

in this lecture i am going to focus on simple harmonic motion which i had motivated in the previous lecture and what i had said is that if i take a particle moving around in a circle and take its x component  $x(t)$  the motion is uniform then  $x(t)$  is given as  $r \cos(\omega t)$  which in general i am going to now write as  $x(t)$  equals some constant  $a \cos(\omega t)$  the corresponding velocity  $v(t)$  is minus  $\omega a \sin(\omega t)$  and the corresponding acceleration is minus  $\omega^2 a \cos(\omega t)$  which is nothing but minus  $\omega^2 x$  itself so what it means is as you know acceleration  $a(t)$  is nothing but the derivative of velocity with respect to time which is the second derivative of  $x$  with respect to time and this for simple harmonic motion is given as  $-\omega^2 x$  so this is going to be our equation for simple harmonic motion that means whenever the displacement is of the form this minus some constant  $c$  where  $c$  is positive times  $x$   $c$  is positive because this is coming from  $\omega^2$  so it has to be a positive number the motion is going to be simple harmonic and the solution of this equation is going to be of the form because remember  $c$  is equivalent to  $\omega^2$  so motion is going to be  $x(t)$  is equal to some constant  $a \cos(\sqrt{c} t)$  and some other constant  $b \sin(\sqrt{c} t)$  ok this is i will develop it mathematically further but i am motivating you how this equation where  $d^2 x / dt^2$  that is the second derivative of displacement or the acceleration is negative is proportional to the displacement itself with a negative sign i am going to get a motion which is simple harmonic what it means is since force is equal to mass times acceleration that means if force on a particle is proportional to minus the displacement where minus sign means let us write that also explicitly minus sign implies force is in the direction opposite to the displacement right so if force on a particle is minus proportional to minus displacement minus sign means forces in the direction opposite to the displacement it will always imply always imply the motion of particle is going to be simple harmonic motion and is going to perform a simple harmonic motion so we have just looked at this particle moving around in a circle and through that we found out what  $x(t)$  or  $y(t)$  or the displacement is took its derivative and then turned the whole argument around we just went back saying that if the acceleration is proportional to the displacement or the forces proportional displacement but in the direction

opposite right  
 the motion is going to be simple harmonic that makes perfect sense because if there is a particle at a certain point if it moves to the right suppose this is a displacement unless the force is in the opposite direction it will not come back right it will not go back and forth so the force has to be in this direction and if the particle is displaced to the left the force has to be in the right direction that is a physical way of looking at it so for it to perform a simple harmonic motion right where it goes back and forth in a very very periodic way the force has to be in the direction opposite to the displacement and i have also shown you how to visualize simple harmonic motion from now on i am going to call it s h m which stands for simple harmonic motion right so how to visualize it i have already given you the tool to do this always think of a particle moving uniformly in a circle right with constant angular speed or constant speed linear speed and take this component on the x axis or the y axis or a combination of the two that is the way to visualize it in fact later i will again come back to this visualization and teach you something called the phasor diagrams which are very helpful whenever we are looking out at periodic motion like this but now lets become slightly sophisticated mathematically all right and develop this further but before that i want to solve some problems related to whatever we have learnt so i am going to ask you problem number one what is the general solution for the following differential equation ok i am giving you an equation which is  $\frac{d^2 x}{dt^2}$  is equal to minus two x this is exactly the kind of equation we were finding out in simple harmonic motion this is exactly the kind of equation where the acceleration is opposite to the displacement and therefore the general solution is going to be  $x = t$  which is equal to a some constant a cosine of square root of two times t and a combination with corresponding sine term sine of square root of two t that is the general solution you take a second derivative you want to get exactly this as i showed you earlier right let us take another example let us say  $\frac{d^2 y}{dt^2}$  is equal to minus five y i am using y just to emphasize these are just symbols what you need to do fundamentally is look at what is the relationship between the symbol which represents the displacement a second derivative and the symbol itself which represents the displacement so in this case again you see that

