

in the previous lecture we talked about super position of waves and that resulted in something called the standing waves as an example we discussed standing waves on a string we found that given a length l of the string certain frequencies only were allowed for it to vibrate with that is because the string if it is of length l and tied at both ends its ends could not move and therefore they were nodes and the wavelength had to be such that the string did not move at these points at all

so either you had half a wavelength on it or one full wavelength on it and so on on the other hand we also considered situation where one end was fixed and the other end was free to vibrate its like if you take a long rope hold it in your hands and move your hands up and down in that case we saw that either one quarter of a wavelength could be excited or three quarters of wavelength could be excited and so on and these gave the frequencies with which the string could vibrate exactly along the same

lines we can also now discuss the vibrations of an air column in a pipe and let me explain what that means what that means is suppose i have a pipe it could be open ended on both sides or it could have a closed end on one side open end on the other and the air inside it can vibrate when air inside it vibrates for example when you see a flute being played the air column inside is vibrating what kind of frequencies can it vibrate at now remember air column vibration is described by

so vibration of an air column is described by pressure variation and if you look at these two pipes we have been discussing that the open end the pressure would be the same as in the atmosphere so pressure variation at the ends would be zero whereas in the middle it could be quite large similarly for the open ended or the closed end pipe on the right side

Δp would not be zero because the closed end out here this wall can withstand any pressure on the other hand the open end Δp will be zero let us go to the next slide and see what does that mean

so if i consider the open ended pipe Δp on two sides is zero and it could be Δp is 0 and in between it could be nonzero if the air column is vibrating this is exactly the same situation as that of a string remember in a string the displacement Δy at the ends was zero and in the middle Δy was not zero Δy or y the displacement

so the string could vibrate in different modes and so on in exactly the same

manner now if i look at this open ended pipe and if i were to plot the variation of the pressure in the middle it will be zero at the ends it could be large right at the center or it could be zero at the ends and a

different kind of variation at the center but you see the change in pressure as a function of length is exactly the same as change of displacement of a string as a function of length on it and therefore their vibration frequency should be the same now how do we deduce that recall that in the case of the pipe i am going to write Δp which would be described by depending on how i choose my coordinates so i will take the left hand to be $x = 0$ and the right hand to be $x = l$ that is the length of the pipe and Δp would be a sine of kx cosine of ωt or any different variations i have chosen $\sin kx$ because this gives Δp to be zero automatically at $x = 0$ now i want Δp at $x = l$ also to be zero so before that i should point out this implies that Δp at $x = 0$ equals zero is automatically zero and this means that sine of kl is equal to zero for at all times and therefore k is going to be $n\pi/l$ over l exactly in the same manner as it was for the string and therefore i have for this open ended pipe uniform pipe one end i have taken to be at $x = 0$ the other end at $x = l$ Δp is 0 here Δp is 0 here and it changes in the middle and i have Δp as a function of x and t as given as some amplitude a of the pressure times sine kx cosine of ωt and what we have deduced just now because Δp at $x = l$ at all times is zero this means sine kl at all times is 0 what it means is that k is $n\pi/l$ k is nothing but $2\pi/\lambda$ which should be equal to $n\pi/l$ i cancel π on both sides and i get $\lambda = 2l/n$ and therefore the frequency ν is going to be equal to v/λ v is whatever it is we have found this earlier square root of b/ρ density or gamma square root of b/ρ but important thing is is going to be $n v / 2l$ so the frequencies are $\nu_n = n v / 2l$ which is $v / 2l$ ν_2 which is $2v / 2l$ and so on so this air column can vibrate as frequencies which are multiples integer multiples of $v / 2l$ these are by the way known as in the case of strings as well as air columns the frequencies which are given as $n v / 2l$ are known as harmonics so $n = 1$ is first harmonic $n = 2$ is second harmonic and so on $n = 3$ will be third and $n = 4$ will be fourth harmonic and so on the frequency $v / 2l$ notice that the expression is the same as in this case of string is known as the fundamental frequency so whether i have a string of length l where the displacement

is 0 at the ends or i have a pipe where the pressure change is zero at the ends

so the boundary conditions what happens at the boundary of this medium whether its a string or a pipe is the same either the displacement

is zero or pressure change is zero the frequency nth frequency of nth harmonic frequency

comes out to be n times v over two l for the length l of the pipe or the string and the

only difference is in terms of v v for the string is nothing but square root of t over mu and v for

the pipe is nothing but square root of the bulk modulus divided by the density times the gamma

factor which comes because we consider adiabatic expansion of the air column so this is what it is

