

we continue with our discussion

on simple harmonic motion recall from the previous lecture that we looked at the equation of motion for simple harmonic motion and

we what we found is that if the displacement is x then so

let me call this displacement then the equation of motion was d^2x/dt^2 is equal to minus some constant $\omega^2 x$ and we identified this ω which is greater than 0

as $\omega^2 x$ where ω is the angular frequency

so looking at this equation d^2x/dt^2 is equal to minus $\omega^2 x$ we also wrote the solution as $x(t)$ is equal to some constant a cosine of

ωt plus b sine of ωt where constants a and b are determined by two conditions which could be the displacement for example could be displacement and velocity

at zero or displacement at two different times and we solved certain examples of this we

also learnt besides the equation

so equation which is d^2x/dt^2 equals minus $\omega^2 x$ remember how we motivated this equation this equation was motivated by

looking at the x and y coordinates of a particle moving in a circle of radius r it is

moving uniformly with either speed v or angular speed ω

so that the angle

it forms in time t is ωt and its x component is given as x equals r cosine

of ωt and the y component is given as r sine of ωt $y(t)$ equals and these

are simple harmonic motions right and then through that we could get to the equation

we also learnt that this is a representation of s h m and finally we saw in the last lecture that physically a spring mass system when the spring follows hooke's law right performs simple harmonic motion

so this is a setup against

which now in this lecture i am going to show you and discuss with you some more examples of where

you see simple harmonic motion

so these are the physical systems where simple harmonic

motion takes place one obvious question is what happens if i take this spring and hang a

mass to it in a vertical situation remember last time what we looked at was a spring and mass which

was moving on the mass was moving on a horizontal frictionless surface now i am going to look at a

mass which is being hanged from a which is hanging from a spring

so what happens if the mass is m

is that initially the spring is going to stretch and the mass is going to come down in some

equilibrium position how much does it stretch lets say it stretches by l which is going to

be mg/k where k is the spring constant which we defined last time m is the mass of the block and g is the gravitational acceleration

so this mass comes down what i am

going to do next is pull it a bit by distance y or push it up by a distance y

and release

so question we ask is what happens if we pull or equivalently push the mass by a distance y and release it

so lets see what will happen

as the mass is pulled down from its initial equilibrium position which was at l by a distance y the net force due to the spring on it is going to be upwards and this force due to the spring is going to be k net displacement

now is l plus y and of course this force is pulling it up so i will just show it by an arrow up and its own f due to m is going to be mg which is going down

so net force is going to be $k(l + y) - mg$

and remember kl equals mg and therefore all i am left with is ky pulling it up notice that the displacement is downwards right and the force is upwards therefore i can write that when pulled down the force on this is

$-ky$ on the other hand suppose this spring is pushed up from the initial position

where the mass was up to stretched from the length of the spring was up by l and now i have pushed it up by y then the force due to the spring is going to be $k(l - y)$ and it is

going to be upwards because i am assuming that l is greater than y

so the spring

is still stretched and f due to the mass is going to be mg and downwards therefore the net force is going to be equal to again $k(l - y) - mg$

minus ky right and this is minus

signs minus ky upwards or ky downwards

so when y is up the force is the other way

right

so net force i can always write therefore whether the spring is pushed up or pushed

down f is always going to be equal to minus ky

so the equation of motion is going to be m

$\frac{d^2y}{dt^2} = -ky$ or $\frac{d^2y}{dt^2} = -\frac{k}{m}y$

omega square still comes out to be $\frac{k}{m}$ so even if you hang the spring vertically right

is being pulled down by constant force the mass is being pulled down by constant force

mg even then omega square remains the same a related problem could be that if i take this

spring mass system put it on a horizontal table and pull it by a constant force f

so all that is

going to happen is going to change its position to a new position right with this displacement

being $\frac{f}{k}$ and after that if i again displace it from that position is going to have again the

same frequency omega square equals $\frac{k}{m}$

so the natural frequency of this system remains unchanged

even when spring mass system is vertical or it is horizontal and still and being applied a constant

force all that happens is the equilibrium point shifts let us take the second example again i am