the acceleration is opposite to the displacement because that negative sign and proportional to the displacement therefore  $y(t)$  again is going to be some constant cosine of  $\sqrt{5}t + b \sin(\sqrt{5}t)$  that's the general solution you can check it for yourself right that if you take the second derivative it will satisfy this equation so this is just looking at the equation where the second derivative of a quantity with respect to some other quantity is proportional to that quantity itself right of which I am taking the second derivative and there is some constant let me also give you a general kind of thing suppose I have an equation  $\frac{d^2 y}{dt^2} + k y = 0$  and this is equal to minus let's say some constant right  $k y$  where  $k$  is greater than zero what is the solution you see this equation if I put it on the side is exactly like  $\frac{d^2 x}{dt^2} = -k x$  its exactly like that I am taking the second derivative of a quantity here I am denoting that by  $y$  with respect to another quantity where which I am denoting by  $x$  and the second derivative is proportional to  $y$  itself its exactly the same structure all that we have done is replaced  $x$  by  $y$  and  $t$  by  $x$  so the general solution for this is also going to be  $y$  as a function of  $x$  is equal to some constant  $a \cos(\sqrt{k}x) + b \sin(\sqrt{k}x)$  so what you have to keep in mind is that the mathematical structure of the equation which is in this case second derivative of a quantity right with respect to some other quantity is proportional to that first quantity itself then mathematical structure tells you that the solution is going to be a linear combination or a combination of cosine and sine terms right and where in all these three examples  $a$  and  $b$  are some unknown the unknown right now I do not know them there is no way from the solution that we can find them unknown but they are constants they cannot depend on  $t$   $x$   $y$  anything they are constants the way to determine constants are if we wish to determine these constants we need further information and these are two constants  $a$  and  $b$  two constants so I need two equations to determine them so the further information should be in terms of two further information right so it could be say this information could be number one displacement and velocity at  $t$  equal to sometime let us say  $t$  equal to  $0$  or it could be displacement at two different times or any information where I need two informations so let us solve an general example so suppose I have this equation  $\frac{d^2 x}{dt^2} + k x = 0$

$d^2x/dt^2$  is equal to  $-\omega^2 x$  I just written it  $\omega^2 x$  to keep things simple

so that  $x(t)$  is given as  $a \cos(\omega t) + b \sin(\omega t)$

so this is it this is

this is I cannot determine  $a$  and  $b$  any further and suppose now I give you that  $x$  at

time  $t = 0$  is  $x_0$  and  $dx/dt$  that is velocity at time  $t = 0$  is some  $v_0$  now I have given you two specific informations

then I can determine  $a$  and  $b$  how do we do that  $x$  at  $t = 0$  is going to be

if I substitute  $a \cos(0) + b \sin(0)$  which is  $a$  and this is given to be  $x_0$ .

so I have already determined what  $a$  is going to be similarly  $v(t)$  at  $t = 0$  which is  $dx/dt$

$v$  is nothing but  $-\omega a \sin(\omega t) + \omega b \cos(\omega t)$  at  $t = 0$

is going to be nothing but  $\omega b$  and this I have been given this to be  $v_0$  and therefore  $b = v_0 / \omega$  now I have the complete solution and therefore I am going to have in general  $x(t)$  is going to be  $x_0 \cos(\omega t)$

plus  $v_0 / \omega \sin(\omega t)$  this is where the displacement and velocities have been

given at  $t = 0$   $x_0$  is displacement at time  $t = 0$  and  $v_0$  is velocity at time  $t = 0$  let us solve an example find the solution of equation  $d^2x/dt^2 = -25x$

with  $x(t = 0) = 3$  meters

and  $v(t = 0) = -2$  meters/second

so I have been

given  $x_0 = 3$  and  $v_0 = -2$  and immediately I can write that  $x(t)$  is going to be  $3 \cos(5t) - 0.4 \sin(5t)$

how do I get this 5 because this 25 is nothing but  $\omega^2 = 25$

so  $\omega = 5$  which then comes out to be  $3 \cos(5t) - 0.4 \sin(5t)$  that's the

solution

so you see I have a general solution of that differential second order

differential equation where the minus sign in front and the second order derivative being proportional to the

displacement itself I have a complete solution if I specify  $x$  and  $v$  at  $t = 0$

which I am taking to be  $t = 0$  right now or the displacement at two different times and

so on ok

with this preliminary introduction let's conclude that force proportional to minus the displacement implies acceleration which is  $d^2x/dt^2 = -cx$

constant  $c$  and this all this together leads to simple harmonic motion and by that we mean that the displacement  $x$

