

in the previous lecture we started our discussion on waves and what I had told you about that waves are disturbances that travel

from one place to the other and particularly the waves that we are concentrating on were dispersionless waves and that is if I take a particular disturbance and let it travel down a string or somewhere it does not change its shape as it travels along and we obtained the form of the function

so if this is a function f then $f(x - vt)$ for a wave the disturbance is given as a function of either $x - vt$ or f some other function f let us call it $f(x - vt)$ this would be for the waves traveling to the right

so waves travelling in the informal language I am calling it to the right but actually waves travelling in the positive x direction and for the waves travelling in the negative x direction for the waves travelling in the negative x direction this function of $x - t$ would be given as some function of $x + vt$ or some other function of $t + x/v$

so these are travelling waves right now it may

happen that at certain place if there is a wave travelling one way and there

is another wave which is coming the other way or I create a disturbance and it meets another disturbance somewhere else so with that suppose there is a disturbance which is sinusoidal

so this disturbance is travelling and

so is the sinusoidal disturbance travelling how does the final displacement look so

the question we are asking is as follows what happens when two or more waves arrive at a place at the same time let me explain what we mean by what happens

so what we are asking is what is the displacement as we saw in the string or pressure as we saw in pressure waves or sound waves when two different waves arrive at a place at the same time

so this is what we want to answer and

this is answered by the superposition of waves what the superposition says so we talking about superposition what the superposition says is that if there are two or more waves arriving at a point at the same time that means there is a wave f_1 arriving

there is a wave f_2 arriving and

so on then the net displacement or pressure these are the two waves that we have studied earlier at that point is given by the sum of individual displacements or pressures I will keep using word displacement I do not want to write pressure again and again

so the net displacement at that point $f(x, t)$ will be equal to $f_1(x, t) + f_2(x, t)$ and so on

so at any point

when I see the net displacement it will be sum of all these different waves arriving

there now this is very particular to waves satisfying linear differential equation I am stating this that

this arises out of waves satisfying linear differential equation for

completeness the waves that we are considering right now are essentially those that satisfy linear differential equations so you can take it for granted that we are going to use superposition principle to describe what happens when two or more waves arrive at a point so principle of superposition gives me the net displacement and the principle of superposition says that the net displacement at that point is going to be the sum of individual displacement displacements or individual pressures for example when i am speaking and somebody else also is speaking at a certain point the pressure difference is going to be some of these two pressure differences if i have a string going and two persons create two different waves on it at any point the displacement of the string is going to be given by the sum of the displacements created by the waves by these two persons so ah let me just briefly re state what i just said is that there is something called the principle of superposition and i will just state it mathematically now that the net displacement at a point is going to be sum of individual displacements given by individual waves at that point and now i am going to specialize to sinusoidal waves and let me remind you what these are in these waves the displacement is given as some amplitude sine of $kx - \omega t$ or $\omega t - kx$ i could equally will write this as amplitude sine of $\omega t - kx$ or some amplitude cosine $kx - \omega t$ and any other form for the waves travelling towards the positive x direction or sine of $kx + \omega t$ for waves travelling to the negative x direction so let me make indicate that by these arrows and let me remind you that k is ω/v k is also same as $2\pi/\lambda$ speed of waves is given by the frequency times λ and all these things we have done earlier so what principle of superposition now says then that at any point the net displacement due to different sinusoidal waves arriving is going to be amplitude one sine of $kx - \omega t$ in fact let me make it $k_1x - \omega_1t + a_2 \sin(k_2x - \omega_2t) + a_3 \sin(k_3x - \omega_3t)$ could be $k_3x - \omega_3t$ the wave could be travelling towards the negative x direction $\omega_3t - k_3x$ right it could be of some other form $b_1 \cos(k_4x + \omega_4t)$ and so on so all these possibilities exist that net displacement is going to be a sum of all these the consequence of superposition principle is that during reflection

