

hello students welcome to a problem solving session on the topic of simple harmonic motion so in this lecture we are going to solve problem on simple harmonic motion and i will take problems from previous years ze advance question papers and will solve problems not necessarily in sequential order from previous years but what we will do is pick up problems randomly and we will try to solve them in fact simple harmonic motion is one of the most important topic in all of physics it is useful not only for mechanics but it helps in solving many problems from other topics as well for example it is pretty useful in solving problems in some problems in say you know related to electromagnetism then some problems with modern physics and so on sometimes concepts are mixed with uh simple harmonic motion concept and then you can solve problems utilizing the idea of simple harmonic motion so let us begin the first problem that i have taken is a simple one it's from 2009 ze it's in this very simple problem you know the displacement time graph of a particle undergoing simple harmonic motion is shown as you can see from the figure kindly please note down the problem you are asked to find the acceleration of the particle at t is equal to four by three second four options are given and out of these four options only one option is correct okay so as you can see you know information that you have at the moment is what is the displacement or position of the particle at different different times so obviously you need to find out the velocity and once you know the velocity then you can find out the acceleration now if you notice you will see that the particle has started from the position x is equal to 0 right and and at time t is equal to 0 so what we will do i will assume basically you can assume that the position is represented by this or displacement is represented by this equation this is the simple harmonic motion equation it is how we represent it is the amplitude and omega is the angular frequency of the simple harmonic motion and phi is the phase now because you know that at time t is equal to 0 x is equal to 0 using this immediately you can find out what is this phase angle phi even before that if you see the diagram carefully you will see that the amplitude is simply 1 okay amplitude is simply equal to one and other information that you can find out is the as i said phase so phase you can just you put down these values there

and you will immediately find out phase to be zero and ϕ is equal to zero there and what other information you can have if you look at it carefully now another information that you can have is the time period from here you see the time period is simply eight second right so T is equal to eight second and if you know the time period then what other quantities you can find out quite clearly you can find out the angular frequency and angular frequency is 2π by time period so angular frequency will turn out to be 2π by 8 and actually okay maybe because of my picture you cannot see it but it will be π by 4 per second right that would be the angular frequency so you have a is equal to 1ω is equal to π by 4 ϕ is equal to 0 so your x is equal to simply then $\sin \pi$ by 4 t right yeah so this is what you have now once you have the appropriate form of the displacement and versus time equation with you okay position versus time you can say that also then you can find out what is the velocity so you differentiate it once to find out the velocity and that turns out to be π by 4 $\cos \pi$ by 4 t and if you differentiate once again then you will be able to find out the acceleration at time t and once you find out the acceleration at time t at any given time t so you can immediately find out what is the acceleration at time t is equal to 4 by 3 second right and you just if you put it it's very simple trivial calculations is going to give you it to be minus root 3 by 32 π square centimeter per second square remember the position was given in the unit of centimeter so if you look at the options basically then one you can immediately say that the correct option should be d okay because the d and a look similar but remember d has this minus sign is there and there is a minus sign there so correct option is d okay let us go to another problem so in this problem uh please note it down in this problem uh what is given a point mass is subject to two simultaneous sinusoidal displacement in x direction x_1 is equal to $a \sin \omega t$ and x_2 is equal to $a \sin \omega t + 2\pi$ three adding a third sinusoidal displacement x_3 is equal to $b \sin \omega t + 5$ brings the mass to complete rest the value of you are basically asked to find out the value b and ϕ okay this question was asked in 2011 ge so how to go for it again you see there are four options are given out of this four option only one option is correct if you look at under if you understand the problem basically what is happening is two forces are actually

applied on the point mass particle and they are applied in such a way that it gives displacement x_1 and x_2 at the same time at a given time t and then if you now add another sinusoidal displacement that is going to that means if you apply another force which basically is going to result in a displacement that will lead to the cancellation of all on this and then the particle will come to a halt complete or to completely rest

