

having discussed oscillations in
this lecture we want to start with waves
so let us first understand what
is a wave
so question we raise is what is a wave is it particles moving from one place
to
the other
so let us ask that question is it particles moving from one place to another
let us explore that
so you have seen waves
like in an ocean or if you take a big pan like a thali or something
right and fill it with water and dip your finger in it
so i am dipping my
finger here what you will see is that the water goes up here comes down and
this thing this ripple
let me give it name ripple travels down it travels does it take water along
with
it and we have seen it does not
so only thing that travels is
that ripple and we call it a wave when i am speaking to you
or anybody across the room i create a disturbance here does it mean that
the air goes from one place to the other if you stand in front of the person
speaking you do
not feel any air coming but you do hear him
so what happens whatever disturbances the
person creates travels to your ear
so when a person is speaking to another person sound travels but the air in
between does not
so certainly wave is not movement of a material from one place
to the other
so let us write that a wave does not represent movement of particles then what
is it
so the question again arises what is a wave and in simple terms what i am
going
to say is wave a wave is a disturbance in any form i mean when i am speaking
to you i am creating a disturbance in the air here and it travels to
the mic from there it gets recorded or if somebody is speaking to you that
person is creating a disturbance in the air and that disturbance travels to
you
so wave is a disturbance that travels from one place to another but you have
heard of standing waves
right
so question next one is does a wave mean that the disturbance has to travel
from one place to another
so this is another
question and let me try to answer that suppose i have a string i tie it at one
end and
give a disturbance here give it a jerk
so what you will see is that this jerk or disturbance
travels down the string
so with time it travels this is certainly a travelling wave but think
of this i tie this string between two ends and start shaking it you see that
it gets distorted like this and then comes down
so its moving up and down like this what we call a standing wave and in this

wave the disturbance is not travelling

so a wave could be a standing wave or a travelling or what we call a progressive wave or travelling wave so what we

have answered now is a wave is a disturbance created at one place and travelling to another place what i will call the travelling waves or it could be an extended disturbance like the string and which i will call the stationary wave and now i want to describe it quantitatively we want to see how really do we describe a wave so what we want to see is how do we describe a wave i will be very general in this because what you have probably heard in your classes is a wave looks like this and it is travelling

so what you have seen is a disturbance $y(x, t)$ equals a sine $2\pi x / \lambda - 2\pi f t$ this kind of wave or it can also be written in cosine terms a cosine of $2\pi x / \lambda - 2\pi f t$ and then you are told that speed is $v = f \lambda$ where λ

is the wavelength and f is the frequency this is a very particular kind of wave we

will come to it but before that since i called a wave is a travelling disturbance let me come to describe that first

so let us say i have a string or as the example we gave is a water surface and i create let us say disturbance like this out here or i create a disturbance like this out here and it travels to understand it properly let me say that the disturbance i create is something like this at $t = 0$ a parabolic one

so let us say this is at time $t = 0$ and the function $f(x)$ this is function $f(x)$ this is x direction

so function $f(x)$ or since i am going to later write y let me say y as a function of x looks like a square minus x^2

so what does it represent it represents at $x = 0$ the height is a and it becomes 0 at $x = \pm a$ for $|x| \leq a$ and it is 0 otherwise just to give you a feeling for the pulse that we have created

so this could also be a disturbance that has been created on the surface of water now this disturbance travels and let us say it has gone towards the right and has come at a point where the peak is at $x = 0$ then the same disturbance at a later time

so let us say this has taken time t

so at time t this disturbance would be given as a square minus $(x - vt)^2$ for $|x - vt| \leq a$ and zero otherwise

so what we have done is either on the surface of water on a string we created a disturbance and at a later time it moved to this

point $x = vt$ and obviously if it travels with speed v this distance $x = vt$

is going to be $v t$ and here it was given as $y(x, t) = a^2 - (x - vt)^2$

square

for $\text{mod } x$ less than equal to a and zero otherwise and the right hand side i am going to write this in red $y(x, t)$ is given as a square minus x minus x^2 for $\text{mod } x$ minus x zero being less than equal to a and zero otherwise which can also be written as a square minus x minus $v^2 t^2$ square for $\text{mod } x$

minus $v^2 t^2$ less than equal to a and zero otherwise what about the case where it traveled to the left