something happens
right and i am going to discuss that standing waves are formed by two waves travelling in the opposite direction that is also we are going to discuss
so all these phenomena in the later lectures i am going to discuss about beats and interference that all these phenomena are basically consequence of superposition principle
so let us take them one by one so let me just write them first the consequences of superposition principle right
so we are going to see reflection at a boundary then we are going to see standing waves and later in your 12th grade you are going to study interference of waves part of it we are going to do here called the beat phenomena
so let us take them one by one number one reflection of waves at a boundary in describing all these phenomena
mathematically i am going to choose my points conveniently
so that mathematics becomes simple
so in this case what i am going to do is let there be a wave which is coming to the right and gets reflected at point x equals 0 by a hard wall all right it could be a string tied on a wall or it could be a pipe which is open at this end
so that Δp will be zero out here there it can sustain open and cannot sustain any pressure difference
so the pressure out here is going to be the same as the atmospheric pressure and that gives you Δp equal to zero
so these are two things which are the same that means a string tied at a hard wall and an open ended pipe where the pressure difference would not be sustained in this case suppose there is a wave coming in
so let me show it although i am going to finally consider a sine wave let me show a displacement coming in it comes in and at any boundary this reflects
so this is going to reflect here
so it gets reflected either in the same form or in the negative direction it will in fact be in the negative direction i am going to show now now at any given time the displacement at this point where the string is tied
so displacement at the point where the string is tied is going to be zero and we will use that to show that during reflection the pulse actually changes sign
so let us do that now
so i am considering now reflection of a wave on a string at the point where the string is tied ok
so the phenomena we are considering is here is a string and it is tied at certain point and there is a wave that comes in

so at the boundary at x equals zero i have conveniently chosen this point to be x equals zero we are going to have net displacement equal to zero and net displacement i know is equal to sum of individual displacements in particular here this is going to be the displacement due to incoming wave plus displacement due to the reflected wave and i know the left hand side is zero so what i have is zero is equal to displacement due to incoming wave plus displacement due to the reflected wave and this is at a hard boundary where there can be no displacement and this immediately tells you that displacement of the reflected wave is equal to minus and that is opposite to the displacement due to the incoming wave so they are always opposite so what will happen is that this wave comes in let me show the incoming wave by red comes in it is coming in towards the wall and when it gets reflected is going to change the displacement direction so reflected wave is going to be in the opposite direction in particular now let us take a sine wave that means i am looking at this string on which a sine wave is coming in so this is the wave coming in and simultaneously when it hits the wall it gets reflected how are the two displacements related so incoming wave now the displacement let me write this as y x t is going to be given by some amplitude a sine of $kx - \omega t$ so this is incoming and let me write y reflected x t as b sine frequency cannot change k cannot change λ cannot change so because the same medium but it is going to be $\sin(kx + \omega t)$ we are looking at x equals zero and therefore y net at x equals zero is nothing but a minus a sine of ωt plus b sine of ωt and this should be zero what does this tell me this tells me that b equals a so what do we have we have incoming wave is equal to $a \sin(kx - \omega t)$ and reflected wave equals $a \sin(kx + \omega t)$ notice that i can write reflected wave at x equals zero as $a \sin(\omega t)$ which is equal to minus of minus $a \sin(\omega t)$ which is minus of y in coming so the two waves are actually opposite the displacements are opposite to each other so i can write that y reflected is equal to $a \sin(\omega t)$ which i can also write as $a \sin(-\omega t + \pi)$ why because a sine of minus ωt plus π is going to be minus of a sine of ωt which is equal to $a \sin(\omega t)$ but the purpose of writing it in this form is as follows that y reflected is equal to $a \sin(-\omega t + \pi)$ this minus ωt actually shows the phase of the incoming wave so we have to add a phase of π to show the displacement of reflected wave so let me write that to get the reflected wave we add a phase of π to the

phase of incoming wave and this is only for when the reflection is from a hard wall

so there is a phase difference

right there is a phase difference implies that there is a phase difference of π between phases of incoming and reflected waves

when the reflection is from a hard wall let us see this pictorially

so what

is happening is here is the wall and at any given time let

us say the incoming wave is like this at this time the reflected wave

so this is incoming wave is $\sin(kx - \omega t)$ at some given time t the reflected wave is opposite

so the red one this is reflected wave which is $\sin(kx + \omega t)$ at that same time