so let us do this problem it's very simple one actually if you if you see the first egg that x_1 is equal to you know a sine ωt and this you can represent in a displacement vector form because it is it will be characterized by this amplitude as well as the phase angle the phase angle here ϕ is 0 so here i can represent that x_1 is a 5 ϕ is equal to 0 x_2 i can use you see this phase angle in this case is 2π by 3 so it would be represented by the amplitude a and ϕ is equal to 2π by 3 so this is the phase angle so and these two things i can plot in a diagram factorial diagram and this is what i have and because x_1 is directed along this direction and x_2 is directed along this direction the resultant direction of this two and that is x okay shown by this rate line now the what is said key you have to add another displacement such that this x actually gets cancelled and obviously that is going to happen provided you add the displacement in the opposite to this x direction right along this vector opposite and its magnitude should also be the same so it's very easy so this is the resultant one and this can be done simply by you know you have to and that is going to have the same amplitude and the angle ϕ is what you have to determine very easy you just have to draw this extend is in the opposite direction from the diagram you can see that this angle ϕ is already you have this π by 3 is there and this angle is π so total ϕ would be this much it's the angle made by this third you know vector with this original direction here that is your x one so that will be π plus π by three so π would turn out to be uh four by three into π okay so amplitude is a again here so this displacement that you have to add here is simply it will be characterized by amplitude a and phase angle 4π by 3 .

so if you go for the

options there you can immediately see the correct option or turn out to be what it would be
 b right it will be correct option will be simply b this can be done in another simple another
 way you just do it algebraically this method that i have shown you just now is a bacterial
 method graphical method
 so you can do it by algebraic method as well
 so here you see that
 you just have to add this third one
 so that this x_1 plus x_2 its effects get cancelled out
 so x_3 is obviously equal to minus of x_1 plus x_2 now it becomes a simpler trigonometric addition problem here because x_1 is equal to a sine ωt and x_2 is equal to
 a sine ωt plus $2\pi/3$ now if we apply this sine c plus sine d formula all of you know it i
 think i'm sure all of you know it you just say apply it there then what you are going to get
 is the resultant would turn out to be this one now this minus sign is coming outside
 so which you
 can take inside if you just add a π there plus π there
 so this is what you are going to have so
 final expression would turn out to be like this now x_2 is in the original equation it was termed
 up to be $b \sin \omega t$ now if you compare this to expression immediately you can see that a
 is equal to b is equal to a and ϕ is equal to $4\pi/3$
 so the correct option is once again it's
 again b okay all right
 so this was a very simple problem now you you can decide which one is the simple simpler method accordingly you can use it okay now another problem is from 2016 and
 this problem yeah this problem has more than one option correct okay in this particular
 problem let me read the problem and you please try to note it down
 so that it will be easy when
 i am going to explain it because i cannot you know solve this problem again and again in a brief
 slide it will be difficult
 so please note it down let me read it first
 so it says that a block with
 mass capital m is connected by massless spring with stiffness constant k okay to a rigid wall
 and moves order friction on a horizontal surface the block oscillates with small amplitude
 a about an equilibrium position x_0 you are asked to consider two cases case one
 is when the block is at x zero that is the equilibrium position and when the block is
 at x zero plus a okay is the amplitude so in both the in both the cases a particle with
 mass m small m is softly placed on the block after which they stick to each

other who is
 of the following statement is or are true about the motion after the mass m is placed on the mass capital okay what are the options the options are the amplitude of oscillation in the first case changes by a factor of this much that is square root of capital m by small m plus capital m whereas in the second case it remains unchanged the final time period of oscillation in both the cases is the same the total energy decreases in both the cases the instantaneous speed at x zero of the combined mass decreases in both the cases i am sure all of you have already noted it down while i actually read this problem now to solve this problem uh you just you just see the options what are the things that is required that is needed you have to look for the amplitude in case one and case two and the time period total energy and the instantaneous speed okay let us do this problem it's a very simple problem initially the situation is this before case 1 and case 2 this is the situation the you know the mass block is attached to a spring and of spring constant k and that spring is attached to a rigid wall here and x naught is the equilibrium position uh equilibrium position here so immediately you can write down what are your you know angular frequency would be square root of k by m and time period is obviously T is equal to 2π by ω okay and another thing is that uh amplitude is a q so therefore the velocity you can write in terms of as ω into a right and total energy is equal to is equal to half k a square i think these are familiar results to all of you so you can utilize this information so this is the initial situations okay initial configuration now we are going to consider case one and case two separately first let us consider case one in case one uh now what is done that in case one a small mass is put gently on the top of this softly on this bigger block of mass capital m because of who is what is happening its velocity simply gets changed to a different one let us say v_1 so once you put that and it gets stick to it okay so then your angular frequency would simply would get changed by this mass of amount k divided by m plus capital m plus m and now in this case linear momentum is conserved by the way because uh you know there is no external force applied so linear momentum is conserved