now similarly i can also consider now the case of closed string ok sorry similarly i can

consider the case of closed pipe in which case delta p at this end will not be equal to

zero whereas delta p at the open end is zero this recall is exactly the same case as the string

where string left hand is tr left end is tied ok left end of the string is tied and the right end is being moved with some amplitude a remember in this

case what happened is that i still can take my y x to be some amplitude sine of k x cosine omega t sin k x is chosen because it automatically

gives me zero at x equals zero however k l sine of it is not zero in fact it is maximum in that case we learnt that k l should

be equal to two n plus one pi by two and we got our answer for k k was two pi over lambda times l is equal to two n plus one pi by two and we cancel pi

on both sides and we get lambda equals four l over two n plus one that was the case of the string exactly the same thing is going to happen in the

present case here is a string which was tied and this end was vibrating

so i could have modes where i had either lambda by four or three lambda by four and

so on exactly in the same manner if i have a pipe is closed on this x equals l and open at x equals 0 i would have

delta p as a function of x and t equals some large value a amplitude sine k x cosine of

omega t with k x such that x equals l that is it becomes k l is equal to two n plus one pi by two

so it gives me exactly the

same answer as that for a string which is tied at one end and is being shaken on the other

side and this again gives me two pi over lambda l equals two n plus one pi by two and i can

cancel a few terms

so pi cancels and i get lambda equals 4 l over 2 n plus 1 and therefore

the frequency nu n is going to be v over lambda which will be equal to two n plus one

over four l times v or i can write this as n plus a half v over two l again v over two l

is the fundamental frequency and rest are higher harmonics

so you see this is exactly the same case as in the case of string except that the velocity now is going to be not square root t by μ which was the case for the string is going to be γ square root b by ρ so let me write this the velocity now in this case is given as γ square root of b by ρ where b is the bulk modulus ρ is the density and γ is the c_p over c_v for the gas so the only thing that changes the velocity rest because of the boundary conditions the same now in the previous lecture i had also given you the physical interpretation of the modes in a string if i should write on a string or in an air column it is exactly the same now when i have this air column the pressure is θ at the ends and in between it varies so it could vary like this maximum in the middle and then it keeps changing with time or it could vary being θ at the ends and in the middle it would be like this and so on so what you have is either λ by 2 exists or λ could exist this is actually 3λ by 2 and so on so the free the the wavelengths that can exist are such that $n \lambda$ by two is equal to l or λ equals two l over n exactly the same answer as before and you can do similar physical interpretation for the pipe closed at one end now one difference is there between the string vibrations and air column vibrations and this is known as the end correction what is that i just want to remind you that when we took this open ended pipe or a pipe closed at one end what we said was right at the end of the pipe Δp is zero and therefore the length that we took for the wavelength was exactly the length of the pipe which is the same answer for this string also what happens in the air column cases that the node or the $\Delta p = 0$ does not come exactly at the end of the pipe but slightly outside and this distance happens to be $\frac{\theta}{6}$ times r you can take it to be an experimentally established fact similarly if the pipe is open at both ends the node on both sides occurs as at a distance of $\frac{\theta}{6}$. r where r is the radius of the pipe so effectively the length for open ended pipe would be equal to l plus $1 \cdot \frac{\theta}{6}$ so i let me write this in capital effective and similarly effective length for a pipe closed at one end will be equal to l plus $\frac{\theta}{6}$. these are the corrections that you got to make in the length and it is this length effective length that you are going to put in the formulas otherwise the formulas

remain

exactly the same velocity is that the velocity equal to the velocity of sound in the air which is $\gamma \sqrt{b \rho}$ and rest of the things remain the same we

have discussed

so far the superposition of two waves of the same frequency and paid attention to the

waves which are travelling in opposite directions and that gives rise to what we have discussed

so far this standing waves i now want to discuss superposition of waves that have slightly very slightly different frequencies what we mean by that is suppose one frequency

is ν_1 the other frequency is ν_2 then $\nu_1 - \nu_2$ magnitude is much much much

less than either ν_1 or ν_2 for example i could have $\nu_1 = 500$ hertz and $\nu_2 =$

