

good morning to all of you today we will continue with our discussion on electrostatics

so today we are going to discuss a very very important law in electrostatics and that is gauss's law gauss's law relates electric field and charges and for this we would need to introduce the concept of flux concept of flux

so this flux actually comes from the word it means flow in latin

so first we will introduce the concept of flux and i will show you that gauss's law relates electric flux and charges

so to introduce the concept of flux let me consider a fluid flowing with a uniform velocity

so i can take for example ah the fluid is flowing like this say let me assume that this is x this is y and this is z

so let me assume the fluid is flowing along the y direction i take a a surface like this c of some length l and with l and this this frame is placed perpendicular to the flow direction

so the liquid is flowing along the y direction and this frame is parallel to the exit plane

so the fluid is crossing this surface and moving from left to right the question i ask myself is what is the volume of fluid crossing this surface per unit time what is the volume

so the fluid is flowing through the surface

so how much volume of fluid is flowing per unit time through the surface now if i look at the fluid flowing through the surface

so here is my surface fluid is flowing like this through the surface now you see if the surface area is s and the velocity is v as i wrote then how much fluid will cross how much volume of fluid will cross to understand that let me take a length equal to v from here

so this length is b

so there is a imaginary plane i consider distance v from the surface this is my actual surface through which i am trying to find out the rate of fluid flow and i consider an imaginary surface at a distance v from this surface now as the fluid flows remember the flow the fluid is flowing the velocity v

so this surface after unit time would have come and into the surface

so this surface i attached with the surface i move with the fluid and in a unit time this surface the back surface would have come and coincided with the front surface

so what it implies is all the fluid contained within this volume would have crossed the surface in a unit time

so volume rate of flow will be the volume contained in this wa in this the volume of fluid contained in this in this volume and what is the volume of this the surface area into this length

so that means v into s

so this is called flux flux of fluid flow through this area is velocity times the surface area and in this case i am considering the area to be perpendicular to the direction of flow

so this implies that v times s volume of fluid is flowing crossing this area per unit time now thats an area perpendicular to the flow direction but suppose my area was not perpendicular to the flow direction suppose the liquid was flowing like this and my my my frame happened to be at an angle

so this returns an angle theta

so this is my frame now i have inclined to play the frame the same area i have inclined and now you see that the rate of flow will change because imagine the limit when this frame becomes parallel to the direction of fluid flow no fluid will be crossing the area because they are all flowing parallel to the area

so now how do i calculate this

so let me look from this side

so i have this line this is my vertical this is theta

so i can take a point which is at a distance  $v$  from here this is my area  $s$

so all the fluid within this volume

so this this is also  $b$  all the fluid contained in this volume would have flown through the surface in a unit time just like in the earlier case the fluid which was lying at a distance  $v$  from the surface would have crossed the surface in a unit time here also all the fluid contained within this volume would have crossed the surface and the volume of this fluid now is nothing but  $v$  times  $s \cos \theta$  this is  $s \cos \theta$  and this is  $v$  that's a parallelogram and its volume its area is  $v s \cos \theta$  multiplied by the other dimension if you multiply by the other dimension this dimension then you get the volume which is  $v s \cos \theta$

so the this is now the flux is  $v s \cos \theta$  this flux is reduced because the this area is now projected with a smaller area as far as the fluid flow is concerned

so this area this area is inclined to the fluid flow

so although the area is  $s$  the fluid is actually only crossing through this area which with its projection

so if you see if theta becomes ninety degrees  $\cos \theta$  becomes zero and the flux becomes zero

so if the if the fluid is flowing like this and your frame happens to be like this obviously there is no fluid crossing the surface it is just grazing the surface and going away

so the flux depends on the angle between the surface and the direction of fluid flow and the flux happens to be  $vs \cos \theta$  now what is  $v \cos \theta$   $v$  is this direction and theta is this angle

so if i draw a normal to the surface this vector is perpendicular to the surface this line is perpendicular to this line