so initially what is the moment linear momentum just simply m into v right and uh and now when this is put there now your new momentum is m plus m into v_1 which has to be the same with the initial one from here you can immediately find out what is v_1 okay and from this expression it's very easy to see that instantaneous speed basically it is v_1 is smaller than v okay in this case 1 v_1 is smaller than v okay that one information we obtained and another one is the new amplitude is a_1 so therefore v_1 i can write in terms of amplitude as this ω_1 into a_1 okay all right so then a_1 you can find out as b_1 divided by ω_1 and you already have worked out what is v_1 okay this using this previous relations we can find out a_1 in terms of original this initial amplitude in the original thing so this is what you have a_1 is equal to square root of capital m divided by capital m plus m and time period is 2π by ω_1 okay now consider the total energy of the system total energy of the system is what half $k a^2$ right here total energy of the system so we have found out what is a_1 so already we know that half $k a^2$ capital a^2 this guy is the energy of the initial system okay when the small mass m was not put so this is the expression we get immediately you can see one thing is this that initial energy is basically getting changed in when i'm considering configure is case one okay total energy obviously decreases compared to the initial total this is obvious from the equation 4 here i hope all of you are getting it so now let us go to case 2 in case 2 this spring is basically this block is now getting extended to $x_{naught} + a$ okay x_{naught} was the equilibrium position and now it is given x_{lambda} so it is basically getting the extreme position extremum position here so in this position obviously the velocity is going to be zero all right and because of the conserve okay i'll come to that and immediately one thing you can see is the uh angular frequency i think i just forgot you to tell you that okay here you are going to put the small mass m here i forgot to draw it there anyway so your angular frequency would be root over k by m plus capital m plus small n yeah i have actually written it here i should have put a mass there so if a mass m is put over this block due to conservation of linear momentum velocity of the combined systems

just

after m is put over m is also zero okay when you have actually it is i hope you are

getting it because when it is getting extended right extended to the extreme position you have only the potential energy

so there is no kinetic energy after that you actually put this mass now because you are already at the extremum position velocity is zero when you put it there because of

conservation of linear momentum no external forces there you're putting just the entry here without

applying any force without disturbing the system

so therefore momentum has to be it has to conserve momentum so it has to get conserved and velocity of the combined system is going to be zero again okay because

of the conservation of linear momentum okay right

so what about the amplitude it's not going

to change it is going to be the same right a_2 is equal to a because you just extended it from the

equilibrium position by a all right

so immediately you can see the total energy is half $k a^2$ square which is exactly the same as the initial configuration that we have that means not case

one that is when no mass is put on the block so total energy in this case is not getting changed

from the original one in case two okay and as i okay that is what i have written here the total

energy of the system remains unchanged in case two whatever the time period this is 2π by

ω_2 and ω_2 is equal to ω_1 one it's not getting seen

so time period is

exactly the same with that of the case one okay okay and uh yeah that's what i have written

clearly the final time period in both cases is the same all right now let us see the options once again

so if you look at the options carefully you will find that options first you see the amplitude

oscillation in the first case changes by a factor of this yeah that is basically the case in case