so suppose it was at x equal to zero and it travel to the left if it travel to the left then $y(x, t)$ you can easily see is going to be a square minus x plus $v^2 t^2$ whole square for x plus $v^2 t^2$

less than equal to a and zero otherwise because now this distance is going to be minus $v^2 t^2$

so what you see in both the cases that the disturbance $y(x, t)$ is a function of either x minus $v^2 t^2$ or x plus $v^2 t^2$ let us understand that

so i will make x equal to zero right here in the middle creates some disturbance now i am not even going to say it like a square minus x but its $f(x, t)$ equal to zero then what

would happen if it is travelling to the right in time t this disturbance is going to be given by $f(x - vt, t)$ equal to zero why because now whatever disturbance is at time t which i am going to call $f(x, t)$ is the same disturbance that was at $x - vt$ which was centered around $x - vt$ at t equal to zero

if on the other hand it travel to the left then the peak to peak distance would again be $v^2 t^2$

so whatever disturbance is at t as a function of x is the same as that which was at $x + vt$ at t equal to zero t equal to zero is fixed

so what we have shown is that any disturbance created at t equal to 0 around the origin it could be any shape if it travels with a constant speed v and undistorted

so let me write this

so a disturbance created as $f(x)$ at time t equal to 0 will be changing or will be given as a function of time as $f(x, t)$ equals $f(x - vt)$ at t equal to zero if it travels to the right right means positive x

axis and it will be given as $f(x, t)$ equals function same function $x + vt$ t equal to zero if it travels to the left because thats where it was now thats where it was at t equal to zero

so a disturbance which has been created at t equal to zero and disturbance i am calling $f(x)$ it could be any disturbance would appear as to the right i am going to

write $f(x - vt)$ t equal to 0 equals $f(x)$ t to the left i am going to write $f(x + vt)$

t equal to 0 equals $f(x, t)$ this is travelled this way the green one has traveled to the right

and under what conditions

so conditions are the disturbance travels undistorted number one number two travels with constant speed v

so two conditions it travels undistorted what is also called

so undistorted also implies what is called in technical term dispersionless you will hear this term quite often during the discussion on waves

so its a dispersionless travel it does not disperse it does not change shape it is undistorted and it travels with constant speed v you can make out that undistorted assumption

is really pretty ok when i speak to some person whether the person is standing at 10 meters from me or 100 meters from me or 200 meters from me if he hears me or if she hears me what all these three persons standing at different distances here is the same word that i have spoken with the same interval and that means whatever disturbance i created goes pretty much undistorted

so its a pretty reasonable assumption there are cases in advance courses you will see where there is dispersion one very obvious example of dispersion is light wave where the speed is different for different frequencies and that's why you see a prism giving you different colors because the refractive index differs but right now we will assume is dispersion less travel

so that is one way of looking at it that is a function of x minus $v t$ or x plus $v t$

another way of looking at the wave travel another is if i am looking at or disturbance at certain point x then what i see exactly what happened at t minus x over v at time ok another way of looking at the phenomena is wave phenomena is let me suppose i am standing at a point x and see a disturbance here this disturbance would have taken time t equals Δt equals x over v to travel from the origin this time taken is x over v

so i can safely say that if i observe a disturbance at x at time t it would have been disturbance at x equal to zero at a time t minus x over v this is another way of describing the wave phenomena if travel is to the right on the other hand if the disturbance travels to the left then at x t what i see is something that would have occurred at x equal to zero at time t plus x over v x is less than zero

so it is travelling to the left

so i can also describe a wave as a function of t plus x over v or a function of t minus x over v and i will just show by arrows minus x over v is travelling to the right plus x over v is traveling to the left which is pretty much the same as writing it as x minus $v t$

and $t - x/v$ because a function f of $t + x/v$ is same as a function of $t - x/v$ which is a function of $x + vt$ remember the variables are x and t

so v can take as a constant and similarly function $t - x/v$ as a function of $t - x/v$ which is same as a function of $t - x/v$ is a constant so i can write the wave disturbance travelling to the left or the right as $x + vt$ or a function of $x - vt$ or a function of $t - x/v$ or a function of $t + x/v$ so let us write that