502 hertz

so the difference is really small what happens in that case and that case gives

rise to something called the beat phenomena and i like to discuss that

so for that let

me consider one wave which is travelling in one direction let us say to the right and

it is given by $y_1(x, t) = a \sin(k_1 x - \omega_1 t)$

another wave which is slightly different in frequency very slight also traveling in the same direction and i have $y_2(x, t)$ let me call this y_2 the first

one y_1 is equal to some amplitude $b \sin(k_2 x - \omega_2 t)$ and i stand at a certain point x

so we stand at a point $x = x_0$ and for simplicity let us take this to be zero

so that my

whole thing simplifies and i have $y_1(t)$ at that point is equal to $-a \sin(\omega_1 t)$

one t let me also take it plus it does not matter because finally i am going to look at

the intensity and $y_2(t)$ is equal to $b \sin(\omega_2 t)$ now these two

waves are going to superpose and therefore the net displacement at that

point is going to be given as $y_1(t) + y_2(t)$ is equal to $a \sin(\omega_1 t) + b \sin(\omega_2 t)$

so when these waves superpose i have $y(t)$ at that

point where i am standing is equal to $a \sin(\omega_1 t) + b \sin(\omega_2 t)$ and this will give

rise to phenomena of beats it is easiest beats can be understood easily if we take $a = b$ that means i am taking the amplitude

of the two waves to be the same if they are different i will work that out

also a little

later but in this case what i will have is $y(t)$ is equal to a and in the

brackets $\sin(\omega_1 t)$

$+ \sin(\omega_2 t)$ which i can write as $2a \sin\left(\frac{\omega_1 + \omega_2}{2} t\right) \cos\left(\frac{\omega_1 - \omega_2}{2} t\right)$ let us check that first if

$\omega_1 = \omega_2$

then i get my answer as $y(t) = 2a \sin(\omega t)$ which is correct now

interesting thing

happens when ω_1 is not equal to ω_2 .

so let us consider the case when ω_1 is not equal to ω_2 and let us take ω_2 to be equal to ω_1 plus some $\Delta\omega$ where $\Delta\omega$ is much much much less than ω_1 .

then i can approximately write my $y(t)$ is equal to $2a \sin(\omega_1 t + \phi) \cos(\Delta\omega t)$ can still be taken as just ω_1 or if i want to be very precise i will write this as $\sin(\omega_1 t + \phi) \cos(\Delta\omega t)$ plus t times cosine of $\omega_1 t - \phi$ where ω_1 plus is $\omega_1 + \omega_2$ over 2

which i can write approximately as ω_1 and ω_1 minus is equal to $\omega_1 - \omega_2$ magnitude for cosine it doesn't really matter divided by two

so what i have is that $y(t)$ is equal to $2a \sin(\omega_1 t + \phi) \cos(\Delta\omega t)$ which is some amplitude sine of ω_1 plus t and cosine of ω_1 minus t ω_1 minus is much much much less than ω_1 plus

so if i were to plot it as a function of time the first term ω_1 plus t is very high frequency

so that is going to vary very fast the time period T_+ plus which is 2π over $\omega_1 + \omega_2$ is much much much shorter than T_- minus which is equal to 2π over $\omega_1 - \omega_2$ minus and if i multiply this by ω_1 minus ω_1 minus is much slowly varying

so the second term varies very slowly and that will go down

so if i multiply the two terms right here i will show this if i multiply the two terms what i am going to get is something like this becomes small again and then will pick up again this term is cosine of ω_1 minus in fact what i have multiplied by sine ω_1 minus this is sine of ω_1 minus t

so you have learnt one more thing if i multiply $\sin(\omega_1 t + \phi)$ and $\sin(\omega_1 t - \phi)$ how do they look this is $\sin(\omega_1 t + \phi) \cos(\Delta\omega t)$ let me correct myself for the other function which is $\cos(\omega_1 t + \phi) \sin(\Delta\omega t)$ if i were to plot cosine term it will be something like this

so $\sin(\omega_1 t)$ is going like this $\cos(\omega_1 t)$ term would be one here and then slowly go to zero and go like this

so the product is going to look like large it will become small and will pick up again become small and pick up again

so this is going to be a profile like this this is the product $\sin(\omega_1 t + \phi) \cos(\Delta\omega t)$

so what you notice is this is a function of time that as a function of time the vibration is

increasing in amplitude and decreasing in amplitude slowly and you can feel it why you

can feel it because this is happening at a much smaller scale than the vibration $\sin(\omega_1 t + \phi)$ itself