taking these examples to show you how different systems perform simple harmonic motion so in this system i am going to take a string pulled so that it has a tension t i put a mass m in between attach another spring identical spring of length l of same tension t and attach it on the other side to a wall so i have two strings of length l both having tension t they have been pulled up with this mass in between and next what we do is displace this is the initial position displays this whole thing a bit tension remains t i will be displacing it such that suppose this distance is x x over l is much much much less than one so the change in the length of the string is not really much therefore tension remains roughly the same what is going to happen though is this mass is going to be pulled like this by the string on this side and like this by the string to this side and therefore there is going to be a net force f_{net} in this direction on the string opposite to the displacement on the other hand if i pull this mass down like this this is tension t this is tension t then the force is in this direction so you can see that when the mass is displaced there is a force opposing that displacement and how much is that force let us calculate that suppose this angle is θ i can write in this side also θ then the net force on this is going to be one component of tension from right side one component of tension on from left side similarly here these two components are going to give me the net force so f_{net} is going to be that vertical component which you can easily see is going to be $t \sin \theta$ plus $t \sin \theta$ which is $2t \sin \theta$ since x is much much much less than l $\sin \theta$ is roughly equal same as $\tan \theta$ is roughly same as θ is x over l and therefore f_{net} is going to be $2t \frac{x}{l}$ and in the direction opposite to the displacement therefore i am going to put a minus sign in front so we figured out that f_{net} in this system is opposite to the displacement and its minus $2t \frac{x}{l}$ so the equation of motion for the mass is going to be $m \ddot{x}$ is equal to minus $2t \frac{x}{l}$ which i can write as minus $\frac{2t}{l} x$ and therefore \ddot{x} is minus $\frac{2t}{l} x$ identifying this term as ω^2 you have the mass now oscillating with angular frequency ω equals square root of $\frac{2t}{l} \frac{1}{m}$ or time period is equal to $2\pi \sqrt{\frac{l m}{2t}}$ and the general displacement y or x of this mass is going to be given as some constant A cosine of square root of $\frac{2t}{l m} t$ over $l m t$

plus $b \sin$ of square root of two t over $l m$ t
 so that's another example just to show you that
 simple harmonic motion occurs almost every day in many many different
 situations in our lives
 let me take a third example example three suppose i have a wooden block or any
 block of
 made any of any material floating in some liquid lets call this block of
 uniform surface area a
 here and it is floating in some liquid submerged by depth l here it is
 so it is submerged this is a surface
 and this depth from here to here is l and from archimedes principle i know
 that $\rho g l$ times a is the weight of the liquid
 displaced is going to be mg where m is the mass of the block all right ρ is the
 density of liquid l is the depth to which the block is submerged g is the
 gravitational acceleration and a i have shown you is the
 this is a area of cross section
 so i have a $\rho g l$ equals $m g$ g cancels
 so m equals $a \rho$ liquid times l that
 is the mass of the block now what we do is we displace it a bit suppose i
 push
 it down all right i push it down a bit push it down by y what happens when i
 push
 the block down by y
 so this is the liquid earlier the block was up to depth l and
 now we have pushed it down further by y
 so the lower surface is going to feel more
 pressure and therefore the net f buoyancy force is going to be the volume of the
 the weight of
 the liquid displaced
 so area remains the same l plus y ρg and this force is up the
 buoyancy force is always towards up and f gravitation force due to gravitation
 is
 mg is down right
 so i can write this as minus write this as plus up sine s plus sine
 and therefore f net is going to be $a l \rho g$ minus mg is 0 right we just found
 out that a
 equals $a \rho l$ and therefore $a l \rho g$ and mg are the same they cancel and
 you get the net
 force $a y \rho g$ up
 so if you are pushing it down there is a force up right
 so force is opposite
 to the displacement let us see what happens if i shift it up
 so suppose in the same
 liquid right the block was initially to depth l but now i have
 pushed it up by y right
 so so now the depth is going to be l minus y and f buoyancy is going to be a
 $\rho g l$ minus y is
 still going to be up and f gravitation or weight is going to be minus mg is
 down
 again when i add the two forces f net comes out to be $a \rho g l$ minus mg cancel and
 you
 get minus $a \rho g y$ up and minus sign means down
 so this is going to be $a \rho g y$ down
 so when