so a wave can be described as a function of $x + vt$ a function of $x - vt$ or a function of $t + x/v$ or a function of $t - x/v$ so all these forms exist and these are all traveling waves

so let us see an example all you have seen is how do i generate $y(x, t) = A \sin(2\pi(x/\lambda - ft))$ this is what you have seen you have seen earlier a wave which is a disturbance like this and it travels i will come to that in a minute but before that i again want to go back and discuss that these functions $f(x - vt)$ or $f(x + vt)$ all these are functions of x and t both so are travelling

or a standing wave is a function of x and t that is the position and time both so

if you want to visualize it you can either look at $f(x - vt)$ as a function of time by looking at a fix position let us say $x = 0$ so what do i do i stand at one point and see as a function of time how does the disturbance occur how does it change with time

so i stand at $x = 0$ and see how wave disturbance is changing with time or what i can do is fix a t $t = 0$ and observe how the disturbance looks as a function of x what would i do for that just take a photograph of the wave at that point and then you can just see that as a function of x you will see that it is spread all right and it is at a particular time let us say $t = 0$ at a slightly later time it could be spread but somewhere else it would have traveled by a distance vd or could have traveled to the left a distance vt undistorted

so this is how i would look at a wave phenomena at a given time and at a given position if i stand there i will just see that things are just moving up or down or left or right depending on what kind of wave it is and we will see how it changes with time now let us see i said

earlier that how is this wave generated right

so $A \sin(2\pi(x/\lambda - ft))$

so suppose i fix my position what do i see if i stand at a

particular point then f at say x equal 0 as a function of time would look like a sine of $2\pi x$ is zero over λ minus $f t$ which is a sine $2\pi f t$ with a minus sign

so i can write this as minus a sine of ωt so as a function of time i will just see that point at x equals zero going up and down what about as a function of position as a function of position if i take a snapshot at a particular time this is what i am going to see that this is spread from left to right all over and let us say the time equals zero then it look like $f x$ at time t equal to zero is equal to a sine of 2π over λx naturally what you see now is that if x goes to x plus λ you get a sine 2π over λx plus λ which is again a sine 2π over λx plus 2π which is same as a sine 2π over λx so after a distance λ the displacement is the same so the λ which is the distance between two similar points because the displacement is the same i am showing this in pink this λ is called the wavelength so what i have shown you is that if i have this wave and i will just take a portion of it this has wavelength λ between two similar points then the distance between two similar points is λ and after certain time t after certain time t it would have moved further so i will just make it this distance would now be $v \Delta t$ after time Δt now the next question i ask through this is how are λ , v and frequency f of the wave related so let us try to answer that question i told you if i stand at a particular point x equal to zero or x equals some other point x zero as a function of time this fellow is going to go up and down which goes as a sine of ωt so after time t equals capital t which is related to the frequency as one over f the displacement is the same now let us see if i took this wave frozen in time and looked at it after time capital t it will look exactly the same except if there was a point a here this would have moved to point a at this point by distance λ so in time t the wave has moved by distance λ otherwise the displacement would not look the same so if it looks the same it must have moved by distance λ so v is going to be λ by t which is going to be f times λ this is a relationship you very well know so the speed of wave is given by f times λ this particular wave which is known as the sinusoidal wave since we are talking about dispersionless waves the speed is the

same for all frequencies how to generate a wave like a sine $2\pi x$ over λ minus

$f t$

so let us look at that i am now focusing specifically on this wave what is called as the sine wave

so this is called ah sign wave

so let us see if i have a

string an extended string and i start shaking this point up and down as a sine of ωt

so that time period is 2π over ω which is equal

to t then as we had argued earlier

so right now at y at x equal to zero as a

function of time is equal to a sine of $2\pi f t$ then y as a function of x and time would be y

at x equal to zero at time t minus x over v if it is travelling to the right and that i know

is going to be a sine $2\pi f t$ minus x over v which is equal to a sine $2\pi f t$ minus $x f$ over v and we have just seen the relationship that v equals $f \lambda$

so i can write this as minus a sine of $2\pi x$ over λ minus $f t$ which is the same form that you have been seeing or which i showed you earlier so

if i take a rope start shaking it at one end with with a sine ωt i create a sine

wave which is travelling to the right if i did this disturbance and the wave traveled to the left then i would have $y(x, t)$ equals y at x equals zero t plus x over

v and this would be a sine $2\pi f t$ plus x over v which is same as a sine $2\pi f t$ plus

x over λ

so these are different forms that you have seen

so let me kind of summarize what

we have done

so far is that number one we have looked at a disturbance travelling with speed v and undistorted and therefore we could represent this as