so what

is happening is when a little time later when the incoming wave has moved forward

so lets say this has become like this it has moved forward

so that this

maximum has advance by some amount at the same time the wave which is coming due to reflection would also move forward but in the other direction so this would have done like this

so that these two displacements remain

exactly opposite you see whatever is the displacement here same displacement is

here

so when this fellow moves this way and this fellow moves this way the displacement

at x equals zero remains zero and this is how the net displacement is maintained

to be zero at point x equals zero

so this is about the reflection now this has

consequences this is consequences that reflection at boundaries creates reflected waves and what we have seen just

now is the displacement at only the point where the reflection is taking place

what we want to know is what happens to the displacement at other points and what we will see is this

gives rise to the superposition of incoming and reflected waves gives rise to standing waves what do i mean by standing

waves these are the waves that are not travelling they are standing right there but before we do that i will just like to leave you with a question

about reflection

i have talked about reflection at a boundary where the pressure difference or the displacement

is zero i would like you to think about what would happen at a point where the displacement is

not zero for example i could have a string here tied to a ring and tie another string on this

side in that case a wave which is coming in would make this ring also go up and down and therefore it will create a wave

here also and a wave which is reflected in this case what will be the displacement here and what would be the ratio of the waves which

are reflected the amplitude of the reflected wave what will be the amplitude

of the transmitted wave

this is something which you will learn in advanced classes but for the time being qualitatively

you should think about what all could happen at the boundary where this ring is there right so

what i am talking about is the net displacement at the boundary is not equal to zero in terms of

pressure waves what it means is that the Δp the pressure difference at the boundary is not equal to zero that would happen for

example if i have a pipe and there is a hard wall on one side here no matter what the pressure

difference is nothing nothing really happens so Δp here need not be zero on the other

hand on the side where the pipe is open Δp is zero

so i let you think about this and

now we will move on to discuss standing waves

so to understand the standing

waves let us consider a wave moving in the positive x direction and this would be amplitude times sine $kx - \omega t$ and superpose this with a wave of the same amplitude

so a sine

$kx + \omega t$ travelling in the negative x direction so

what we have is we have a wave which is travelling to the right and i am

superposing this with another wave which is travelling to the left and let us see what is

the net result keep in mind that the amplitudes of the two waves are the same

so the net result

$y(x, t)$ at any given point is going to be a sine of $kx - \omega t$ plus a sine of

$kx + \omega t$ and i can expand this and write this as a sine kx cosine of ωt

minus a cosine of kx sine of ωt plus a sine kx cosine of ωt

plus a cosine of kx sine of ωt and if i add these i get y

x, t which is of the form $2a \sin kx \cos \omega t$ because the second

term cancels now this is not of the form this is not of the form $f(x - vt)$ or $f(x + vt)$

or

so x and t are not coming in this form in the combination of $x - vt$

or $x + vt$ or $t - x/v$ or $t + x/v$ but they are separated

so what does it represent this represents represents not a travelling wave but it represents a standing wave

so you understand what we mean by standing

waves it is a wave which is superposition of two waves of equal amplitude

going in opposite

directions

so that the net result is that nothing is travelling because if it traveled it

would have had the form of a function of $x - vt$ or $x + vt$ or $t + x/v$ or $t - x/v$

it does not have that form nonetheless there is a displacement which is a function of

time and x and therefore we call it a standing wave let us make a picture of it and see what

it means

so i have the form $y(x,t) = 2a \sin(kx) \cos(\omega t)$ let me call this two a just another constant $b \sin(kx) \cos(\omega t)$

so if i look

at it what it is is that the displacement at any given time is a function of x

so the displacement is like this at any given time it could go to the left and to the right

so this is $b \sin(kx)$ as

time changes each point performs a simple harmonic motion with frequency ω

and time dependence is given as $\cos(\omega t)$

so this point for example the one shown by

red arrow will go up and down with frequency ω the point next to it shown by green would go

up and down with frequency same frequency ω this will also have a time dependence of $\cos(\omega t)$

so with time what you will see is this suppose i

were to just focus on this segment in the middle what you are going to see is that this whole