y is positive that means is being pushed up the force is down
 so the net force the expressions
 are the same f_{net} irrespective of whether the block goes up or comes down is
 $-\rho g y + \rho g y$
 $\rho g y - \rho g y$
 so the equation of motion is going to be $m d^2 y / dt^2$ is equal to minus $\rho g y$ or $d^2 y / dt^2$ is equal to
 $-\rho g y / m$ the number ρ times l was m
 so therefore i can write this as g / l y / m over ρ is nothing but l and
 therefore
 ω^2 is going to be equal to g / l or time period is going to be
 $2\pi \sqrt{l / g}$
 so if it goes down by depth l the time
 period is going to be $2\pi \sqrt{l / g}$ next example i am going to take oh by
 the way before i take the next example you can also generalize it to other
 shapes
 so for example if i have this water and i submerge something else in it right
 that has a uniform cross section
 so for example i could have a bottle that is submerged in that case when
 the bottle is pushed down or up the relevant cross sectional area is
 going to be this cross sectional area A ok
 so you can work this out i have
 given you already the idea how to work out these problems next example
 number four i am going to take is a youtube of uniform cross section in
 which we have filled some liquid and let us say this entire
 length of this liquid column is l what we do now is push this liquid
 down push it down
 so that the new position the liquid is lets say goes up on one side and comes
 down by the same amount on the other
 so this was the initial equilibrium position
 so it goes up by say amount y and therefore it comes down by an amount y
 so that the total height
 out here is $2y$ it could be the other way also the liquid could have been
 pushed up the other side by amount y this is y this is also y
 so what
 happens now this extra two eye height applies a pressure extra pressure and
 that pushes the liquid column down as the
 height decreases the pressure also goes down and therefore the force also goes
 down nonetheless
 when the height is h the force due to this pressure is going to be the cross
 sectional area
 A the cross sectional area A times ρ_{liquid} times $2y$ times g that is the
 net force and
 is in the direction opposite to the displacement
 so if displacement to the liquid is higher on
 the left hand arm then the force is pulling it down this way as shown here the
 left arm it
 comes down right arm it goes up on the other hand if the liquid is higher on
 the right
 arm then the right arm it tends to go down and here it goes up
 so the force is
 opposite to the displacement the mass of the liquid is the cross sectional area
 A times l is the volume times ρ_{liquid} that is the mass
 and therefore the equation of motion is $m d^2 y / dt^2$

is equal to minus two a rho g times y that's what we had
 calculate the force to be two a rho y g two a rho y g and mass is a l rho
 so a l
 rho is equal to minus two a rho g y a cancels
 so does rho and you have the equation
 of motion or there is a t two y by d t square you have the equation of motion
 d
 two y over d t square is equal to minus two g over l times y this is exactly
 the same
 equation as that for simple harmonic motion and therefore you are going to
 have the
 frequency of simple harmonic motion given as two g by l or time period of
 oscillations is
 equal to two pi square root of l over two g
 so these are some examples that you see in daily
 life you can do it even at home and measure this time and see that simple
 harmonic motion indeed
 takes place in this i am now going to come to a very specific example of
 simple
 harmonic motion called the simple pendulum and derive the time period for
 oscillation of a simple pendulum whatever we do here you can check at home also
 because a very easy thing to make all you do is take a string and tie a mass
 at the bottom
 the extent of mass or the size of the mass should be much less than the length
 of the
 string and then it becomes a simple pendulum if you displace it to one side
 the mass comes back swings to the other side
 the energy is not lost to the same height and does a back and forth motion
 so first
 thing you notice is that the motion of the pendulum is periodic but the question
 is is the motion simple harmonic motion and you will see and it will
 become clearer if it may take this pendulum to be that made of a rod that for
 large theta if the
 displacement from the vertical is large it is not simple harmonic on the other
 hand if theta
 is very small then the motion is simple harmonic
 so let us see how does that happen if the theta is
 small then for small displacement i can consider this to be almost moving
 horizontally all right so
 that if the displacement is x all right the force in the x direction i need to
 calculate why
 is that force there let us analyze that when the pendulum is displaced
 and here i am going to make an angle is weight mg can be written as two
 components
 one component is along the string and the other one is perpendicular to the
 string and it is
 in the direction opposite to the displacement and this component if this angle
 is theta
 this component f perpendicular to string is equal to m g sine of theta ok
 so here is the pendulum it has been displaced by an angle theta here is weight
 mg it has a component
 parallel to the string and the other component perpendicular to the
 string and f perpendicular to string is mg sine of theta
 so force is