so this angle is also theta

so this is nothing but  $v \cdot n$  into  $s$  because  $v \cdot n$  is  $v \cos \theta$   $n$  is the unit normal to the surface  $s$  and

so  $v \cdot n s$  is the flux now

so this tells me that the flux now flux of fluid flow depends on the angle made by the surface with the direction of the fluid flow now i can write this in a more compact form by introducing what is called as a vector area

so if i have an area if i define an area like this suppose the area is  $s$  and this is the direction of the normal to this area i define the vector area  $s$  vector is equal to  $s$  times  $\hat{n}$  vector area is a vector whose magnitude is equal to the area of the surface and whose direction is the normal to the surface of course for a surface like this whether i choose this normal or that normal there is an ambiguity but later on we will be discussing closed surfaces in which this ambiguity is resolved

so a vector area contains not only the magnitude of area but also the unit vector perpendicular to that area

so vector area not only gives me the area but also its orientation for example if i take an area suppose i have the three axis like this  $x$  by  $z$  if i take an area here in the exact plane its normal will be pointing like this

so the area is this is vector area will be  $s$  times  $\hat{j}$  if you take another area the same area and put it in another direction say for example i put it like this this area is this is  $\hat{i}$

so vector area here is  $s$  into  $\hat{k}$

so the magnitudes of these areas are equal but the directions are different and

that is contained in the unit normal

so vector area is a very useful concept that you will be using in many different subjects but this vector area contains not only the magnitude of the area but also the direction of that is the direction of the normal to the area

so what we now like to do is i would like to calculate what is the flux

so although i have introduced the flux as the flow of a fluid this flux concept can be generalized to all vector fields

so i can define an electric flux with respect to the electric field vector we know the relative field is the vector and

so if you have for example a surface like this of area  $s$   $s$  vector is like this and if the electric field is pointing in the direction this uniform electric field i will define the electric flux  $\phi$  as  $e$  times  $s$  actually it is  $e \cdot s$  where  $s$  is the vector area and  $e$  is the electric vector and

so  $e \cdot s$  is nothing but because in this case  $e$  and  $s$  are parallel  $e \cdot s$  is  $e$  times  $s$

so that's the electric flux in the case of fluid flow there was actually a fluid which is flowing through the surface but in the case of electric field there is nothing which is flowing i have extended the concept of flux to a vector field like electric field

so i can this will give me what is the something like the number of electric field lines crossing the surface area but this is there is nothing flowing in the electric field case it is simply represents a quantity

so now let me try to calculate i am more interested in a calculation in which i want to see what is the flux crossing a closed area

so i am taking a closed surface which is closed

so as an example let me take a cube of side  $l$  cube of side  $l$

so this tube has and this cube is oriented along the  $xyz$  axis like this which i have shown and let me assume that i have a constant uniform electric field  $e$  vector along the  $y$  direction

so maintaining vector is pointing like this along the  $y$  direction it is supposed to be uniform

so i want to calculate what is the net flux of electric field passing through this closed surface this is the closed surface as you can see here it is a cube which i am taking as an example and the cube encloses a volume and the cube is a completely closed surface now although the normal to the surface was ambiguous in the earlier case here we always defined the normal to be the output normal

so outward means a normal direction which is pointing out of the volume

so for example on this surface the outward normal is in this direction on the surface the outward normal is up on the bottom surface the outward normal is downward on the side normal the normal is here and on the surface the normal is another direction and behind the normal is like this

so there are six surfaces and six normals and all the normals are taken to be outward normals

so i want to calculate what is the net flux of electric field through this volume through all the surfaces

so what i will do is i will calculate the net flux through the surface through this surface through this side surface here and through the back surface the top surface and the bottom surface

so there are six surfaces we will individually calculate the flux cross flux of electric wheel crossing each and every individual surface add them up and get the total flux

so let me start calculating the flux and let me show you some slide

so in the slide as you can see here the this is the cube this is the electric field which is pointing along the  $y$  direction

so the electric field is  $E$  in the  $y$  direction these are the electric field lines and this is the cube the same cube shown with all the surface normals