1 right we have found that a_1 is equal to square root of m capital m divided by small m plus m into

a whereas in the second case it remains unsafe we have seen that a is equal to a_2 is equal to e

just now we have seen right okay and what about the time period final time period both the cases

are the same that's also we have seen just now what about the total energy total energy decreases

in both the cases no it do not decrease in the second case it remains the same but in the first

case total energy decreases compared to the initial one and then uh instantaneous speed at x_0

of the combined mass decreases in both the cases all right

so yeah that is that is

correct right

so options a b and d are the ones you have to actually select these are the correct option correct options are a b and d okay now let us go to another problem this was

asked in 2009 it's a simple problem but a diagram may look you know but it's simple

you see what is let me read it and you may note it down in the meantime as i read it a uniform rod of length l capital l okay this rod has length repeated

l and mass m is pivoted at the center its two ends are attached to two springs of equal

spring constants k okay the springs are fixed to rigid supports as shown in the figure and the

rod is free to oscillate in the horizontal plane the rod is gently pushed through a small

angle θ in one direction and release you are asked to find out the frequency of

oscillation it is basically when you you know gently push it to a small angle it's going to

undergo simple harmonic motion you just have to get the simple harmonic motion equation and

then thereby you can predict what would be the frequency was a simple problem so options are

given four options are given out of these four options only one option is correct this was

asked in 2009 okay let us see how to do it

so this is the original situation now you just

give a push to this one say if you push it to this direction and then by a very small angle

θ then because of this what you are going to have is that both the springs are going to

get stressed by how much can you make it up both the springs will get stressed out

by displacement of distance right

so it would be this is $\frac{l}{2}$ this is θ

so this

distance is going to be $\frac{l}{2} \sin \theta$ okay $\frac{l}{2} \sin \theta$ because this θ angle

is small

so i can write $\frac{l}{2} \sin \theta$ as $\frac{l}{2} \theta$

so if that is

so and this

similar is the case with this spring also

so therefore there will be a restoring

force on it the restoring force on the rod by each of the spring would be how much

that would be the spring constant into this what is this the stressed amount so

that would be k into $\frac{l}{2} \theta$ okay so yeah

so this is what i have written

both the springs gets traced by this mass the restoring force

on the rod by each spring is k into $\frac{l}{2} \theta$ okay

so there would be going to be torque because

there is a rotation of this rod about this point o here and restoring torque

about θ is going to be in the anti-clockwise direction right it is going to be in the anti-clockwise direction so this would be how much this would be simply this is for one spring this is for the another spring ah this is the force and this is distance okay you you know it is a simple reveal thing to work without this total torque is due to this voltage springs will be half $k l^2 \theta$ so you know the moment of inertia also moment of inertia of the rod of this rod you know about this oscillation of axis would be simply $\frac{1}{12} m l^2$.

so this is a this i think everybody knows this is a very well known expression for a moment of inertia for a thin rod like this of length l and the angular acceleration of the rod would be simply $I \ddot{\theta} = \tau$ okay you will not be able to see it but okay θ is the angular displacement so minus sign is given because it is i'm writing it minus sign because it is taking the torque is in anticlockwise direction so torque is equal to $I \ddot{\theta}$ into angular acceleration so already i have known torque i have no θ i know the expression for $\ddot{\theta}$ if i put everything okay i'm putting moment of inertia here and $\ddot{\theta}$ i'm putting minus sign i'm taking to the other side torque expression already i have worked out so it's very easy to see if i just do the manipulation then i will get an equation of this form where ω^2 would turn out to be $\frac{6k}{m}$ divided by m so once i know ω so i can immediately find out the frequency ω is the angular frequency what is asked in the problem is the frequency okay so it is $\omega = 2\pi \nu$ so we have to find out the new so new is equal to simply this okay so therefore which is the correct option option c is the correct one okay it was a simple problem now um in this problem uh in promise this kind of paragraph type questions are very interesting because why interesting because you can learn many inter new things generally they are out of they may not be in your syllabus but what is acts is very simple kind of concept okay let us read this problem uh the phase space diagrams are useful tools in analyzing all kinds of dynamical problems there's pc especially useful in studying the changes in motion as initial position and momentum are changed