$x(t)$  is going to be of the kind  $a \cos(\omega t) + b \sin(\omega t)$  where  $\omega$  would be determined by  $c = \omega^2$

is equal to square of  $c$  plus  $b$  sine of  $\omega t$  how do we get this mathematically

so let us now develop that this is going to be slightly on the advanced side but i think you will enjoy it because by now you must be i am just giving you arguments

that this is how the solution comes out but if you are you know getting that feeling how

does it come out let me just give a mathematical digression detour ok so i have been

given  $d^2x$  by  $dt^2$  is equal to minus  $c x$  i am taking  $c$  greater than zero ok

so in this case i can the equation i have  $d^2x$

by  $dt^2$  is equal to minus  $c x$  i will assume solution of the form  $e^{\lambda t}$  to lambda somewhere

lambda is some constant  $t$

so i am assuming  $x(t)$  can be written as  $e^{\lambda t}$

raised to lambda  $t$  and therefore  $\frac{dx}{dt}$  is of the form  $\lambda e^{\lambda t}$  to lambda

$\frac{d^2x}{dt^2}$  is of the form  $\lambda^2 e^{\lambda t}$  substitute

this in the equation  $um$

so substitute this in the equation and then what you get  $\lambda^2$

square  $e^{\lambda t}$  is equal to minus  $c$  times  $e^{\lambda t}$

lambda  $t$  these two terms cancel and i get  $\lambda^2 = -c$  lambda equals plus or minus  $i \sqrt{c}$  and

therefore the general solution is either of the form  $e^{i \sqrt{c} t}$  or  $e^{-i \sqrt{c} t}$

and the most general solution is going to be a combination of the two this is

going to be some constant  $a_1 e^{i \sqrt{c} t}$  plus some other constant  $b$

one  $e^{-i \sqrt{c} t}$  ok so  $x(t)$  is of the form  $a_1 e^{i \sqrt{c} t}$

plus  $b_1 e^{-i \sqrt{c} t}$  and you will learn or if you have not already learned  $e^{i \sqrt{c} t}$  or some constant  $t$  is nothing but cosine of  $\sqrt{c} t$  plus  $i$  sine of  $\sqrt{c} t$  right

so therefore  $x(t)$  if i combine the two  $i$

can write as some constant  $a$  cosine of  $\sqrt{c} t$  terms plus some other constant  $b$  sine of  $\sqrt{c} t$

where  $a$  you can make out is going to be  $a_1 \cos + b_1 \sin$  and  $b$  is nothing but  $i$  times  $a_1$

one minus  $b_1$

so i can write the solution i have given you mathematical way of getting the solution

this also teaches you something interesting if  $c$  is negative ok that means the

force is proportional to displacement but no minus sign here notice no minus sign here

that means if you displace a particle to a certain distance force is in the same direction if you

displace in the negative side forces in the same direction you can already see physics wise that