so the normal to this surface which I have called  $bcgh$  is along the  $x$  direction  $i$  direction the one which is behind which is  $adif$  is minus  $s$  in the  $x$  direction the top surface  $ghif$  has the normal along the  $k$  direction the bottom surface  $bcd a$  is minus  $k$  direction and similarly this surface  $hc d i$  has plus  $s$  in the  $y$  direction and this back surface  $g b a f$  has minus  $s$  in the  $y$  direction

so these are all the six surface normals

so I need to calculate what is the flux crossing each one of the surfaces

so let me calculate on this sheet of paper here

so let me consider the surface

so let me draw the figure here

so I have this cube  $x$  by  $z$  and

so electric vector is  $E$  in the  $y$  direction

so plus let me calculate what is the flux through the surface

so this surface has a unit vector which is  $j$  direction

so this of area  $s$

so the

so let me call this flux  $\phi_1$  let me use the same indices this is called  $bcd a$  and this is  $ghif$

so this is a flux  $\phi_1$  through surface  $hc d i$

so this will be  $E \cdot s$  which is  $E$  in the  $y$  direction dot  $s$  in the  $y$  direction which is nothing but  $E$  times  $s$  the surface is  $s$  in the  $y$  direction electric vector is  $E$  in the  $y$  direction

so the flux through the surface is  $E \cdot s$

so  $E$  in the  $y$  direction dot  $s$  in the  $y$  direction which is  $E$  times  $s$  now what is the flux through the back surface that is this back surface

so this is  $afgb$  this is equal to again  $E$  in the  $y$  direction dot  $s$  which is equal to  $E$  in the  $y$  direction now remember the back surface has a unit vector which is  $s$  times minus  $j$  direction

so this is equal to minus  $E$  times  $s$

so you can understand that the flux is negative because the surface area is pointing towards minus  $j$  direction the electric vector is pointing towards plus  $j$  direction and the dot product of these two is minus  $E$  times  $s$  we can similarly calculate the flux through the remaining surfaces

so let me take one more example

so flux through let me calculate the flux through the surface which is  $bcgh$

so this is equal to let me call it  $\phi_3$  which is equal to  $E \cdot s$  which is equal to  $E$  in the  $y$  direction dot  $s$  in the  $y$  direction now  $s$  vector is what let me see here

so this is  $s$  and this  $s$  vector is actually  $s$  times  $i$  direction because this normal is pointing towards  $i$  direction dot  $s$  in the  $y$  direction which is equal to zero because  $j$  direction dot  $i$  direction is zero  $j$  and  $i$  are perpendicular to each other

so that is zero and you can again understand this because as I mentioned before the electric vector is pointing along the  $y$  direction and the surface is actually in the  $y$  direction is parallel to the surface

so there are no electric flux lines crossing the surface

so similarly you can show that the flux crossing the back surface the bottom surface and the top surface are all equal or zero or equal to zero and

so the total flux is nothing but the sum of these two and that happens to become equal to zero

so total flux for this example total electric flux is equal to  $E$  times  $s$  minus  $E$  times  $s$  which is equal to zero

so there is no flux electric flux crossing through this is zero please note electric field is not zero electric field is finite it is uniform but it

so happens that the amount of electric field lines entering one surface is equal to the electric field lines leaving the surface

so this particular flux is from the front surface here this flux is from the back surface and they assume they are equal to each other and opposite of each other of opposite sign and

so the total becomes zero and there is no flux entering the remaining four surfaces and

so the net flux becomes zero

so i can actually calculate using this flux formula what is the flux of a vector field through any closed surface now let me show you another slide

so this in this case i had a cube which was exactly oriented with the axis now let me take a situation where the cube let me look show you the slide in which the cube is now not placed along the axis but inclined

so i have rotated the cube around the z axis

so that the line a b makes an angle of theta with respect to x axis

so now i need to again i want to again calculate the total electric flux

so for which i need to draw all these surface normals

so here is a slide which shows you

so the the front surface which is shown as red here has an electric flux pointing the surface area is pointing in this direction theta is the angle between the x axis and this plane

