

hello i am going to give you a few lectures on oscillations and waves and what this kind of motion oscillatory motion and wave motion what kind of motion is this we are going to look into that and how we describe it mathematically and where does it occur so oscillatory motion comes a class of motion called periodic motion so let us understand what does periodic motion mean periodic motion is something is a motion that repeats itself so let us understand what does that mean if a particle is going around in a circle so let us say it started from here it goes around in a circle and then keeps repeating it and every time it goes around it does the same motion then the motion is periodic let us see if i want to calculate the x component or the y component of this motion what does it look like so if the particle is going around in a circle and let us say it covers a distance θ in time t right starting from here on the x axis then $x(t)$ would be a projection of this on the x axis this will be $x(t)$ and this would be if the radius of the circle is r $r \cos$ of θ and similarly the y component of this motion $y(t)$ y of t is going to be $r \sin$ of θ and suppose this completes an entire circle in time T right so it comes back in time T then you can see x starts from some distance r initial distance if it starts from this point is r then it changes in a certain manner and when it reaches here at the upper point which i am doing number two this was number one started from number one reaches number two then x starts becoming negative becomes minus r at number three goes up becomes zero again at number four number three number four and could do something like this back to number one and if this motion repeats itself over time that means θ is going to be exactly the same after it has completed one cycle then the motion is going to be periodic so if i were to plot it carefully let me show it here here is x and y axis this is a particle going around the radius is r i am plotting $x(t)$ i plot it in a certain way but could be any general function however it repeats itself exactly in the same manner then it is a periodic motion with time period being from here to the point it reaches point one again so let me show this this was point one this was point two this was point three maximum this is point four and this is point five which is again point one and the motion repeat itself if θ is in general changing very arbitrarily then the motion would not be periodic however if it repeats itself after a certain time it is periodic it could so happen that the particle goes around and after it completes two circles once and twice then the motion repeats itself then the periodicity

of

the time period will be twice as large but the main point is that if the motion repeats itself if a motion repeats itself after time t it has to be exactly

the same then the motion is periodic with time period t other examples of this would be examples of periodic motion earth rotation on its axis right and this motion repeats after t roughly

equals 24 hours other example would be moon going around the earth and this motion repeats itself after roughly 29 days

so this is a periodic motion with

time 20 time period 29 days or the earth going around the sun with time period roughly equals 365 days or a year and the motion repeats itself after that much time

so these are some examples of periodic motion

so what i have defined for you is time period which is t whatever units we use a

related quantity is going to be frequency and what frequency means is in per unit time how

many times the motion repeats itself

so frequency usually is denoted by f and this is one over t in time t the motion repeats itself

so obviously in per unit time its one over t times the motion takes place right

so this is the frequency third we also define something called the angular frequency meaning of it would be clear

after some time and this is 2π times f usually denoted by ω and this is 2π over t right

so this is the angular frequency time could be given in seconds then

frequency is going to be per second sometimes also written as

hz or what it means is hertz if time is in hours then frequency is per hour and time is in days then frequency is going to be per day

so we talked about i

will go back to now motion of a particle in a circle and are now specialized to motion where the speed of the particle is constant sometimes also called uniform what it means is that when this

particle is going around in a circle its speed here is v is the same all the time and therefore if the circle radius is r the time taken for the particle to go

around is going to be $2\pi r$ divided by v and this is the time period you can see it for yourself if the speed is

uniform the motion is going to repeat itself after the particle reaches its initial

point it is going to repeat itself so time it takes to go around once is going to

be the time period and that is $2\pi r$ over v the frequency f of motion is going to be

one over t which is v over $2\pi r$ v over r i am going to call ω which is angular

frequency divided by 2π now you understand the meaning of angular frequency it takes time t

to go around angle 2π right

so angular speed is nothing but 2π over t and thats

the same as angular frequency and which is nothing but 2π and f

so we all

these things are related as you can see now why this motion is interesting is because if i look at this motion a particle going around in a circle of radius

r with uniform speed v then the angle it covers in time t the angle is going to be measured in radians as you can see earlier i said is two pi when it goes around once in full circle this theta theta is going to be equal to this distance

here is going to be v times t this radius is r

so theta is going to be v times

t the arc length divided by r we can you can see this omega times t which is a well known relationship and therefore x t the x component here is going to be equal to $r \cos(\omega t)$ y t is going to be $r \sin(\omega t)$

so in this

case if i plot the motion

so let me show this to you again i am considering the case where the particle is going around in a circle right of radius r this speed is v and this angle theta is ωt where ω is v over r x as a function of time is $r \cos(\omega t)$