$f(x - vt)$ or $f(t - x/v)$ if it travels to the right that means positive x axis and it is represented as $f(x + vt)$

or $f(t + x/v)$ if travelling to the left

so you have seen both ways all

right and then we specialize to sinusoidal wave which is given as a $y(x, t)$ as amplitude a sine of x over λ minus $f t$ times 2π

so it keeps

repeating at along the x axis it keeps repeating at intervals of λ in time at a fixed point

it the displacement keeps repeating itself in ah with frequency f in time one over f and

the way this is generated is by shaking at one point with a given frequency in a

simple harmonic manner

so that the wave created something like this and is travelling

either to the right or to the left we also see in this case that the speed v of

the wave is given as the frequency times λ which can also be written as 2π

frequency times lambda over two pi this i can write as omega and now i am introducing

a new quantity $2\pi/\lambda$ equals k which is known as the wave vector or wave number

$2\pi/\lambda$ is that many waves in 2π interval
so ω/k

so ω equals $v k$ that's

a new relationship i am giving you and in terms of this new relationship $y(x,t)$ can also be written as

$\sin(kx - \omega t)$ this is another form you will see in your books or places where waves are discussed

so once we have understood this now let me give you two kinds of waves known as number one transverse waves these are waves where displacement $y(x,t)$ is perpendicular to the travel direction

so examples would be waves on a string or example of transverse waves the other kind is longitudinal in which if the wave is traveling

in the x direction the disturbance is also along the same direction

so in this the disturbance is in the same direction as the direction of motion of wave for example sound wave where i create

a pressure difference that is the disturbance is an example of this

so sound waves are longitudinal where the pressure is actually in the same direction as the propagation of wave one question we had asked in the beginning and

i am going to address it now question do waves carry particles as they travel and the answer is no i had argued earlier you can

also see it you know a string if you just shake it the string does not move from one

place to the other the disturbance does if there is a water wave going you can leave a

leaf or a piece of paper there and you will see it just moving up and down but it does not move

with the wave

so waves do not carry particles as they move

so they do not carry particles

and also i can write do they carry material and they do not question number two do waves carry energy from one place to another and the answer to this is yes the simplest

possible way i can answer this is when i am speaking to somebody the person hears whatever

disturbance i have created whatever pressure difference i have created whatever movement local

movement of particles i have created by speaking travels to the other place creates the same

disturbance on the ear drum or in the ears that means it has the capability of taking

that energy from one place and transferring it to another place

so yes waves do carry

energy from one place to other third question how do i calculate speed of waves and this is a question which i

am going to address now

so let us first calculate the speed of waves on a string for this let us take a string and give it

a disturbance let us say this is the disturbance a sine wave right and

so let us say that $y(x, t)$ is given as a sine $kx - \omega t$ and I know which I have said earlier that v the speed is ω/k or $\omega = vk$ I am going to need this now let us see as a particular portion of this goes up and down what happens so if I take a particular piece of this string and assume so let us write on the side assume that y is much much much less than λ this is tight so at a given time I have taken the snapshot I have taken a picture of this this feels a tension T to the right and a tension T to the left now this angle that it makes with the horizontal is very small so let's call this θ_1 here and θ_2 here let's call this θ_1 at point one and θ_2 at point two so we are going to say that θ is much much much less than one implies $\sin \theta \approx \theta$ $\tan \theta \approx \theta$ $\cos \theta \approx 1$ for this given curve at this time $\cos \theta$ is approximately equal to one so now this tension T at position two has two components one going up one going to the right this is $T \cos \theta_2$ which is roughly equal to T moving up is this $T \sin \theta_2$ which is roughly equal to $T \theta_2$ at point two on the lower side at point one this is tension T it has a horizontal component and a vertical component the horizontal component again is given by $T \cos \theta_1$ of θ_1 which is roughly same as T and $T \sin \theta_1$ in the vertical direction which is $T \theta_1$ at one so on this section between point one and point two I have net vertical force in the y direction which is $T \theta_2$ at this given time θ_2 at two minus $T \theta_1$ at one in the y direction and zero in the x direction let us say this distance between one and two is Δx then by Taylor's theorem or by simply taking derivatives I can write θ_2 is equal to θ_1 plus take the derivative of θ by x which will be $\frac{d^2 y}{dx^2} \Delta x$ therefore the vertical force on the section of the string is $T \frac{d^2 y}{dx^2} \Delta x$ what would this force do it will make it accelerate up or down so if I equate this to the acceleration let's see what happens so acceleration of this section is going to be nothing but I will freeze at that particular point at x and see it moving up and down so at that given position I will calculate $\frac{d^2 y}{dx^2}$ square that's the acceleration so the force which is $T \frac{d^2 y}{dx^2} \Delta x$ at this given time which is also written as $\rho \Delta x a$ I will just use a technical term the partial derivative of y with respect to x because y is a function of both x and t by writing partial