so we can write the surface area area vector for this surface similarly the area vector for this surface which is exactly opposite of this vector because it is in the opposite direction the area vector for this surface the area vector for the back surface the area vector for the top surface and the area vector for the bottom surface

so this for example this vector you can you can see but this line is parallel to this line here and

so this makes an angle theta with x axis

so this unit vector has a component  $\cos \theta$  along x direction and  $\sin \theta$  along the y direction and that is why the area vector is given by this the magnitude of this vector is same as s and the direction is given by  $i \cos \theta + j \sin \theta$

so i can actually find out the unit vectors of all surfaces and then from these unit vectors i can calculate the total flux

so for example let me calculate the flux through the front red surface is shown in the slide

so the electric flux

so electric flux now ah i have gone back to the surface v c h g that is this of this surface here front surface and let me show you the slide in this in this slide here

so this is let me call this  $\phi_1$  which is equal to  $\mathbf{e} \cdot \mathbf{s}$  which is equal to  $e \cos \theta$   $j \cdot s$  into  $i \cos \theta + j \sin \theta$  which happens to be equal to  $e \cos \theta s \sin \theta$  because  $j \cdot i$  is zero this flux becomes  $e \cos \theta s \sin \theta$  the flux through the back surface which if you see the slide here this back surface now the flux flux through a d i f which is exactly the surface opposite to the surface in this in the slide you can see the ah area vector

so  $\phi_2$  is equal to  $\mathbf{e} \cdot \mathbf{s}$  which is equal to  $e \cos \theta$   $j \cdot s$  into  $-i \cos \theta - j \sin \theta$  which is equal to  $-s$  into  $e \cos \theta \sin \theta$

so you got the flux through the back surface similarly you can calculate the flux through the remaining surfaces the flux to the top surface will be zero the flux through the bottom surface will be zero and the other two fluxes let me

write down the expression here the other two fluxes will be if i call this  $\phi_3$  is equal to  $ah$

so if you look at this this surface the surface which is shown blue in the slide here if you can see the slide it is shown as the blue surface here and that is through the through that is  $\mathbf{E} \cdot \mathbf{s}$  which is  $E \sin \theta$  plus  $\mathbf{j} \cdot \mathbf{s}$  into  $\mathbf{i} \cdot \mathbf{s} \sin \theta$  plus  $\mathbf{j} \cdot \mathbf{s} \cos \theta$  which is equal to  $E \sin \theta$  and finally the flux through the surface which is opposite to this surface is  $\mathbf{E} \cdot \mathbf{s}$  which is equal to  $E \sin \theta$  plus  $\mathbf{j} \cdot \mathbf{s}$  into  $\mathbf{i} \cdot \mathbf{s} \sin \theta$  minus  $\mathbf{j} \cdot \mathbf{s} \cos \theta$  which is equal to  $-E \sin \theta$  and flux through top and bottom surfaces equal to zero because the the normal the surface normals or the surface areas are perpendicular to the direction of electric factor

so we got four fluxes too

so you have flux through the  $ah$  four surfaces one is  $E \sin \theta$  the other is  $-E \sin \theta$  one of them is  $E \cos \theta$  the other one is  $-E \cos \theta$  and you can now see the total flux will be the sum of all these four fluxes and that happens to be again zero

so  $E \sin \theta$  minus  $E \sin \theta$  plus  $E \cos \theta$  minus  $E \cos \theta$  which is equal to zero

so the flux crossing each surface has changed but the net flux is still zero

so this is the technique to calculate the flux through any close surface i draw i look at the normal to the closed surface and then calculate  $\mathbf{E} \cdot \mathbf{A}$  for each of these areas and i will get the total flux

so now we will come to gauss's law

so once having defined electric flux electric field flux now let us look at gauss's law

so as an example to to calculate electric flux let me consider the charge  $q$  and let me take a sphere around the charge sphere of radius  $r$