$mg \sin \theta$ all right which for θ much much much less than one
 ok if the length of the pendulum is l and this displacement in the horizontal
 direction is x then x or this arc length does not really matter for θ much
 less than
 one therefore this can be written as roughly $m g$ which is equal to $mg x$ over l
 and now i
 have to be careful and put a minus sign here because the force is in the
 direction opposite
 to x all right and therefore equation of motion $m d^2 x$ over $d t^2$ square is
 equal to minus
 $m g$ over $l x$ this is the equation of motion i will cancel m from both sides
 and i get $d^2 x$
 over $d t^2$ square is equal to minus g over $l x$ this is the equation for simple
 harmonic motion notice
 that this is simple harmonic motion only under the approximation that θ is
 much much much less
 than 1 or the displacement x is much much much less than l θ is much less
 than
 one

so what i have is for the pendulum θ much much much less than 1 or this x
 ah θ much less than one or x much much much less than l i have $d^2 x$
 over $d t^2$ square is equal to minus g over $l x$ and therefore ω^2 is g over l
 or ω
 is square root of g over l and the time period T equal 2π square root of l
 over g notice that the time period does
 not depend on the mass of the bob or mass of that point mass that you
 are hanging at the end of the string it depends only on the length of the
 string an

interesting problem would be find the length of a pendulum that has time period
 of one second we will go to the formula one equals
 2π square root of l over g gives you l equals g over $4\pi^2$ which
 i can roughly
 roughly because g is roughly π^2 is one over four meters or roughly
 twenty five
 centimeters and you see your wall clock at homes the pendulum that swings
 there is roughly has
 the length of 25 centimeters roughly because i have taken g to be π^2
 so it will be
 very close to 25 centimeters and that has a time period of one second next
 what i am
 going to do is

so far we have looked at
 so so far we have looked at point masses performing s h m now we are going to
 go
 generalize and see what happens if we deal with extended bodies rigid bodies and
 so on
 so as the first
 example i am going to take a rod hanging with a pivot hanging pivoted at one
 end and hanging vertically and when it is displaced from vertically starts
 going back and forth
 around the equilibrium position
 so it does perform periodic motion is it simple harmonic motion thats the
 question we

are asking and let us see its very much similar to the simple pendulum that we discussed but you

have to be careful when you are dealing with rigid bodies remember this whole body is

one whole body right

so let me write when dealing with extended rigid bodies what is the equation that we use we use torque equation

so in this case i am no longer going to use the force equals $m \times$ equation but rather torque equals $I \alpha$ where α is the

angular acceleration equation

so let us now formulate the problem and say a uniform rod of mass m and length l is pivoted at one end

so let me make a

picture here it is pivoted at point it is hanging vertically in equilibrium if displaced by an angle θ

from the vertical write its equation of motion when it is released

so what we

are doing is we are displacing it by an angle θ and releasing it and i want to

write the equation of motion number one number two find if for θ much less than one it performs simple harmonic motion and find its time period

so lets see this is a rigid body its

a uniform rigid body and therefore the force acts at the center of mass at a distance $l/2$ the force acts this way mg and it applies a torque how much is the torque

torque is going to be perpendicular distance right

so let me show it in different

color the perpendicular distance which is shown in red $l/2 \sin \theta$ and that pulls it back

so the torque on the body is $mg \cdot l/2 \sin \theta$ and it is in the direction

opposite to the displacement an equation of motion therefore is going to be moment of inertia I

of the rod times angular acceleration α is equal to minus $mg \cdot l/2 \sin \theta$

why this minus sign because the torque is acting in the direction

opposite to the displacement recall that α is equal to $d^2 \theta / dt^2$

and therefore the equation of motion is $I \cdot d^2 \theta / dt^2$ is

equal to minus $mg \cdot l/2 \sin \theta$ or $d^2 \theta / dt^2$ is equal

to minus $mg \cdot l/2 \sin \theta$ this is the equation of motion notice that

on the right hand side i have $\sin \theta$ now if θ is much much much less than one then