i mean t is kept fixed and the acceleration is $\frac{d^2 y}{dt^2}$ at this position which is also written as partial derivative of y with respect to t this automatically means it is a given x and the relationship between the two is going to be force equals mass of the string $\mu \Delta x$ times acceleration where μ is mass per unit length of string

so i get $t \frac{d^2 y}{dx^2}$ at that fixed time is equal to $\mu \Delta x$ there is a Δx here there is a Δx on top $\frac{d^2 y}{dt^2}$ at fix x now let us take $y = x$

t equals $a \sin(kx - \omega t)$ then what do we get so what we have written is that $t \frac{d^2 y}{dx^2}$ which correctly actually i should write as a partial derivative times Δx is equal to $\mu \Delta x \frac{d^2 y}{dt^2}$ at a given x Δx Δx cancels and this is the equation and i am taking a sinusoidal wave

so y equals $a \sin(kx - \omega t)$ $\frac{d^2 y}{dx^2}$ for $f \times$ time is going to be $-\omega^2 a \sin(kx - \omega t)$ and $\frac{d^2 y}{dt^2}$ at a fix x is going to be $-\omega^2 a \sin(kx - \omega t)$ and i substitute it back in this equation and i get $t k^2$ is equal to $\mu \omega^2$ and this gives me $\frac{\omega}{k}$ is equal to square root of $\frac{t}{\mu}$ recall what we had said earlier earlier i had said that v equals $\frac{\omega}{k}$ and this we have calculated to be $\sqrt{\frac{t}{\mu}}$

so you see how taking newton's equation of motion for a particular section and relating how ω and k should be related to the speed of wave that we did earlier although for a sinusoidal wave we could get what the speed of wave should be in terms of t and μ so this also implies that wavelength for a given frequency will be $\frac{v}{f}$ which will be $\frac{1}{f} \sqrt{\frac{t}{\mu}}$

so this is the derivation that we have done just by simply looking at how newton's equations are applied for a section of wave another example i am going to take calculation of speed of waves is the sound waves in this what we do is suppose i have a air column where i take a particular section let us say of length Δx and create an extra pressure p at this point so when i am speaking for example i create a pressure and this pressure changes with distance so it becomes $p + \Delta p$ on the low next side in creating this pressure i have also moved this wall by a distance let's say z and therefore here on the other side it moves by $z + \Delta z$

so let me look at this shaded portion of air and look at this point which i am showing by arrow and i will see the movement of this whole shaded section as a whole and relate it calculate this acceleration related to the force that it is feeling now let us see the force it feels

so if i look at this portion there is $p + \Delta p$ on this side p on this side

so if this cross sectional area is A then the force it feels is to the left which is Δp times area and what would this force do this force would give it an acceleration

so we are going to write that $\frac{d^2 z}{dt^2}$ at this point x lets say this point is x times the mass of this portion shaded portion

is going to be equal to minus Δp times A from this i should be able to get the speed of

the wave now what does this pressure also do

so this Δp the pressure difference makes it accelerate on the other hand the pressure p on both sides changes the volume through that volume change i can actually calculate relate Δp to some other property of this gas and lets do that

so in this gas or

in this air i have this small portion on which pressure p is here pressure $p + \Delta p$ is

here the length is Δx i am looking at point z and what we have seen is that mass which

is going to be the density of this air times its volume $A \Delta x$ then acceleration

we have figured out is $\frac{d^2 f}{dt^2}$ then we have seen that the Δp times A is the

force and therefore we get $\rho \Delta x$ is equal to $\rho \Delta x$ times the acceleration $\frac{d^2 f}{dt^2}$