here we consider
some simple dynamical system in one dimension for such system phase space is a
plane in which
position is plotted along horizontal axis and momentum is plotted along the
vertical axis the
phase space diagram is x versus px curve okay in this plane the arrow on the
curve indicates
time flow okay this is what we're talking about indicates time flow for
example the phase space
diagram for a particle moving with constant velocity is a straight line as
shown in the figure
okay we use the sign convention in phase space position and momentum position or
momentum upward or to the
right is positive and downward or towards left is negative
so i think all of you are getting it in
this example uh this is the phase space diagram basically phase diagram for a
particle moving
constant velocity
so if it is moving with constant velocity momentum is going to be constant at
all
time okay
so that's why this constant you see it started and at all position
so that's why this
diagram will look like this
so simple thing simple problem simple idea it's all about relation
between uh momentum and position all right let us do problems now what they
are asking there
are actually three questions they are asking based on this concept first
question is this phase space
diagram for a ball thrown vertically up from the ground is
so which of this trajectories there are
four options uh they're giving
so out of these four options which one is the proper trajectory
okay
so ball is basically thrown upward thrown up uh from the ground
so when it is thrown up from
the ground
so obviously it is going to have some you know it's up uh particularly up from
the ground
so how to do it i think it's what you have to do is find out this kinematic
equation all of you know suppose
the ball is thrown up with some velocity v_0 and has a mass m right and then
you can you know this equation very trivially equation you know v^2 is
equal
to $v_0^2 - 2g(x - x_0)$ that is the initial velocity minus because it is showing up there
and from this
equation this velocity versus position is there
so you can have you have to simply have
to convert it to momentum versus position equation
so you multiply both sides by m square
so if you do that then what you are going to get is this equation you are
going to get right you
are multiplying all the things
so now immediately you get a relation between momentum and

position

so this is what you get

so plus and uh p is equal to plus minus square root of n square

this is this now to get the plot what you have to do what is happening at say x is equal to 0 and

as the ball is going up then it is reaching a you know maximum then it's again coming down when

it is going to reach the maximum you know what is going to happen the ball will move instantly

yes it will be zero there

so momentum would be momentarily zero then it will go down so it's simple okay

so you can analyze it from this equation itself when the ball goes up then x your momentum is as you see from here at x is equal to zero when the momentum is go

go up momentum direction is you can take plus so m into v zero is the momentum all right and at u

again when the ball is coming back the momentum is changing its direction and it is minus

$mv = 0$ and then at the maximum height you know that it will be zero

so momentum is

simply zero

so this information are good enough for you to plot the trajectory now if you look at

these options here you can see that you know the correct option is obviously going to be d because

of the fact that at x is equal to 0 momentum when it's going up you see it's going up here from

this position x is equal to 0 here the momentum is going up here it's increasing and it's going

to be you know you know it's sorry it's basically here it has some momentum $m v = 0$ and it decreases

decrease it and it becomes 0 at the maximum position and then it changes its direction right

and it changes this direction and it becomes minus $m v = 0$.

so therefore this is what the correct

option is if you look at the other position immediately you will see and all the positions all

the other plus don't give you the correct distance okay correct trajectory

so it's trivial

problem i think it's option d is correct okay you can actually immediately by looking at the diagram also you can you know first of all if you look at diagram a

here the position is shown to be negative and here you you cannot go the bottom of the ground

right you cannot go below the ground

so therefore you cannot start here

so this option

is obviously not correct similarly option a and b immediately you can strike off then you can

have to think about option c and option d and then you can again option c also so

i think you get it what i mean to say all right

so correct option is d and this one uh second problem 5 this part is of the problem is this phase space diagram of simple harmonic motion is a circle centered at the origin please note it down in the figure two circles represent the same oscillator okay but different initial conditions and e_1 and e_2 total mechanical energies respectively now when it is said it is the same oscillator what does it mean that means that it has the same k can say spring constant let us say that means mass and the angular frequency is the same so it's not really a problem if the energy is simply dependent on the amplitude so you can and what is basically the relation between the energies of these two situations so simply you can apply is equal to half $k a^2$ which is half $m \omega^2 a^2$ square in the first case e_1 is this amplitude is $2a$ so this is half $m \omega^2 (2a)^2$ square e_2 is this so you just take the ratio it's really trivial so e_1 it turns out to be $4 e_2$.

