have to be in this direction

so let me call this  $E$

so I will have the electric field produced by an infinitely long line charge distribution is at this point will be like this and  $E$  will be like this and that is equal to  $\lambda$  by two  $\pi \epsilon_0 r$  where this distance is  $r$  and this will be  $E$  and this is the line charge distribution which are charged per unit per unit length as  $\lambda$

so compared to a point charge where the electric field decreased as one by  $r^2$  in this case the electric field decreases as one by  $r$  distance or the distance of that point from the line charge and it is along the direction of the perpendicular drawn from the point where you are calculating the electric field to the line charge distribution

so you have seen a little bit of mathematical calculation to find out the total electric field at this point because of the entire line charge distribution now let me try to solve the problem using Gauss's law

so I go back and look at my problem again

so I have this infinite long line charge distribution charge density  $\lambda$  and I want to calculate the electric field at some point now what do I do as before I must use some symmetry arguments to find out the possible direction of the

electric vector and choose a gaussian surface on which the electric field remains constant ok

so first thing i notice is as i as you see here electric field cannot have this component as you already seen by symmetry arguments because of electric fields from two different elements cancelling of the normal component they cannot be this component even otherwise they cannot be component here because there is nothing in the line charge distribution which differentiates the upper direction from the lower direction because this is fixed there is nothing which says that this is different from this

so there cannot be a component of electric field in the vertical direction there cannot be a component of electric field perpendicular to the plane of the page because if it is coming out of the page why cant it go into the page

so there is no difference between coming out and going in

so there cannot be any electric field distribution in that direction

so the only possibility is electric vector is like this it has to be along the direction ah if i drop a perpendicular from here on the line charge electric field here electric field will be if i drop a perpendicular it will be like this here at this point i drop it will be like this

so electric field has to be pointing away from this line charge because it is a positive line charge distribution if it was negative all the vectors will be pointing towards the line charge and

so electric field will be pointing away from the line charge that's first thing second you see here that if you take a circle with the line charge at the center circle having a radius  $r$  electric field at these points must be the same because there is no difference between this point and this point at this point this point they are all the same because there is no difference there is nothing which differentiates this position from this position or this position

so electric field magnitude must be the same at all these points they are they are in the respective directions away from from the line charge but they have to be equal in magnitude

so i have got a line in which i know the electric field is the same i have got the vector direction of electric field

so now i choose my gaussian surface

so this is my line charge i choose a gaussian surface like this

so this is  $\lambda$  and this is my gaussian surface its cylindrical with the center of the line charge of radius  $r$  this is the upper surface there is a lower surface here and this is a cylinder surrounding the line charge distribution of radius  $r$  with the line charge at the center of the cylinder

so now i know first thing is that the electric field at this point will be pointing away from the surface of the cylinder normal to the surface of the cylinder they are all second the electric field on the upper surface is tangential to the surface the electric field of the bottom surface is tangential to the surface

so remember when we introduced flux we said it is dot product of the electric vector and the area vector on the upper surface area vector is like this electric field is parallel to the surface

so the dot product is zero here the outward normal is like this and the electric field is like this

so there is no flux crossing the upper surface of the cylinder and the lower surface of the cylinder the only flux that is crossing is through the cylindrical surface of the cylinder and on all points on the surface of the cylinder the electric field magnitude is the same number one number two electric field is always at all points along the normal to the cylindrical surface

so all i need to do is because the electric field is constant on the

cylindrical surface and it is along the same direction as the normal at every point on the surface the gauss's law will tell me the magnitude into the surface area of the cylinder which is  $2\pi r l$  if the length of the cylinder is  $l$  is equal to charge enclosed by  $\epsilon_0$  and what is the charge enclosed charge per unit length is  $\lambda$  this is the length  $l$  sorry large charge per unit length is  $\lambda$  this is the length  $l$  of the wire

so charge enclosed  $\lambda l$  by  $\epsilon_0$

so that gives me immediately an expression for the electric field  $E$  is equal to  $\lambda$  by  $2\pi \epsilon_0 r$

so this is this is the distance  $r$  and the magnitude the vector is along this direction

so let me go back and look at what was the expression I got before this was the expression which I got by integration of the total charge total electric field from coulomb's law that is the expression which I got from gauss's law and you can see how much more simplified gauss's law application was in this case and that's because I am using some symmetry arguments I am finding out from the symmetry arguments what could be the orientation of the electric vector I am then taking a gaussian surface on which the electric vector magnitude remains constant and that helps me to pull out the electric field out of the integral of the gauss's law and help me to calculate the gauss's helping calculate electric field using the gauss's law

so this is a very very very powerful method to calculate especially when there are symmetries in the system now let me go to another interesting problem which is another example field due to an infinite sheet of surface charge  $\sigma$

so I am taking an infinite series a sheet with surface charge  $\sigma$  charge per unit area and let me again assume this should be positively charged what is the electric field produced by this