so if there are two sound waves of slightly different frequencies and if i stand at one point and hear them what i am going to hear is that suddenly the the loudness of the wave is going to loudness of the sound is going to be large then it will go down it will come up again it will go down will come and this is known as the phenomena of beats there are these beads taking place and what is the frequency of beats now notice that my amplitude $y \times t$ is two a sine of $\omega_1 + t$ cosine of $\omega_2 - t$ in the case of sound is going to be $\Delta p \times t$ is going to be some large pressure p sine of $\omega_1 + t$ cosine of $\omega_2 - t$ and the energy that you are going to get is going to be proportional to Δp^2 what happens is this frequency is $\omega_1 - \omega_2$ divided by 2 so the pressure difference is going to be large small again large small again but you are going to hear loud sound at this point where i am drawing a vertical blue line at this point at this point and this difference is twice the frequency itself this is half this time period is t by two t minus by two which is one half of 2π over $\omega_1 - \omega_2$ which is 2π over $\omega_1 - \omega_2$ magnitude so the beat frequency which is how many times you going to hear the sound go up and down is going to be equal to $\omega_1 - \omega_2$ because every time the amplitude goes up whether it is on the negative side or the positive side you are going to hear a loud sound so beat frequency is equal to the difference in the frequency keep in mind that the moment ω_1 becomes equal to ω_2 beat frequency is equal to zero now you may ask i took a the amplitude to be the same for both waves this was to give you an idea as to what happens if the amplitude is different that means i have sine $\omega_1 t + b$ sine of $\omega_2 t + a$ i could write this as $\frac{a+b}{2} \sin \omega_1 t + \frac{a-b}{2} \sin \omega_2 t$ and this is going to give me $\frac{a+b}{2} \sin \omega_1 t + \frac{a-b}{2} \cos \omega_2 t$ and in fact i should put a minus sign here so you notice again this is a superposition of two waves of different amplitudes but both showing phenomena of beats

so you again going to hear beats and what i like you to do is try to plot this as a function of time and see how many beats are you going to hear is it exactly $\omega_1 - \omega_2$ or something more happens but the idea is now clear idea is that when you mix two waves which have slightly different frequencies the amplitude of the superposition varies with time very slowly and you hear amplitude going up and coming down and this is the phenomena of beats and finally in these lectures on oscillations and waves we discuss something called the doppler effect what it concerns itself with is suppose we have a source of waves and it is moving with some velocity v source and i as an observer observe it from a certain point now observing means i could the most common thing that i could do is that the source is a sound source and i hear it the sound is standing some some point so this is moving with v source and i am standing at an angle from the direction in which it is moving or it could so happen that the sound source is stationary and i as an observer am moving or a combination of the two in that case what is seen is that the frequency observed and in this case in the case of sound when i say frequency observed it is the frequency heard is different from the frequency emitted by the source and this is known as doppler effect what we are going to study now is how the observed frequency or the frequency that we hear how much does it differ from the frequency that is being emitted by the source we are going to confine ourselves to the cases where θ is 0 that means that the source and observer are right along the line of movement of the source or the observer they are not at an angle from it so let us consider these cases one by one now case one i am going to study is source moving towards towards the observer so let us take the source and here is the observer it emits a wave at a particular frequency let us call it ν or i will not put ν i will just call it ν because i otherwise get confused with when i put ω for the observer so this is some frequency ν so let a particular wave be emitted from here let us take a particular point the maximum displacement so when maximum displacement is given out it travels towards the observer and the distance between two maximum displacement is λ right which is given by v divided by ν where v is the speed of wave this is when the source is stationary now let us see what happens if the source is moving so here is the observer here is the source and it gave out a particular maximum this maximum starts

travelling and after a time period t it again gives out a maximum
 so this maximum is travelling
 towards the source or towards the observer by the time the next maximum is given
 out in the case of stationary source this distance was λ
 now this distance is going to be reduced because the source has moved and this
 is going
 to be reduced by velocity of the source times t okay if i make it on the left
 hand side
 so this maximum travelled or distance λ and by the time the next maximum
 was emitted the source
 has moved by a distance of v source times t because it emitted that
 after time t
 so effectively λ has become this one shown by purple which is λ'
 $\lambda' = \lambda - v$ source times t
 so the λ that the observer
 is receiving is λ' which is equal to $\lambda - v$ source times t
 so the frequency that person is going to here which i will call ν
 one is going to be the speed of wave divided by λ' which is v
 divided by $\lambda - v$ source times t let us write everything in terms of the original frequency
 so v is v λ is
 nothing but v divided by the frequency minus v source times t is one over the frequency
 which is equal
 to v divided by v minus v source times the frequency
 so the frequency at which now i hear these maxima
 of the wave coming to me is slightly larger so what we have deduced is that if
 there is a source
 which is moving towards an observer along the line of the line which is between
 the source
 and the observer then the frequency that the observer hears is v divided by v
 minus
 v source times ν which is greater than ν by the same logic if this fellow
 source was moving
 the other way along the line joining the observer and the source then ν would
 be v over v
 plus v source times ν which is less than ν
 so i am going to hear either a higher
 frequency or a lower frequency this you see very often why when you standing
 near a railway gate if the train is approaching you
 and with blowing a vessel you hear much higher frequency and it passes through
 you and goes away
 from you you hear the frequency which is lower the way you feel it is that the
 quality of sound
 changes
 so this is case one case two is when an observer is moving towards the source
 so in this case again here is the source and
 this observer now is moving towards the source
 so here are these maxima which has
 been emitted at regular intervals of time difference t and the
 distance between them is λ but because this observer is moving towards the
 source he
 or she is going to hear or see the difference between them to be slightly
 shorter and therefore