so this is a sphere of

so my problem is to calculate what is the electric flux crossing the surface what is the electric flux crossing this sphere because of the point charge placed at the center of the sphere now what is the electric field generated by the point charge we know this one by  $\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$  where  $\hat{r}$  is this direction and  $r$  is the distance from the center

so this is the electric field at any point at a distance  $r$  from the charge and  $\hat{r}$  is a unit vector from the charge pointing out like this along the radial direction now i am assuming a positive charge here

so the unit vector the  $\hat{r}$  vector  $\hat{r}$  is along this direction now to calculate the total flux through the sphere i must know the area vector

so for example at this place the area vector will be pointing like this at this place the area vector will be like this at this place the area vector will be like this they will all be radially pointing away from the center it is a sphere

so the area vector of any patch of the sphere will be pointing away from the center and

so this will be the area this will be the direction of the area vector

so what you can see is unlike the example before the area vector the area direction of the area vector keeps on changing as you move along the surface but at all points the area vector is along the line joining the center to the center of that area vector now as you can see here the electric field is radial along the line joining the charge to this to the point on the sphere and at that point

so suppose i want to calculate the flux to a small area here this is the electric field direction and the area vector is also in the same direction

so what i need to do is because of a surface which is not flat what i must do is i must take a small area here a small area  $d\mathbf{s}$  vector then i calculate i know

the electric field at that point

so i calculate the small flux through that which is  $\mathbf{E} \cdot d\mathbf{s}$

so  $d\mathbf{s}$  is a small area  $d\mathbf{s}$  vector is a small area vector  $\mathbf{E}$  is electric field at that point i calculate  $\mathbf{E} \cdot d\mathbf{s}$  there that gives me the flux through this small area

so i calculate like this i divide the entire sphere into areas at around all around the place and add up all the fluxes to get the total flux now as i mentioned at every point the area vector points along the directional electric vector because electric vector is radial and

so is the area vector because the this charge is located at the center of the sphere

so  $\mathbf{E}$  and  $\mathbf{s}$  become equal in the same direction the other thing i notice is that electric field across all the points on the sphere is the same because the charge is centered at the in at the center of the sphere the electric vector electric field at all points on the sphere is exactly the same and electric field on the sphere will be the magnitude will be one by four pi epsilon zero q by r square

so electric field is the same at all points on the sphere the electric vector is parallel to the area vector at all points on the sphere

so the total total flux will be electric field into the area of the sphere because electric field is the same at all points on the sphere

so the total flux will be electric field into four pi r square which happens to be q by epsilon zero

so if you have a point charge placed at the center of a sphere the net electric flux flowing through the sphere is q by epsilon zero that's a statement of gauss's law if you have a point charge at the center of a sphere then the total flux electric flux crossing the sphere is q by epsilon zero now what will happen if the surface which i am considering i am let me take the same point charge but a surface which is not a sphere

so the problem would be what will happen to the flux

so i want to calculate the flux through a surface which is not a sphere

so i have the point charge here this was the sphere which i drove to earlier and i have some some arbitrary surface

so let me show you a slide here in a slide i have i am showing you a charge placed at the center of a sphere in the slide you can see the center of the sphere having a charge

so this is the sphere here which i am drawing as  $S_1$  and there is some arbitrary surface here which i am calling  $S_2$  and these lines represent the electric field lines from this point charge they are all radial pointing away from the point charge

so i take a small area here and i draw the electric field lines crossing that area now as you can see here these lines the electric field lines come out and hit the other area the  $S_2$  surface over some other magnitude of area and orientation please remember this arbitrary surface is has its normal at every point in different directions

so here i have drawn a small patch having its area vector shown as an arrow here the area vector on the surface of the sphere is along this direction because this is the center of the sphere and electric field is also parallel to it here the electric field is pointing like this and the area vector is pointing in some other direction

so i need to calculate for the flux through this i had simply multiplied by electric field by the area here i need to remember that this is makes an angle

so i will have to calculate the dot product of this electric vector and this area vector now as you can see here if i imagine electric field lines coming