the particle is going to run away from that point let us see that

mathematically in that case the

differential equation is going to be  $d^2x$  by  $dt^2$  which is going to be  $-cx$  where  $c$  is again positive this minus sign is gone and if i again take solution  $x(t)$  to be of the form  $e^{\lambda t}$  you will find that  $\lambda^2$  is equal to  $c$  or  $\lambda$  equals plus or minus square root of  $c$  and therefore the solution  $x(t)$  is going to be of the form  $e^{\sqrt{c}t}$  or  $e^{-\sqrt{c}t}$  the general solution  $x(t)$  is going to be some constant  $a$   $e^{\sqrt{c}t}$  plus  $b$   $e^{-\sqrt{c}t}$  and you can see as  $t$  increases as  $t$  increases the first term increases exponentially and therefore the particle is just going to go run away  $x$  is going to be keeping on increasing further and further so that minus sign in front is very important which we understand physics wise is that if  $f$  is proportional to minus displacement right and  $f$  is proportional to displacement the motion is quite different in this case displacement is this way force is the other way the displacement is this way force is the other way so this is  $x$  this is  $f$  in this in the other case force displacement is this way force is further this way so it makes the displacement grow faster if the displacement is the other side force is also in that side so it makes the displacement grow farther and that is shown by this exponentially increasing term so this is the mathematical digression that tells you how the solution arises and how if that minus sign is not there the the solution instead of being oscillatory it will be exponentially increasing and there will be no oscillatory motion so lets summarize what we have learnt so far in this lecture what is new that we have learnt is that if force is proportional to minus the displacement the equation of motion is of the form  $d^2x$  by  $dt^2$  is equal to minus  $c$   $x$  and this general solution of this equation is of the form  $x(t)$  equals some constant  $a$  cosine of root  $c$   $t$  plus some other constant  $b$  sine of root  $c$   $t$   $a$  and  $b$  are determined by some conditions in fact two conditions given this could be  $x$  and the velocity displacement the velocity at certain time or two displacements at certain time and so on if force is proportional to displacement that means no minus sign in front  $d^2x$  by  $dt^2$  is equal to  $c$   $x$  i must write  $c$  greater than zero  $c$  greater than zero and the particle tends to move away when displaced so this first thing leads to what is called as the simple harmonic motion now the question we ask if you set all this mathematical device where or in which systems does simple harmonic motion occur that's one question and two what is important about simple harmonic motion that we are paying

so much attention to it

so i'll answer the first question

and then we will go to the second one in which systems the simple harmonic motion

occur we have already seen if force on a particle is proportional to the displacement but in the direction opposite

to it simple harmonic motion takes place

so one place where this happens is a spring mass system because you know in a spring which is say of natural length or unstretched

length  $l_0$  by Hooke's law if the spring is stretched beyond this  $l_0$  by a displacement

$x$  then the force that it applies is proportional to the displacement and spring pulls you back

on the other hand if the spring is compressed by a distance  $x$  the force again is  $kx$  and

this is in the positive direction

so it's always opposite to the displacement all right this is Hooke's law where  $k$  is known as spring constant and its dimensions are newtons per meter  $ok$

so if i displace it by one

meter how much force applies divide that force divided by the displacement gives you

the spring constant

so spring mass system if i take a spring mass system let us

say on a horizontal frictionless table and put a mass  $m$  here right and let my coordinate system be such that  $x = 0$  is where the equilibrium point is where the

spring is has its natural unstressed length if i displace this mass by  $x$  then the force on

the mass is  $-kx$  and the equation of motion is mass times the acceleration  $d^2x/dt^2$

is equal to  $-kx$  this is the equation of motion

so in a

spring mass system where i have a spring and a mass attached to it the unstressed length is  $l_0$  and i measure my displacement from  $x = 0$  this

unstressed length if i displace it to the right by  $x$  right it experiences a force to the left which is

$f = -kx$  or on the other hand if i compress the spring by  $x$  it experiences a force to the right

so this is again going to be  $-kx$   $x$  is negative

so  $f$  will become positive if i write it as  $-kx$  and the equation of motion is  $m$

$d^2x/dt^2$  is equal to  $-kx$  or if i divide by  $m$  i get  $d^2x/dt^2$  is equal to