and if i were to plot x t versus t at t equal to zero the displacement

is r it comes down as a cosine curve goes to zero that is let me show

this again by positions position one position 2 is going to be here position 1 is here

and then x becomes negative and at position 3 is maximum negative

so it goes

like this this is position three and after this the x starts reducing again becomes

less negative it comes here at position four becomes zero again and then goes up and back to five or one

so this entire time that it takes to come back

is time period T and this is a cosine curve

so its x t equals $r \cos(\omega t)$ similarly if i were to plot y t right if i were to plot y t this is going to be $r \sin(\omega t)$ and if i were to plot it at this point position one y zero this is position one and then it goes up reaches the maximum y equal to r at position two comes down and then goes

up again negative and then goes up again this is position 3 position 4 position 5 and then it repeats

so this is also periodic

with the same time period this point is supposed to be right here this is this is position

five this supposed to be same same point t ok

so it repeats itself this is a very specific kind of motion

so what is happening here is if a particle is going around in a circle with uniform

speed then x t is equal to $r \cos(\omega t)$ y t is equal to $r \sin(\omega t)$ t moves around with

exactly one frequency right this is called simple harmonic motion all right

so this this is motion

which is simple harmonic it contains a cosine ωt or sine ωt time dependence and this is going to be the focus of our study in these lectures but before that i just

want to give you some more periodic motions and how to represent them on the

displacement

versus time graph motion and their representation on displacement versus graph
ok

so let us see look at that number

one let me take a particle which can move along the x axis

so let us say this is the x axis from

0 to l and then there are these some hard walls at these positions

so that this particle goes

from here with the uniform speed v goes here hits this wall and immediately returns back

so you

can see is going to be going back and forth and repeating its motion

so this is a periodic motion

let us see how does x versus t graph look for this

so if i were to plot x t versus t and lets say t equal to 0 it was at this point

on the left hand x equal to 0 then x increases goes to uniformly goes to a value l

so this is l and as soon as it reaches here it starts coming back when it starts coming back with the same speed without losing energy

it goes down x decreases goes to zero right and then the motion again repeats itself you see again that motion is repeating itself it is exactly the same triangle that keeps coming back again and this time from

here to here is going to be the time period what is t equal to the total distance traveled is $2l$ divided by v that is going to be the time period let us take the second example a ball is released from height h comes down and without losing energy it bounces back

so it goes up to height h again

and then it comes down and goes up and this motion keeps on repeating this is also a periodic motion because exactly the same motion is taking place after certain time

and if i were to plot the height of the ball y t versus the time t it starts at a height

h comes down and you know from your equations that the height is going to be like this because i am going to have $y = h - \frac{1}{2}gt^2$ so it looks like this now it hits $y = 0$ and starts moving up is exactly the

same speed goes up goes to height h and then again it starts coming down

so you see motion repeats

after this much time this is the time period how long did the ball take to come down well

we know that $y = 0$ implies $t = \sqrt{\frac{2h}{g}}$ but that is not the time

period because that is the time it took to reach this point here where i am making a big blob so

total time period t is going to be twice as much which is $2\sqrt{\frac{2h}{g}}$ that is

the time period whenever the motion is periodic all the related quantities are also periodic so

what we have done in the examples i have shown you we have shown the displacement of

a particle x t which is periodic that means this is changing i am also giving

you all sorts of words changing periodically with time period t
 so in the first example let me
 just go back again we took a particle which was going back and forth
 between two rigid walls and x plotted against time look like x was
 increasing went all the way
 up to l came back x was increasing came back i want to show you now how other
 quantities are
 going to look like
 so suppose i want to plot this velocity versus time right i am saying it
 velocity because i am going to take negative and positive both now as the
 particle
 moved up to this point capital t by 2 it was moving with positive velocity
 and the velocity was certain value v as soon as it reached this point the other
 wall it got hit and started moving the other way so
 velocity became negative and then it came here this was the velocity from this
 point
 onwards up to this point and then the velocity became positive
 again up to this point and then it became negative again sorry it is red color
 up to this point
 so velocity of this particle going back
 and forth between two walls is like this this i am showing with the dotted line
 its not
 well defined then again like this here then the dotted line then like this
 dotted line like
 this and you can see that after time period t velocity is repeating itself
 so velocity is also
 a periodic function of time
 so if x vs t is periodic
 so is the velocity and this is given as $\frac{dx}{dt}$ slope of x
 with respect to time that is also periodic
 so what i have shown you in this example
 again particle going back and forth between x equals zero and x equals l i
 have shown
 you x curve which i will quickly plot now like this i have shown you the
 velocity curve or let me put here x vs t versus t i have shown you a velocity
 versus time curve which looks like this and
 so what about the acceleration let us plot the acceleration also you see when
 the particle was moving with
 uniform speed the acceleration is zero and then suddenly it has a huge
 negative acceleration
 so that the velocity becomes negative is zero again
 then a positive acceleration and 0 again negative acceleration 0 again and
 so on positive acceleration 0 again negative acceleration and 0 again what you
 can see
 is this acceleration is also repeating itself from here it becomes negative
 positive and then zero
 again this is the time period in real life when this ball hits it is going to
 be squeezed a bit
 for a short time there is going to be some force which we are showing that
 short time to be roughly
 zero but in real life is going to be slightly different the actual curve may
 look
 something like this 0 then it hits the wall then it hits the wall gets a huge
 acceleration