from the point charge all those lines which are crossing this small area here will also be crossing the same area here although this area is larger it is oriented in a different direction and

so its projection will be along a direction perpendicular to this electric field line and as we discussed before it will have a component  $d s \cos \theta$  and if I can imagine electric field lines starting from the point charge and moving out the number of electric field lines crossing this area on the sphere will be equal to the number of electric field lines crossing this area on the arbitrary surface

so I can extend this argument and for every patch of area on the arbitrary surface I can make a projection back to the point charge and that projection will intersect the sphere in a small area and

so what I will see is for every area on the arbitrary surface I have a small area on the sphere and they will have the same flux of electric field passing by

so what this argument implies is that the net flux crossing the sphere is exactly equal to the net flux crossing this arbitrary surface area you can also imagine this by understanding that these are electric field lines all the lines which are emanating from this point charge which are crossing this surface spherical surface will also be crossing this other surface area and

so the net flux of electric field through the arbitrary surface is the same as the flux through the sphere and the flux through the sphere we have just calculated as  $q$  by  $\epsilon_0$

so if I come back to the slide here as we see the point charge has the flux  $q$  by  $\epsilon_0$  whether you take the sphere or an arbitrary surface surrounding the point charge

so this is the generalized Gauss's law

so Gauss's law states that the flux through this through an arbitrary surface which encloses this point charge  $q$  is  $q$  by  $\epsilon_0$

so this implies essentially that whether the point charge is at the center of the sphere or anywhere if you put the point charge here even then the flux it will be  $q$  by  $\epsilon_0$  this is point charge  $q$  irrespective of where you put the net flux will be zero sorry two by seven zero and independent of position of this charge because this this will appear like an arbitrary surface surrounding this point charge now what will happen if I have more charges

so suppose I have a charge  $q_1$  one another charge  $q_2$  the total flux will be equal to  $q_1$  by  $\epsilon_0$  because of charge  $q_1$  plus  $q_2$  by  $\epsilon_0$  because of charge  $q_2$  if I have another charge  $q_3$  plus  $q_3$  by  $\epsilon_0$  because of charge three etcetera

so this will be nothing but  $\sum q_i$  by  $\epsilon_0$  which is equal to  $q$  by  $\epsilon_0$  where  $q$  is the total charge enclosed enclosed by the surface let me write it again here

so if I had if I had a number of charges  $q_1$   $q_2$   $q_3$  etcetera

so if I consider any surface  $A$

so total electric flux is equal to the sum of the charges all the charges inside by surface which is  $\sum q_i$  and this  $\sum q_i$  is charges imposed and that is Gauss's law

so Gauss's law states that if you have any surface which surrounds a set of charges the total electric flux crossing that surface is equal to the sum of all the charges present within this within the enclosed by the surface divided by  $\epsilon_0$  this is a very very important law and this law is used to solve problems in electrostatics especially when you have certain kind of symmetries in the problem as I will show you through examples it is very easy to use Gauss's law to calculate electric fields or if I have if I know the electric field in certain situations I will be able to calculate what is the charge

distribution

so what this gauss's law states it tells me is the relationship between the electric flux crossing a surface and the charges enclosed by the surface now i must mention a couple of points here in this gauss's law these are the charges enclosed by the surface

so suppose i have a surface suppose i have  $q_1$  one charge here another charge  $q_2$  another charge  $q_3$  the total flux is equal to  $q_1 + q_2$  by  $\epsilon_0$  because  $q_3$  is not enclosed by the surface  $q_3$  is not enclosed by the surface

so as you can see here the electric field lines will go like this and as we saw in the case of cube what will happen is there are field lines entering the surface and they are also field lines same field lines will leave the surface

so the net flux because of the charge lying outside the volume enclosed by the surface will not contribute to the total flux

so in this flux equation  $\phi$  is equal to charge enclosed by  $\epsilon_0$

so in this flux equation we are only adding up the charges which are enclosed within the surface or by the surface and any charge lying outside the surface does not contribute to the flux at the same time please remember that the electric field at any point is the sum of the electric field produced by all charges whether inside or outside