so i think you you know paragraph kind of questions are highly scoring because generally conceptor used to be difficult concepting but if you read it little bit carefully i think you will be able to make it out so i would suggest don't always try to attempt a paragraph type of questions they're not as you can see from this particular problem it's a easy problem and options are given so of course option would be simply c all right so another part is there here the spring mass system is given and the mass is submerged in water as shown in the figure the phase space diagram for one cycle of the system is what so these are the options given to you out of this which one is the correct one so quite clearly what is happening is the mass is oscillating simple harmonically only thing is that is put in a water submerged water now if you look at the diagram in all the diagrams you see the position is started it is starting from a in non-zero hello position so what we can do we can assume the position to have an expression like say x is equal to $a \cos \omega t$ now i just need to find out the momentum so i differentiate it once and then i get the momentum expression immediately i get that this is minus $m a \omega \sin \omega t$ so if i plot x and momentum here versus time with respect to time position

and momentum

you see as x is increasing with time you know as time goes on as x goes on the momentum is

going in the other direction uh in the negative direction i think this hint is good enough to

find out the phase trajectory because as you will see another thing is happening is that it is the

system is submerged in water and because of that damping causes continuous reduction in the

amplitude okay

so now if you look at the diagram carefully what about this

so option a all

right

so it is starting here and as its momentum is going increasing but momentum is showing in

a positive but that is not the case right what we had is momentum is in the negative direction

so therefore this cannot be the correct option similarly option d also cannot be

correct what about option b option b you see yeah here position as it is going on you

know changes with time momentum is going in the negative direction that is correct and ultimately

what is happening that it is returning back to different position with a reduced position

amplitude basically right and which is basically the case because it is submerged in water

so i

think option quite clearly option b is the correct but what about option c here also the similar

but here you see it's coming with an enhanced position but which cannot be the case because

amplitude has to decrease

so quite clearly option b is the correct option all right

correct option is b okay this is a nice problem

so now let us work out another one here what is

given a simple pendulum has a time period t_1 the point of suspension is now moved upward according

to relation y is equal to kt^2 k is equal to 1 meter per second square

here y is the vertical

displacement the time period now becomes t_2 the ratio of t_1^2 okay a simple problem because

you see okay this was actually in 2005.

i hope all of you have noted it down okay

so if you see that

y is equal to kt^2 it immediately gives you velocity would be k twice kt

if you differentiate it

once then acceleration would be twice k

so that means point of sub suspension was moving upward uh

with an acceleration a is equal to say $2k$ and k is equal to 1 meter per second square

so 2 meter

per okay let us do it

so it's this problem can be easily solved by using the concept of pseudo

force

so this is the situation you have originally right now this point of suspension or is moving in the upward okay time period is given this is the situation this is the time period now the point of suspension moves in the upward direction with this according to distillation as i said y is equal to as given y is equal to kt^2 and acceleration is here $2 \text{ meter per second square}$

so this is

basically you have the situation now if you go into the frame of reference of this point

of suspense and basically then it's very easy to solve this problem by applying this

concept of pseudo force as you will see okay

so right the acceleration of

the pendulum with respect to the point of suspension is what it is simply a plus z right because if i go to this frame here to a plus z that is a is $2z$ is 10

so $12 \text{ meter per second square}$

so therefore time period will be 1 divided by this acceleration

so this is the time period original time period was p_1 is equal to 2π by 1 by z

so therefore t_1^2 by t_2^2 is immediately it's very simple problem this one you

can solve with a plus if i just have to where it turns out to be six by five if you look at the

option here

so cardiac option is obviously a okay okay you cannot see it because of my picture