$\frac{d^2 f}{dt^2}$ is equal to minus Δp times A there is an A also on the

left hand side and this gets cancelled and you get ρ times $\frac{d^2 f}{dt^2}$ is equal to minus Δp over Δx let us see how do we calculate this Δp over Δx

so pressure p changes the volume mind is pressure p is

not the existing ambience pressure is the extra pressure that i am creating so that is why

this changes the volume how much does it change

so i know that the bulk modulus

is equal to minus $v \frac{dp}{dv}$ or minus v whatever extra pressure i am applying and how much is the change in the i am calling it Δp bar over Δv

so Δp bar is the pressure Δp bar is not the Δp that i am taking is the pressure that i am applying Δv is going to be equal to initial volume

is a Δx the point on the left moves by z point on the right moves by z plus

Δz

so Δv is going to be $A z$ plus Δz minus $A z$ which is nothing but a Δz which is nothing but a change in z as a function of x times Δx that's the change in the volume

so therefore i am going to have p over this change Δv times v of this the shaded area is equal to b is equal to minus v is a Δx times pressure p divided by Δv which is a dz by dx Δx Δx and Δx cancels a and a cancels and i

get therefore p is equal to minus $b \frac{dz}{dx}$

so what we have found is that $\rho \frac{d^2 f}{dt^2}$ is equal to minus Δ

p over Δx which is like minus $\frac{dp}{dx}$ and we have found that p is nothing but

equal to minus $b \frac{dz}{dx}$ what is z z the displacement of this what is f f is also

displacement of the this left hand portion

so f is same as z which is giving me the acceleration of the body

so i am going to have $\rho \frac{d^2 z}{dt^2}$ is equal to minus $\frac{dp}{dx}$ again this is no time

so this is partial

derivative which i can write now as plus $b \frac{d^2 z}{dx^2}$ and where did i

get this one this i got from here

so i have $\frac{d^2 z}{dx^2}$ is equal to

$\frac{\rho}{b} \frac{d^2 z}{dt^2}$ or $\frac{d^2 z}{dt^2}$ is

equal to $\frac{b}{\rho} \frac{d^2 z}{dx^2}$ this is the equation we have got now

what we saw earlier is that p is proportional to z

so i could also have written this in terms of

p but this is what it is the displacement that we create by changing p follows this

equation and again taking sinusoidal wave that means i am taking $z(x,t)$ to be a sine $kx - \omega t$ i get $\frac{d^2 z}{dx^2}$

at a fixed time to be equal to minus $a k^2 \sin(kx - \omega t)$ and $\frac{d^2 z}{dt^2}$

square at a fixed x to be minus $\omega^2 a \sin(kx - \omega t)$ and substituting these in the equation i get $\frac{d^2 f}{dt^2}$ or $\frac{d^2 f}{dt^2}$

which is the same as $\frac{d^2 z}{dt^2} = \frac{b}{\rho} \frac{d^2 z}{dx^2}$

or minus $\omega^2 a$ equals $\frac{b}{\rho} k^2 a$ or $\omega^2 = k^2 \frac{b}{\rho}$

square which is nothing but $v^2 = \frac{b}{\rho}$ implies speed of waves is square root of $\frac{b}{\rho}$

over ρ this is a well known result now b is the bulk modulus and what has been argued is that

at high frequencies this is the adiabatic bulk modulus because ah when there is a high

frequency wave passing through there is not enough time for the heat to dissipate

so its not constant

temperature but adiabatic bulk modulus

so what we obtain now for that v for air or a material is square root of $\frac{b}{\rho}$

so let me just conclude this second portion by saying that we introduced the sinusoidal waves then calculated speed of wave by relating acceleration of a portion of medium and we considered string and bulk medium for sound waves

relating acceleration of
portion medium to the force generated due to wave disturbance in the next coming
lectures we
will now explore these concepts related to you

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