so the electric field here will consist of electric field because of  $q_1$  plus electric field because of  $q_2$  plus electric field because of  $q_3$  the flux which is crossing this will only depend on  $q_1$  and  $q_2$  because  $q_3$  the flux from  $q_3$  actually the net amount of flux entering the surface because of  $q_3$  will be equal to the electric flux leaving the same surface

so  $q_3$  does not contribute to the flux while  $q_1$  and  $q_2$  which are enclosed within the surface are actually contributing to the flux

so we must remember that electric field at every point is determined by all charges present in the system while the flux through any closed surface is determined only by the charges enclosed by that surface

so this particular law is valid for any arbitrary surface and is useful in situation where there is symmetry as we will discuss later as an example wherever there is symmetry in my system i can use this eq this law to find out the electric field because of a charge distribution in some situations i can use in the reverse case when i have an electric field i know the electric field i can that use that to calculate the charge distribution also remember that this law is based on the inverse square law of coulomb's inverse square law like in the example of a charge at the center of a sphere which i calc which i calculated the flux in this remember we had done the electric field was varying as  $1/r^2$  the area was increasing as  $r^2$

so the flux is independent of the radius of the sphere

so whether you take a small sphere or a big sphere the electric flux remains the same now this is based on the fact that electric field goes as  $1/r^2$  the inverse square law if the electric field has was not following the inverse square law then the flux would have depended on the radius and this things would have been very different also remember that because the flux law depends on inverse square law all vector fields which follow the inverse square law will satisfy a gauss's law

so gravitational field which also decreases as  $1/r^2$  we also have a law which is similar to gauss's law it also satisfies law similar to gauss's law now let us calcu lets use some of this discussion to look at some examples the first example i want to see is conductors

so we saw earlier that a conductor is a is a medium in which there are free electrons which can flow and because of this in a static situation there can be

no electric field within the conductor because if an electric field existed within the conductor that will push the electrons that will force the electrons to move and it will not be in a static situation

so once it has reached equilibrium there can be no electric field no electrostatic field within a conductor now suppose I consider the following problem I take a conductor solid conductor and put some extra charges into the conductor

so these are called excess charges these are charges beyond the electron and proton that present in the conductor that are present in the conductor

so I put some extra charges now the question arises where are these charges sitting are they inside the volume of the conductor or are they on the surface of the conductor or are they in both places

so we will use Gauss's law to solve this problem

so now I have a conductor in which I have thrown some excess charge

so I throw a charge  $q$  some excess charge  $q$  into the conductor

so the question is where are they sitting now from my earlier argument in electrostatic situation  $E$  must be zero inside inside the volume of the conductor there is no electric field inside the conductor

so what I do is I take a surface inside the conductor I take a  $cr$  inside the conductor the entire sphere is inside the conductor now this is called a gaussian surface to employ Gauss's law I consider a surface an imaginary surface of any arbitrary shape which is which suits me which is called a gaussian surface

so in this case I take a sphere for example the sphere encloses the entire conductor and I want to apply Gauss's law

so first thing is the electric field at all points on this must be zero because there is no electric field within the conductor

so the net flux must be zero because net flux is equal to charge enclosed  $\epsilon_0 \oint E \cdot dA = q_{enclosed}$  and because electric field at every point on the surface is zero the net flux crossing the surface is zero and that means the net charge enclosed by the surface is  $0$ .