higher frequency let us see how much shorter does he see them to be
 so we will apply the same logic
 as logic as before and what we are going to see is here is the source and
 there is this
 maxima which is moving towards the observer here is another maximum which is
 also
 moving towards the source both by velocity v and the observer is moving
 towards them with velocity v_o zero the difference between them is
 λ
 so question we are asking is what is the effective distance that the observer
 sees or feels between the two maxima and i need one more thing for this is that
 these two are emitted at time interval t for this what we are going to
 consider now is let the first maximum be emitted at time t_1 and the second one
 at time t_2
 so that t_2
 minus t_1 is equal to t now let the observer here the first maximum at t_1'
 and second maximum at t_2'
 so what is going
 to happen is here is this wave coming these are the maxima and at t_1'
 this observer is here and the observer is moving in this direction
 so you are going to have $t_1' + \lambda / (v + v_o)$ because
 the relative
 speed between this maximum moving this way with speed v and observer moving
 with speed v_o is $v + v_o$
 plus v_o and they have to travel a distance of λ before the observer
 hears the second
 maximum is going to be equal to t_2'
 so the observer hears the second maximum at t_2'
 so the time interval or time period felt by the observer is equal to t_2'
 minus
 t_1' because this is the interval in which he heard the two maxima
 so $t_2' - t_1'$
 one prime from the equation above comes out to be $\lambda / (v + v_o)$
 and this is t
 ν' which is $1 / \nu'$ where ν' is the frequency that the
 observer feels and
 this is equal to $\lambda / (v + v_o)$ observer and let me write this
 as $v + v_o / \nu'$
 $v + v_o$ and this immediately gives you that ν' is equal to $v + v_o$
 plus v_o
 divided by v times ν which is greater than ν
 so observer moving towards a source also hears a
 higher frequency because the the the maxima are coming at a faster rate
 so what we saw is that if
 a source is stationary and an observer is moving towards it because the
 observer sees maxima coming
 at a faster rate he hears a new frequency which is $v + v_o / v$
 times ν of course
 if the person is moving away then ν' would become $v - v_o / v$
 ν which is less
 than ν
 so what we have learnt
 so far is source observer moving towards each
 other moving away from each other and

so on we can combine all these and write that ν' is going to be equal to $v \pm v_{\text{observer}}$ divided by $v \pm v_{\text{source}}$ times the source frequency ν where v_{observer} plus sign is for

observer moving towards the source and plus v_{source} is for source moving away from the observer and you can fill in the rest of the combinations an interesting variation of

this problem comes when we consider a source moving towards a wall and the source hears his or her own sound

so what happens in this case is that the wall first receives the frequency from the source as source moving towards observer

so in this case if the source frequency is ν the wall is going to receive a frequency ν' which is going to be v divided by $v - v_{\text{source}}$ times ν and this frequency is emitted back or reflected by the wall and now

the person hearing it becomes the observer

so it is now the observer moving towards the source and this new double prime is therefore ν'' is going to be v over $v + v_{\text{observer}}$ times ν' which is

the frequency coming towards the person times v plus v_{observer} divided by v

so this v cancels

and the person is going to hear ν'' which is equal to $v \pm v_{\text{observer}}$ divided by $v \pm v_{\text{source}}$ times ν

a variation of this could be that the wall is moving towards the person towards

the source and source is just standing in that case also you can show that this is going to be

$v \pm v_{\text{wall}}$ divided by $v \pm v_{\text{wall}}$ times ν that is the frequency that person is going

to here

so let me finally conclude the lecture by stating that we have considered frequencies of an air column in a pipe pipe could be closed at one end or open at both ends we have also considered the phenomena of beats in which two waves of roughly the same frequency superimpose and third we have considered doppler effect in which the frequency that we hear because the source or observers are moving are different you