$-(k/m)x$  this is exactly the equation that we have introduced you to in discussing simple

harmonic motion this is of the form of the form  $d^2x/dt^2$  is equal to  $-\omega^2x$

square  $x$

so for if i identify  $\omega^2$  as  $k/m$  i have the equation of motion  $d^2x/dt^2$

is equal to  $-\omega^2x$  it is also written as  $d^2x/dt^2$

by  $d^2x/dt^2 + \omega^2 x = 0$  and we immediately know from this that the solution  $x$  is going to be a cosine of  $\omega t$  plus some other constant  $b \sin \omega t$  so what I have shown you is in a spring mass system where spring follows the Hooke's law that is the force is proportional to the displacement you get simple harmonic motion right so in a spring mass system if the mass is displaced it is going to perform simple harmonic motion so here is a spring and this is the mass right  $x$  equals zero the solution is  $x(t)$  is equal to  $a \cos \omega t + b \sin \omega t$  where the angular frequency  $\omega$  is given by square root of  $k/m$  right where  $k$  is a spring constant and  $m$  is the mass of the particle so to perform this motion the particle has to be displaced some motion has to be started so if I pull the mass and leave it right so if I do this let me just show it to you in the picture this is my equilibrium position  $l$  zero what I will do is I will stretch the spring by some distance  $x_0$  from here to here and leave it so I pull it up to this point and leave it so that  $v_0$  is zero all right then the motion as we discussed earlier is going to be  $x(t) = x_0 \cos \omega t + \frac{v_0}{\omega} \sin \omega t$  the second term is 0 let me show it to you explicitly so I have  $x(t) = a \cos \omega t + b \sin \omega t$   $x$  at zero is going to be  $a$  and that is given to be  $x_0$   $v$  at zero is going to be  $\omega b$   $\sin \omega t$  plus with the minus sign plus  $\omega b \cos \omega t$  and that is given to be zero at  $t$  equal to zero at  $t$  equal to zero the  $\sin \omega t$  term is already zero cosine  $\omega t$  term is one and this immediately implies  $b$  equals zero and this leads to the solution that is one possibility other possibility is I take this spring mass system and give it a hit so that when it was at  $t$  equal to zero at  $x$  equals zero it got an initial velocity  $v_0$  say in the positive direction right so then from  $x(t) = a \cos \omega t + b \sin \omega t$  and from this condition that  $x$  at  $t$  equals zero is zero and  $v$  at  $t$  equals 0 is  $v_0$  I am going to get  $x(t) = \frac{v_0}{\omega} \sin \omega t$  that is going to be the description of the motion both are simple harmonic motion lets quickly solve a couple of examples so example one a mass of two kg is attached to a spring of spring constant  $k$  equals 500 newtons meter inverse what will be the frequency of oscillations if the mass is displaced from its equilibrium position and released

so what is given to you is that  $k$  is 500 newtons per meter inverse the mass is given to be 2 kg  
so the angular frequency  $\omega$  is nothing but square root of  $k$  over  $m$  which is square root of five hundred over two which is square root of two fifty and that is going to be five square root of 2500 sorry 5 square root of 10 radians per second  
or if the frequency is needed  $\omega$  over  $2\pi$  that is going to be 5 square root of 10 divided by  $2\pi$  which is 2.

5 square root of 10 over  $\pi$  hertz or per second that is the frequency example two a mass of 5 kg is attached to a spring of spring constant 400 newtons per meter when it is pulled from its equilibrium position by

5 meters and released on a frictionless horizontal table what will be its displacement as a function of time

so what you are given is a spring mass system on a frictionless horizontal table the mass is 5 kg and the spring constant  $k$  is 400 newtons per meter

so you are given  $k$  equals 400 newtons per meter mass is 5 kg

so  $\omega$  is going to be square root of  $k$  over  $m$  which is square root of 400 over 5 square root of eighty radians per second which

is four square root of five radians per second general motion  $x(t)$  is going to be some constant

a cosine of  $4\sqrt{5}t$  plus  $b$  sine of  $4\sqrt{5}t$  however what you are given is that it is pulled

by a distance of zero point five meters and released

so that means  $v$  at  $t$  equals zero is zero

so you just pulled it and released it what is the subsequent motion

so  $a$  is going

to be come out to be zero point five meters because on the side i will tell you  $x$  at zero

is equal to  $a$  plus zero which is given to be zero point five and  $x$  dot that is  $\frac{dx}{dt}$  not