now please remember when we calculate the net flux we have to be aware that the charges can be negative or positive

so for example if I have if I take a positive charge at the center of a sphere the flux will be  $q$  by  $\epsilon_0$  zero if the charge is negative

so here the electric field lines are going like this here if it is a negative charge the electric field lines are moving inside

so the flux will be minus  $q$  by  $\epsilon_0$  zero if I take a situation where I have a plus  $q$  and a minus  $q$  for example a dipole and this is my surface the net flux will be zero because plus  $q$  by  $\epsilon_0$  zero because of this

so we have seen the electric field lines like this as many electric field lines will come out as will get back in

so the net flux because of the presence of these two charges become zero and that is because the total charge enclosed by the surface becomes zero

so in flux calculation I must keep track of the sign of the charges

so let me come back to the conductor here I am taking a gaussian surface and I find that electric field at every point on the surface is zero

so the net flux must be zero which implied that the charge enclosed is zero now I reduce the radius of the sphere to smaller and smaller values the electric flux continues to be zero until I reach almost a point which means that the charge enclosed by that sphere becomes is always zero no matter what the size of the sphere is which means that there can be no excess charge within the conductor

so I can take the sphere at different points on my conductor wherever I want

and i find that the net flux enclosed by this sphere is zero by the sphere is zero but this fear is zero by this fear is zero

so and because the net flux is zero and that is because electric field is zero and i can reduce the size of each sphere to smaller and smaller values until i reach a point there can be no charge inside the volume of the conductor no excess charge

so all it implies is whenever you put an excess charge on a conductor all the excess charge remains on the surface surface

so by excess charge as i said the charge which we add to the conductor these do not include the electrons and protons which are part of the material of the conductor

so here is an example where i have used gauss's law to find out whether there are charges within excess charges within the conductor which means knowing the electric field to be zero i have argued using gauss's law that there can be no excess charge within the volume of the conductor all the excess charge that you put will be sitting on the surface

so that is an interesting result that i get by using gauss's law

so here is a this was an example where i used gauss's law to calculate the charge distribution from known electric field distribution

so electric field was zero inside

so that gives me no charge inside now let me calculate let me take another example

so let me take a sphere ah conducting sphere with charge plus q

so here is a sphere a solid sphere and i put charge q on this now from my earlier argument all this charge must be sitting on the surface of this conductor there is no no charge inside the conductor all this charge is sitting on the surface of the conductor now symmetry plays a very important role in many of these problems

so the question is when i put the plus q charge excess charge on the conductor where is it on the surface

so first thing is from gauss's law i have shown that the charge must be residing on the surface it cannot be within the within the volume of the conductor

so where is it sitting now if you see a sphere is completely symmetric there is no preferential point anywhere on the sphere which means that the charge must be equally distributed everywhere across the sphere on the surface of the sphere there cannot be any point on the sphere which has little more charge because all points on the sphere are equivalent to each other

so when i put a charge plus q on the sphere that will be distributed uniformly right across the surface of the conductor and

so this charge will generate a surface charge density remember i had called it sigma before  $q$  by  $4\pi r^2$   $r$  is the radius of the sphere

so the charges are all sitting on the surface of the sphere as a surface charge density  $q$  by  $4\pi r^2$  now i want to calculate what is the electric field produced by this conducting conductor with an excess charge plus q which is uniformly distributed on the surface what is the electric field that this conductor will now produce as far as outside region is concerned

so i will now use again gauss's law first thing i could have in principle i have to solve the problem by taking every charge on this surface of the conductor calculate the electric field at a point suppose i want to calculate the electric field here i must take the charge here find out the electric field from here what the electric field from here part of the electric field of this point from here and

so on all the electric fields i must add at this point to calculate the total

electric field that problem becomes little more involved we can use gauss's law to calculate the electric field of this ah charge conductor

so this i will discuss in the next class where i will calculate what is the electric field produced by this conductor which is a spherical conductor in which i have thrown an excess charge capital  $q$  and we will use gauss's law and we will see how the calculation becomes very simple i would like to end the discussion here with one problem for you to think about

so let me consider a plus  $q$  here a minus  $q$  here a minus two  $q$  here and a plus two  $q$  here

so i draw two surfaces this i call  $s$  one and this i call  $s$  two

so first is calculate the electric flux through  $s$  one and  $s$  two to draw a closed surface through which flux is a maximum and b and negative and another one is positive and maximum

so i want you to draw surfaces gaussian surfaces through which the flux is positive and maximum a surface in which the flux is negative and maximum thank you very much you