thing is just oscillating back and forth compare this with the travelling wave where what you would have seen with time

is given a displacement like this as time progressed this would have shifted

this would have shifted by $v \Delta t$ in time Δt

that is not happening in the wave shown above this right this maximum point is not shifting all

that is happening it is oscillating back and forth right at that position it is as if there

are many many simple harmonic oscillators at each point each oscillating with different

amplitude but they are all related with each other

so that the amplitudes change as $\sin(kx)$

so this is known as the standing wave and it is a superposition of two waves travelling

in opposite direction

so it is as if each point is oscillating with this amplitude given by $b \sin(kx)$

so what you will see in books is that when they show a standing

wave it is usually shown like this and you also see a picture like this what it means is that this is showing the net displacement as a function of time the

each point is

going back and forth with the frequency ω

so this is representing the

displacement $y(x,t) = \text{some amplitude } b \sin(kx) \cos(\omega t)$ we could

also have taken other forms sinusoidal though and could have written them as some

constant $c \cos(kx) \cos(\omega t)$ or some other constant $d \sin(kx) \sin(\omega t)$

of ωt all depends on what phase are we choosing at time $t = 0$ what displacement we

are choosing but this is how it looks notice that while oscillating there is going to be a time

when this entire string is going to be flat but all the points would be moving

down or moving
up at that point

so this is just oscillating back and forth and that is a standing wave now what we will consider are different examples of standing waves in all this unless required by certain displacements i am going to assume that my displacement $y(x,t)$ is going to be of the form

$a \sin(kx) \cos(\omega t)$ or $b \cos(kx) \sin(\omega t)$ i might take depending on

where i choose $y(x)$ some other form according to my own convenience but that does not matter so

i could also choose for example a cosine of kx sine of ωt or some other combination but this is the general form

so let us

take the first example as standing wave on a string what i mean by that is i have a string which is tied at one end i could either tie it at the other end and

there is some tension in it or i could also have the string tied at one end and the

other end is being moved up and down

so let us say this point is $x=0$

so what would happen is when i create a wave in this suppose i create a pulse is going to move this way and then move back

net displacement is going to be given by some of these two reflected and an incoming pulse at $x=0$ the net displacement is always zero particular

if i now specialize to sinusoidal waves in the string tied at both ends there is going to be a sinusoidal wave incoming and outgoing and the superposition is going to give me a standing wave and net displacement at both points is going to be zero

so as i said earlier standing waves are generally shown like this and what is happening in this case is each point is going to be oscillating back

and forth at this amplitude given by $\sin(kx)$ and

so on the other hand if i

have this string tied at one end and vibrating on the other side i would have a zero net displacement at this point and the largest possible displacement at the open end this is going to be moving up and down and all

these points are then going to be moving up and down by the same amplitude so these

are two different kinds of standing waves one with the string tied at both ends and

the other with the string tied at one end and free to move at the other end i am

taking these points to be at $x=0$

so let us analyze these mathematically

and then also see what does it mean physically

so mathematically i will

take first the case of string tied at both ends in this case as i said earlier the

displacement $y(x,t)$ is going to be given by $a \sin(kx) \cos(\omega t)$ i am taking

the left hand and left hand end of this string to be at $x=0$ and the

right

hand to be x equals l that means the length of the string is equal to l

so when i take this displacement y x

t equals a sine $k x$ cosine of ωt at x equals zero y is zero which should be the case because that's what i want now we also have that y at x equals l at any time

should also be zero why because this string is tied at that point and therefore i should have a

sine of $k x$ cosine of ωt at l which is equal to a sine of $k l$ cosine of ωt equals zero

for the string tied at both ends

so what we have is that for this string which is tied at

both ends i have $y x t$ equals a sine $k x$ cosine of ωt and i have a sine $k l$ cosine of ωt equals zero ah cosine of ωt varies with time

so it

cannot be zero a is the amplitude

so the only term that can make it zero is that sine $k l$ must

be zero and this implies that $k l$ is equal to some integer n times π and therefore in

this case i am going to have $k l$ equals $n \pi$ or k equals $n \pi$ over l now k we have said earlier