so in fact I can again use a coulomb's law to calculate the electric field of the charge distribution but I will use gauss's law now to calculate what is the electric field produced by this infinite charge distribution surface charge distribution of  $\sigma$  now again I must use some symmetry arguments about the direction of the electric field and the magnitude of the electric field as you can see because the surface charge distribution is infinite in size all points at a given distance from the surface charge they are all identical there is no difference between this point at a distance  $d$  from the surface charge this point which is  $d$  from the surface charge please remember I am considering an infinitely large surface charge distribution it's not a finite charge distribution it's an infinitely large surface area

so I can actually ah I first thing I know is the electric field should only depend on it cannot depend on this position here

so anywhere in front

so if I had a plane surface like this if I had a plane of charge density charge distribution like this the electric field at this point at this point at this point they must all be the same in the same direction ah same magnitude because there is no difference in this point and this point at this point similarly on the other side because it will just depend on the distance from the surface charge distribution what about the direction of electric vector now again as before you can see electric vector cannot have this component because if it had that component why doesn't it have this component y density of this component why is it other component because all directions are exactly the same there is no difference between up and down or between left and right if there is an infinitely big surface also distribution

so there cannot be a component which is perpendicular

so if I draw a perpendicular from this point on the surface charge then there

cannot be a component perpendicular to the line that line because of symmetry simply

so the electric vector has to be pointing perpendicular surface charge

so if i again draw the surface charge distribution then at any point here electric field has to be like this at this point it will be like this at this point it will be like this and

so on and similarly on the other side if i look at like this and this is positive

so symmetry tells me electric field has to be normal to the surface symmetry tells me electric field can only depend on the distance if pos if at all from the plane and using these two i have to judiciously choose a gaussian surface which will help me to calculate the electric field

so the gaussian surface i choose as follows

so i take a cylindrical box of area  $a$  and the cylinder is normal to this plane surface  $ah$  and it is a cylinder intersecting the surface perpendicular

so this line is perpendicular and it is this this plane is passing through the center of the cylinder

so this length is equal to this length

so now let me look at what is the flux coming out of this closed surface the closed surface consists of these two flat surfaces here and a cylindrical surface connecting these two surfaces passing through the surface charge density such that the center the the plane surface charge density is intersecting the cylinder right through the center of the cylinder now as we have already argued electric vector has to be normal to the plane

so electric vector at all points will be like this and you see on the cylindrical surface the normal is perpendicular

so there can be no flux coming out from the cylindrical surface of the gaussian surface because the normal to the cylindrical surface is at every point perpendicular to the electric field vector and

so  $\mathbf{E} \cdot d\mathbf{a}$  will be zero at every point on the cylindrical surface

so the only flux that can be coming out is from the two areas on either side and

so the total flux will be and secondly i also know that the electric field at every point on the flat surface will be the same because they are all at the same distance from the cylinder from the sorry from the surface charge density

so all these points are the same distance from the surface charge density all these points are the same distance from the surface charge density the electric field magnitude here and electric field magnitude here are equal the electric field magnitude at every point on the surface is equal to electric field magnitude at every point on the surface

so the total flux total electric flux will be equal to electric field into one area here and one area there here the electric flux is coming out here there are triples coming out

so the total electric flux is  $E$  times two  $a$  and total enclosed charge  $\sigma$  times  $a$  because  $\sigma$  is the charge per unit area

so this cylinder will intersect an area  $a$  on the surface charge density which will then carry a charge  $\sigma a$

so i get if i use gauss's law this is the total flux this is the total charge enclosed

so total flux must be equal to total charge enclosed by epsilon zero which gives me  $E$  is equal to  $\sigma$  by two epsilon zero and  $ah$  if i draw this surface like this left it on the surface like this and if this is the direction  $ah$  if i call this as some end cap direction

so this is nothing but

so this end cap vector is perpendicular to the flat surface

so this is the plane here

so electric field at every point is pointing away from the flat surface charge distribution and as a magnitude  $\sigma$  by two  $\epsilon_0$  its interesting to note that the electric field is independent of the distance from the surface charge now you might ask how can this happen because if i am very far away from the surface charge the electric field must be zero but because this is happening because i am taking an infinitely sized surface charge distribution the surface charge is everywhere in the entire infinite plane and the electric field remains constant as you move away from the surface charge distribution and has a magnitude of  $\sigma$  by two  $\epsilon_0$

so there is electric field

so if i had this if i had a flat here

so here at this point the electric field is  $\sigma$  by two  $\epsilon_0$  in this direction at this point electric field is  $\sigma$  by two  $\epsilon_0$  in this direction at this point is  $\sigma$  by two  $\epsilon_0$  adjust that in this direction and

so on

so every point electric field is  $\sigma$  between  $\epsilon_0$  pointing away from the flat charge distribution if it was a negative charge distribution they will all be pointing towards the charge distribution

so as you can see here because of gauss's law i could very quickly calculate so i have to use initially some symmetry arguments which will help me to choose an appropriate gaussian surface and once having chosen appropriate gaussian surface on which the electric field magnitude remains constant i can then take the integral take the electric field out of the integral in gauss's law and it is easy for me then to calculate the total flux electric flux coming out of the closed surface and with that calculation i am able to immediately estimate what is the electric field