i can write $\sin \theta$ approximately equal to θ and then the equation of motion becomes $d^2 \theta / dt^2$

is equal to minus $mg \cdot l/2 \theta$ this equation is exactly

the equation for simple harmonic motion recall that what we had written earlier was $d^2 x / dt^2$

equals minus some constant times x that x is replaced by θ now except that except that

there is no other difference this is a constant right
 so for small theta theta much less than one
 this is going to perform a simple harmonic motion with omega square equals $\frac{mg}{l}$
 so if i take a rod uniform rod of length l mass m then displaced it performs
 simple harmonic motion with omega square being given as $\frac{mg}{l}$ and for a uniform
 rod i is $\frac{ml^2}{3}$
 so omega square becomes $\frac{3mg}{2l}$
 which is m cancels three g over two l this one of the
 else
 also cancels and the time period t is going to be equal to $\frac{2\pi}{\omega}$
 which is $2\pi \sqrt{\frac{2l}{3g}}$
 so this is slightly different the time
 period is slightly different than that for a simple pendulum where all the
 mass was concentrated at the end another example of this is suppose i have a disk and i
 pivot it at a point on its periphery
 so a disc a uniform disc of mass m and radius r is pivoted at a point on its
 periphery and now we displace it slightly with a small angle by small angle
 theta
 so the question is what is the frequency of its oscillations when it is
 displaced by a small angle theta from the vertical and released
 so what we are doing is we are taking this disc displacing it slightly from the
 vertical position where it is in equilibrium and releasing
 it we want to know the frequency
 so again here is the disc and its been displayed slightly from its
 equilibrium position and all the mass is acting at the center of mass
 pulling it down and that provides a counter torque to this
 so that it is pulled back how much is the counter torque this is r then the
 torque is going to be this perpendicular distance from the pivot and the
 vertical line vertical line passing through the pivot torque
 is going to be $mgr \sin \theta$ and for a small theta this can be written as
 $mgr \theta$ and therefore the equation of motion is going to be $I \alpha = -mgr \theta$
 or $I \frac{d^2 \theta}{dt^2} = -mgr \theta$ by parallel axis theorem i is going
 to be the moment of inertia about the centre of mass which is $\frac{1}{2}mr^2$
 plus mr^2 which is $\frac{3}{2}mr^2$ and therefore the equation of motion is $\frac{3}{2}mr^2$
 $\frac{d^2 \theta}{dt^2} = -mgr \theta$ lets cancel m on both sides lets cancel
 one of the r's on both sides and therefore i get $\frac{d^2 \theta}{dt^2} = -\frac{2g}{3r} \theta$
 so omega square in this case is going to be $\frac{2g}{3r}$ and the time period t
 is going to be $2\pi \sqrt{\frac{3r}{2g}}$ another

problem i am going

to now do just to show you how wide this concept of simple harmonic motion is something called the plasma oscillations and if you recall from your 12th grade book these occur or you may have heard of plasma or plasma frequency while talking about radio waves in ionosphere

so what a plasma is is a collection of positive and negative charges and when they are displaced with respect to each other as i will show in the problem they start oscillating

together and that is called the plasma oscillations and the natural frequency for this is

denoted by ω_p which is known as plasma frequency

so the question that

we are going to ask is problem shown below is a collection of positive and negative charges we can say positive ions and electrons let me show this

so this is like the positive charge shown by black and on top of it is negative charge shown by red and this is shown in slab geometry the positive charges are fixed while negative charges are mobile if the slab of negative charges is displaced as shown it starts oscillating find the frequency of oscillations

so what we are showing is that this negative charge is now going to be displaced slightly outside this this is what is shown so

that on this side there is negative charge on what is left behind here on the back

side is here is all positive charge and then obviously this sets up an electric

field which pulls the negative charge back

so let me now explain the problem further

so what we are doing is we have the slab on which we have this negative charges that have been displaced slightly

so what is happening is here is this this charge becomes negative and what you are left behind with is this positive charge and this sets up an electric field like this out here and this electric field is going to pull this red thing back

so there is a restoring force due to the electric field if this restoring force is

proportional to x or the displacement then i know there is going to be simple harmonic

oscillations let this distance be x let the width of the slab be l let this area out here area be A now let's see if this charge has been displaced by x how much is the surface charge out here