$x$  dot let's write  $v$  at  $t$  equals zero is equal to minus  $\omega a$  sine of  $\omega t$  plus

$\omega b$  cosine of  $\omega t$  times zero and this is given to be zero this term anyway is zero

so  $b$  comes out to be zero and therefore  $x$  as a function of time is going to be zero

point five cosine of  $4\sqrt{5}t$  we have been talking about simple harmonic

motion and looking at the equation  $x$  double dot equals minus  $\omega^2 x$

we have shown that the solutions are  $a$  cosine  $\omega t$  plus  $b$  sine  $\omega t$  where  $a$  and  $b$  constants what we want to show now is that the solution can also be written in the form  $x(t) = a$  cosine of  $\omega t$  plus  $\phi$  or

equivalently some amplitude  $a$  by the way this  $a$  is not the same as the previous  $a$

so this should

not be confused maybe i should just write it  $a$  bar  $a$  bar cosine of  $\omega t$  minus  $\phi$  or some  $a$  bar

sine of  $\omega t$  plus  $\phi$  or minus  $\phi$  does not matter

so first thing that you check if you want to show that this satisfies the simple harmonic equation is  $x \cdot t$  is going to be equal to minus let us take the first function first  $a \bar{\omega} \sin(\omega t + \phi)$  and therefore  $x \cdot \ddot{t}$  is minus  $\omega^2 a \bar{\omega} \cos(\omega t + \phi)$  which is exactly minus  $\omega^2 x$

so it satisfies the equation more interesting however is to see how this  $a \bar{\omega}$  and  $\phi$  are related to the constants  $a$  and  $b$

so let us look at the solution  $x \cdot t$  equals  $a \cos(\omega t + \phi) + b \sin(\omega t)$  and let us write this slightly

differently as multiplied by  $a \sqrt{a^2 + b^2}$  square and in the bracket i am going to write

$\frac{a}{\sqrt{a^2 + b^2}} \cos(\omega t + \phi) + \frac{b}{\sqrt{a^2 + b^2}} \sin(\omega t)$  now notice that  $\frac{a}{\sqrt{a^2 + b^2}}$  square

plus  $\frac{b^2}{a^2 + b^2}$  square is always less than equal to one and

so is  $\frac{b}{\sqrt{a^2 + b^2}}$  square is always less than one furthermore

$\frac{b}{\sqrt{a^2 + b^2}}$  square is equal to square root of one minus  $\frac{a^2}{a^2 + b^2}$  square

plus  $\frac{b^2}{a^2 + b^2}$  square that you can check very easily so i can write  $\cos(\phi) = \frac{a}{\sqrt{a^2 + b^2}}$  and  $\sin(\phi) = \frac{b}{\sqrt{a^2 + b^2}}$

of a square plus  $b^2$  square and therefore  $x \cdot t$  is equal to square root

of a square plus  $b^2$  square  $\cos(\omega t + \phi) + \frac{b}{a} \sin(\omega t)$  which is nothing but square root of a square plus  $b^2$  square  $\cos(\omega t - \phi)$  what i have shown you is that  $x \cdot t$  can be written

as  $\sqrt{a^2 + b^2} \cos(\omega t - \phi)$  where  $a \bar{\omega}$  is square root of a square

plus  $b^2$  square  $\cos(\phi) = \frac{a}{\sqrt{a^2 + b^2}}$  square or equivalently  $\frac{a}{a \bar{\omega}}$  sine of  $\phi$  is equal to  $\frac{b}{a \bar{\omega}}$  tangent of  $\phi$  is equal to  $\frac{b}{a}$

so we have shown that the solution can be written

of the form  $a \bar{\omega} \cos(\omega t - \phi)$  i could have taken  $\cos(\phi) = \frac{b}{a \bar{\omega}}$  and  $\sin(\phi) = -\frac{a}{a \bar{\omega}}$  and then the solution

would have been  $x \cdot t = a \bar{\omega} \cos(\omega t + \phi)$

so depending on how i choose my sine and cosine and those signs i can easily see that the solution can be written in the desired

form  $\phi$  by the way is known as the initial phase of the motion because it is really related to the displacement

and velocity and everything at time  $t = 0$  equals zero let me show that

so if i take the solution

$x \cdot t = a \bar{\omega} \cos(\omega t + \phi)$  then  $x$  at zero is nothing but a