but okay the correct option is a all right now this is a another problem it's from 1998 j

so a particle of mass m is executing oscillations about the origin O on the x axis its potential energy is kx^3 where k is a positive constant if the amplitude the oscillation is a then the what is the time period

so basically

it is asking the relation of the time period with this amplitude of oscillation this is

a nice problem but this problem is exactly this situation is not simple harmonic because

simple harmonic you know you know the potential is half kx^2 and here it is kx^3 so

obviously this is not a simple harmonic motion not exactly but under certain approximation you can

always consider it to be simple remember anyway if you have this kind of a potential you know simple

harmonic potential looks like this and if you on the other hand here kx^3 mod x^3

if you plot it this would look like this it is basically y you see and if you if you

from here you can immediately see the total energy and when the amplitude is a it would be

simply kx^3 at x is equal to $\frac{1}{2}mv^2$ is equal to $\frac{1}{2}mv^2$ and kinetic energy would be 0 here because this is the extremum here the total energy would be simply kx^3 all right so this much information you have now from here and then if you look at any point at suppose at a given position x between 0 and a if the velocity is say that the potential energy in that case would be useful to kx^3 and the velocity is v then the kinetic energy would be $\frac{1}{2}mv^2$ so at any given position x between zero and a so total energy according to the total energy would be kx^3 plus $\frac{1}{2}mv^2$ now because of conservation of energy this has to be equal to the total energy kax^3 so using this you can find out the what is the velocity right velocity $\frac{dx}{dt}$ you get it so you want to find out the time period so you get an equation of this form and you just have to say okay i will show you an easy way out into $\frac{dx}{dt}$ you have worked out this way and then you can find out what is the time period so you have to just enter dt you take it this side this integral you can see it you have to integrate this expression and because of the symmetry what i can do i can just take it from say 0 to a and it will take the because of the symmetry it will take half of the time t by 2 so 0 to t by 2 dt and this this integration you have to solve this can be done easily not that easily but you you'll see you just take x is equal to $a \sin \frac{2\pi}{3} \theta$ here if we put it there okay so what you are going to get if you put all these things i think it's this expression you are going to get and ultimately you will be left out of this expression but you don't bother about it here because what is asked is the relation of the time period to the amplitude so even if you don't solve this intriguing it's not going to harm you because from here you immediately see the time period is dependent as one inverse of the square root of the amplitude so from here you can see that t is directly proportional to $\frac{1}{\sqrt{a}}$ square root of a in fact if you can numerically solve it that is not needed it will turn out to be 2. 1 this integration so option if you look at it the correct option would turn out to be a correct option will be a this problem

this way it looks very very difficult but very easily solved if you just apply the dimensional analysis that's really that can be done

so in dimensional analysis you know the time period is going to depend on the mass spring constant is k okay stiffness constant k and amplitude a

so you know how to do dimensional analysis so let me say m to the power is raised to for α this k is raised to β and this γ and from this expression of energy potential energy expression you know that of this energy expressed it has dimension of this must from here you can find out the dimension of k and dimension of k this is for energies you know m mass into acceleration into distance right that way you can make it

so $m l^2 t^{-2}$ from here you can find out the dimension of k so that is what is needed here amplitude dimension you know that the mass dimension is known so if you put it all these things there amplitude dimension is obviously determined by k dimension is this one you have and now time this one you can write in in dimensional this form you can write all of you are all of you are quite good in dimension analysis i'm sure

so now you just you are going to get users equate both sides you are going to get three set of equations you have to solve it immediately you can see that β is equal to minus half and like this

so you are basically bordering about this amplitude part because what is asked in the equation is how time period related to the amplitude

so you better worry about what the value of γ is if you find out what is γ it would turn out to be minus up

so therefore l to the power minus f so it obviously amplitude is a then time period is proportional to e to the power

so as we have found that very rigorous way here dimension analysis is giving you in very two three steps you are going to get the answer

actually it doesn't mean to be done like this only i suppose there is no need to go for the rigorous method okay

so i think i'll stop here thank you