so let n be equal to number density of the charges all right

so when i displace this by x the volume

that is out this is shown here which i am putting my pen over which has been shown by purple and all

that is going to be a times x

so number of charges is going to be n times A times x so

charge because its a negative charge is going to be minus $n e A x$ where e is the let us say electronic charge if these things which are being moved are electrons exactly by the same amount the charge on the other side is going to be on this side

is going to be $n e a x$

so charge per unit area on both sides which I call σ is going to be

$n e a x$ divided by a which is $n e x$ this is σ

so on the left hand side I have plus σ on

right hand side I have minus σ

so electric field set up in between is going to be σ over

ϵ_0 which is $n e x$ over ϵ_0 pointing to the right and this

is going to apply

a force on these electrons how much is the force force is going to be number
of electrons $n a l$ we

are assuming x is much much much much less than l $n a l$ times the charge is e

I can put a minus sign

but since we are only calculating the magnitude I can put a plus times the

electric field which is n

e divided by ϵ_0

so this is going to be n square $a l e$ square x divided by ϵ_0 and

this force is going to be the mass of the electron which are going to move

times the acceleration

mass is going to be number of electrons $a l$ times m_e mass of the electrons x

double dot is

going to be minus n square $a l e$ square x divided by ϵ_0 this minus sign

shows that this is a

restoring force now I can cancel a from both sides I can cancel $n l$ from both

sides I can cancel

one of the n 's from both sides and I'm left with $m_e x$ double dot equals minus

$n e$ square x

divided by ϵ_0

so I am left with $m_e x$ double dot equals minus $n e$ square x

double dot equals n divided by ϵ_0 $n e$ square over $m_e \epsilon_0$

times x and

this is nothing but ω plus m square so ω plasma square is $n e$ square

over m_e

ϵ_0 and this is known as the plasma frequency

so this is a different

application of the concept of simple harmonic motion to a

collection of charged particles as the final problem in this lecture

I am going to take a square of side a pivot it vertically at one of the corners

and

then displace it slightly from that position by an angle θ from the vertical

position and we want

to know what will be the frequency of oscillation again since it's an extended

body the equation that we are going to use is going to be $I \theta$ double dot

which is same

as if the angular acceleration is equal to τ the torque the torque again just

like in the

disc problem is coming because of this weight mg

so if we calculate it this is the displaced square from the vertical this is mg

distance

from the axis is the half the diagonal which is a over root two sine θ

so the torque is $mg a$

over root two sine θ and I put a minus sign in the front because it's in

the

direction opposite to the displacement which in small angle approximation it

becomes

minus $m g a \over \sqrt{2} \theta$ if θ is small and therefore the equation of motion is
 $\ddot{\theta} = - \frac{m g a}{\sqrt{2} I} \theta$ and ω^2 therefore
 is $\frac{m g a}{\sqrt{2} I}$ that's it now we would got to do is substitute for I
 for the square
 now I about the pivot is going to be I of cm about the pivot plus I about cm
 so I of cm about
 the pivot is going to be m times a square by two plus I about the cm about
 this axis
 perpendicular to the paper is going to be $m a$ square by six which then becomes
 $\frac{2 m a^2}{3}$ and therefore ω^2 is nothing but $\frac{m g a}{\sqrt{2} \times \frac{2 m a^2}{3}}$
 square over 3.
 we can cancel m on both sides one of the a 's goes and therefore the answer
 you
 get is $\frac{3 g}{2 \sqrt{2} a}$ that's ω or the frequency ω itself
 is $\frac{3 g}{2 \sqrt{2} a}$ raised to one half
 so to conclude we have taken this equation $\frac{d^2 y}{dt^2} = - \omega^2 y$ you can have anything for y it could
 be angle it could
 be the displacement of water liquid or whatever as long as the equation is in
 this form this implies
 simple harmonic motion and you can derive it derive the equation of motion
 through
 consideration of the force and displacement in point masses or liquids being
 displaced in rigid
 bodies you can you can derive the same equation using the torque and angular
 acceleration equation
 as long as the equation comes in the form where the acceleration is some
 proportional
 to minus the displacement that constant gives you the frequency of simple
 harmonic motion
 and the body performs simple harmonic motion you