is ω over the speed of waves and this should be equal to $n \pi$ over l

so ω is two π times

the frequency of oscillation and therefore i have two π frequency of oscillation over v is

equal to $n \pi$ over l π cancels from both sides and the frequencies that i get then ν is

going to be equal to n times v over two l

so notice that all frequencies are not allowed the string can vibrate only at certain

frequencies and i am going to call them ν_n and they are going to be multiples of v over two l

so ν_n we have found to be equal to $n v$

over two l and v in this case happens to be square root of tension over mass per unit length

so the frequencies at which the string can vibrate as n over two l square root of t over μ this

is based on the fact that sine $k l$ is equal to zero and therefore k is equal to two π over

λ is equal to $n \pi$ or λ times l λ is equal to $2 l$ over n let us understand this

physically what it means is that for the string tied between two ends since the end points are

at zero displacement the only way that could happen for a given frequency either the wave

is like this or it makes two loops or it makes three loops and each of this half loop is nothing

but λ by two

so what i should have is that λ by two times n where n is an integer and

could be one two or three should be equal to l which implies that λ is equal to two l over

n

so we have seen mathematically now that to keep the two ends the same k

should be of the form k equals $n \pi$ and physically what it means is that n could have only integer number of half wavelengths through this length and that f determines the frequency of this string so it cannot vibrate at any other frequency but certain frequencies now these points which are always at zero displacements are known as the nodes

so for this string which is vibrating at certain frequency with both ends tied let me make a general standing wave like this the points which are always at zero displacement these ones which I am putting these big dots at are known as nodes the points which are at maximum displacement maximum amplitude are known as antinodes distance between nodes is equal to $\lambda/2$ distance between two antinodes that means the adjacent density nodes is also $\lambda/2$ and we have seen that the frequency at which it vibrates is going to be $n/2l \sqrt{t/\mu}$ the other example I am going to now take is suppose I take the same string tie it at one side and on the other side I attach it with a vibrating could be oscillator I could be vibrating with my hand and in that case the standing wave which is of the form $y = a \sin kx + b \cos \omega t$ $y = c \sin kx + d \cos \omega t$ at zero would still be zero but what I want to have now is that y at x equals l and t which is $a \sin kl + b \cos \omega t$ of ωt would have maximum displacement at x equals l and that means kl should be $2n + 1$ times $\pi/2$ and k being $2\pi/\lambda$ should be equal to $2n + 1$

$\pi/2l$ I cancel π on both sides and get $\lambda = 4l/(2n + 1)$ in this case λ is slightly different and the frequencies ν_n which is v/λ I am doing derivation slightly differently this time just to give you different ideas is equal to $4l/(2n + 1)$ which is going to be $v/(2l(n + 1/2))$ so this time the nature is slightly different and I could again write this as $n + 1/2$ $v/(2l)$ so in this case the frequencies are slightly different because one end is open and one is not and again I have a physical interpretation that in this case the waves are going to be of the form where the open end has maximum displacement

so I am going to have $\lambda/2 = 2l/(2n + 1)$ the n segment is going to be of the length $\lambda/2$ so what I am going to have is $\lambda = 4l/(2n + 1)$ and that immediately gives me $\lambda = 4l/(2n + 1)$

so that is a physical interpretation mathematically we could just write what kl should be and get our answer so I will conclude this lecture now

by summarizing what we have done in the next lecture i am going to consider
an open pipes and organ pipes and the the oscillation of air columns in them
and beats and
doppler phenomena

so let me conclude this lecture by summarizing what all we
have learnt we learnt about principle of super position that said
that displacement at any point is the sum of displacements due to individual
waves

arriving there then we learnt about reflection of a wave from a boundary in
particular we learnt

that there is a phase difference of π between the phases of incoming and
reflected waves when the boundary is hard that means no displacement at the
boundary and finally we learnt about standing waves that are like simple
harmonic

oscillators with different amplitude oscillating at each point all with the
same frequency and we learnt about standing waves on a string and obtain the
frequency

at which the string can vibrate both mathematically and looked at
it physically also what does it mean you