so one example which we looked at was a spherical charge distribution one was the line charge distribution and this is a flat charge distribution i can extend this to little more interesting problems for example if i take a thin conducting plate see if my conducting plate is like this and i have thrown a surface charge  $\sigma$  into it

so this is conductor

so as we discussed before if this is plus  $\sigma$  there are plus charges sitting here on the surface there are plus charges sitting here on the surface

so it produces some surface charge density  $\sigma$  here and  $\sigma$  here everything everywhere is positive charge

so assume that this is a very huge plate thin plate and i am sort of neglecting the ends of the plate and assuming that there is a surface charge density  $\sigma$  on the left surface and the surface has density  $\sigma$  on the right surface now you see this surface charge density produces an electric field this surface charge density produces an electric field and what you observe is the sum of the electric fields produced by this surface charging density and the surface does density

so let me look at the surface charge density on the left

so this produces  $\sigma$  by two  $\epsilon_0$  here it produces  $\sigma$  by two  $\epsilon_0$  here

so let me draw a another bigger figure

so there are plus charges sitting here the plus charges sitting here

so this produces  $\sigma$  by two  $\epsilon_0$  in this direction  $\sigma$  by two  $\epsilon_0$  in this direction this charge produces  $\sigma$  by two  $\epsilon_0$  in this direction and  $\sigma$  by two  $\epsilon_0$  in this this charge also produces

$\sigma$  by  $2\epsilon_0$  zero here this charge also produces  $\sigma$  by  $2\epsilon_0$  zero here

so what's going to happen inside you see here that this  $\sigma$  by  $2\epsilon_0$  zero produced by this surface charge density and this  $\sigma$  by  $2\epsilon_0$  zero produced by this sufficient density are exactly equal and opposite to each other canceling off and producing electric field is equal to zero inside the conductor

so as you can see here in this problem the charges if it is a solid conductor the charges in this problem would equally distribute on the front and the back surface this surface and this surface

so that the field produced by this surface charge density and the field produced by this surface charge density are exactly equal and opposite and cancel each other off to produce zero electric field inside here the total electric field will be sum of the electric fields which is  $\sigma/\epsilon_0$  zero here and there is  $\sigma/\epsilon_0$  zero here the net electric field is this here on this side and  $\sigma/\epsilon_0$  on the other side

so from these charge distributions actually i can calculate the net charge distributions now let me look at another example which we will come to ah later in capacitors

so i have the following problem i have two plates plus charge density here and minus charge density here

so  $\sigma$  and  $\sigma$  is minus  $\sigma$  now these are two conductors

so this produces an electric field  $\sigma$  by  $2\epsilon_0$  zero here this produces electric field  $\sigma$  by  $2\epsilon_0$  zero please note this is positive charge

so electric field is pointing away from this charge this is negative charge electric field is pointing towards the charge at this plane this plus produces  $\sigma$  by  $2\epsilon_0$  zero

so and this one produces  $\sigma$  by  $2\epsilon_0$  zero on the other side

so the sign i am taking by drawing an arrow in the opposite direction ok the magnitude of  $\sigma$  by  $2\epsilon_0$  zero one is positive charge produces electric field like this negative charge produces like this similarly here the positive charge produces  $\sigma$  by  $2\epsilon_0$  zero here and the negative charge produces  $\sigma$  by  $2\epsilon_0$  zero

so you can see that the net is zero everywhere including inside except in this region where the field becomes  $\sigma/\epsilon_0$  the two fields add here the two fields cancel everywhere else

so we see a similar situation in a capacitor problem where we bring into ah conductors facing each other carrying some charges and as you will see that the charges

so this positive charge attracts the negative charge to the side the negative charge attracts the positive charge to this side the charge is set up such that electric field inside the conductors is zero and the electric field exists only within the spacing between the two conductors and that will as we will see later on forms a very very important element in electrostatics which is the capacitor problem

so what we have seen is that you can use gauss's law to calculate electric fields produced by charge distribution by choosing an appropriate gaussian surface we must use the symmetry present in the problem to help us to calculate the integral total total flux and once i know the total flux if the ah the electric field i do not know i still can calculate the total flux and by symmetry if i can do that then i can calculate the electric field produced by the charge distribution and as we have seen in some of these

so it is useful in symmetric situations but let me again tell you gauss's law is always valid whether there is symmetry or there is no symmetry the electric flux coming out from any closed surface if i take any closed surface  $s$  integral

$\oint \mathbf{E} \cdot d\mathbf{a}$  is always equal to  $q_{\text{enclosed}} / \epsilon_0$

so the total flux coming out any from any closed surface is the charge enclosed by  $\epsilon_0$  if the flux is zero it does not imply zero electric field it only implies that the net charge is zero

so i will leave a problem at the end of my talk here for you to think about positive charge  $q$  is distributed uniformly throughout the volume of an insulating sphere of radius  $r$  using gauss's law obtain the electric field inside and outside the sphere and you can compare this problem with the gravitational field produced by a mass spherical mass which is uniformly distributed across the volume of the sphere thank you very much you

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