$\bar{a} \cos(\phi)$   
 and  $\dot{x}$  at zero is nothing but  $-\omega \bar{a} \sin(\omega t + \phi)$  at  $t = 0$  which is  $-\omega \bar{a} \sin(\phi)$   
 so the velocity and displacement at time  $t = 0$  are related to the amplitude  $\bar{a}$  and the initial phase  $\phi$   
 so there is another way of writing the solution for simple harmonic motion  
 second problem  
 i am going to take in this involves two springs attached to a mass  
 so the problem says that if we have two identical springs and attach a mass  $m$  to them in the following two configurations  
 so in one case i attach spring one then spring two and mass  $m$  in the other case i attach two springs in parallel and mass  $m$   
 find this is one this is two and we say find the frequency of oscillation of mass  $m$  in the two cases keep in mind whether i keep this spring vertical or horizontal it does not really matter  
 so let us take the first case when one spring and the second spring are attached to this mass all we want to do is displace this mass by amount  $x$  and find out how much is the restoring force on this because of the two springs the springs are massless  
 so let us see what happens when i stretch this the first spring lets say gets stretched by an amount  $y$  this end of the spring has been this mass has been moved from the initial position by  $x$  and therefore the stretch in the second spring is  $x - y$  ok  
 so the spring gets stretched by  $x - y$  let us now look at the force on the second spring second spring has gotten stretched by  $x - y$  and the force on this on this side due to the first spring which has gotten stretched by  $y$  is  $k y$  and the force on this because it has gotten stretched only by  $x - y$  is  $k(x - y)$  now since the spring is massless the net force on it must be zero if it is not the string will require an infinite acceleration and this implies that  $k y$  is equal to  $k(x - y)$  or  $y$  equals  $x/2$   
 so now we found that as this two identical springs are stretched the if if the entire displacement of the mass is  $x$  and each spring gets stretched by  $x/2$   
 so let us make this again if this is  $x$  this has gotten stretched by  $x/2$  and this has gotten stretched by  $x/2$   
 so the force on the mass which is only due to the second spring is going to be  $k(x/2)$  and therefore  $m \ddot{x} = -k(x/2)$  because  $x$  after all is the displacement of the mass is going to be equal to  $-\frac{k}{2}x$  or  $\ddot{x}$  is equal to  $-\frac{k}{2m}x$  and therefore  $\omega^2$  in this case is going to be  $k/2m$  or  $\omega$

is going to be square root of  $k$  over  $m$  one over root two

so the frequency in this case

if two identical springs are attached in series is reduced compared to a single spring by a factor

of one over root two the second case is simpler in the second case the two springs are attached

together

so if the mass is displaced by  $x$  each spring gets stretched by  $x$  and therefore applies a force  $kx$

so  $f_{net}$  in this case is going to be  $2kx$  and

therefore  $x \ddot{\phantom{x}}$  or  $m x \ddot{\phantom{x}}$  is going to be  $-2kx$  or  $x \ddot{\phantom{x}}$  is

equal to  $-2k/m x$  and therefore  $\omega$  is square root of  $2k/m$  or

square root of  $2$  square root  $k/m$

so  $\omega$  in this case goes up compared to a single spring by a factor of root two

so let me just summarize the realization physical realization of simple harmonic motion right one

possibility that we have discussed is a spring mass system where the spring follows the hooke's law all right that means force  $f_x$  is equal to  $-kx$  in that case the frequency  $\omega$  of oscillation is given by square root of

$k/m$  and the general displacement  $x(t)$  is  $a \cos(\omega t) + b \sin(\omega t)$  you

plus  $b \sin$  of square root of  $k/m t$  you