for some time and huge acceleration for some time the actual curve may look something like this nonetheless the point is that even acceleration is repeating

so $x(t)$ is periodic velocity $v(t)$ is periodic with the same period and

so is the acceleration i let you think what this integral if i were to take an integral from time t_0 up to this time t equal to 0 to say this time of the acceleration $a(t) = 2$ let us say this is t_1 t_1 $d t$ what it would be i will give you the answer the answer would be $\frac{1}{2} v^2$ you figure it out why it is

so is basically

related to that the acceleration $a(t)$ let me write it on this side $a(t)$ keep writing on the right hand side this is $x(t)$ the next point is $v(t)$ is nothing but $\frac{dx}{dt}$ and the acceleration $a(t)$ is $\frac{dv}{dt}$ which is $\frac{d^2x}{dt^2}$ and this is what this is what leads to this integral let us look at the

next example that i did and that was a ball dropped from height h and bouncing without losing any energy and in this case when i plotted $x(t)$ versus t or height $y(t)$ versus t it look like this where this up to this point is the time period let us now plot as velocity also $v(t)$ versus t and you know from your daily or earlier exercises in this case it started with zero velocity

and out here by the time it reaches this point let me show it by black the velocity is negative

so its velocity goes like this and keeps on increasing up to this point then the ball bounces and this velocity just changes direction and therefore it becomes positive the same magnitude and then it again it starts slowing as it moves up velocity decreases reaches zero when the ball reaches the highest point because it stops

momentarily and then it comes down again and then the motion repeats itself and

so on this is the velocity curve

so you can see that velocity goes up to this point and then changes it is like this and the time period time over which

it repeats itself is from here to here what about the acceleration again if i were to plot acceleration is $-g$ up to this point it minus g throughout minus g but at this point the body changes velocity

so it goes up a huge acceleration

in the positive direction and then minus g again all the way up to this point changes again all

the way up to this point changes again and this is the time period you see motion is repeating itself

so it does not matter how i write the time period i can write it between these two points

these two points and

so on in real life again the acceleration would look something like this it will go like this goes up comes down goes up comes down this will be real life acceleration

so peaks are not that sharp they are spread a bit and you have if i were to take the

integral of time let us say t_1 to t_2 just like earlier integral t_1 to t_2 $a dt$ would be equal to $v_2 - v_1$ where v_1 is the velocity when it hits the ground which is going to be $2\sqrt{2gh}$ and all this follows from that this is of course the first term is $y(t)$ the second term is $v(t)$ which is dy/dt and the third term acceleration $a(t)$ is dv/dt which is same as d^2y/dt^2 so if you integrate this equation you get this answer the third motion which I said we will be interested in the lectures that follow is the simple harmonic motion which we said is nothing but if I take a particle going around in a circle of radius r then $x(t)$ which is equal to $r \cos(\omega t)$ right this $x(t)$ is called simple harmonic motion because it contains the term $\cos(\omega t)$ or equivalently I could also call its component $y(t)$ which is $r \sin(\omega t)$ which is also called simple harmonic motion if I were to plot $x(t)$ versus t it looks like cosine curve with this maximum displacement being r or minus r the corresponding velocity $v(t)$ versus t is initially slope is almost zero so its going to be zero however as time increases as the particle goes around like this is going into the negative x direction so velocity increases and it reaches a maximum here you can see at this point two is maximum then it starts decreasing and becomes zero again at this point so the velocity looks like this and then it reaches a maximum positive here so let me show the corresponding point this is point one point two point three point four this is point one point two point three point four and then zero again at point one or point five and then it starts repeating itself $v(t)$ here is dx/dt which is nothing but minus $r \sin(\omega t)$ there is a ω here minus $\omega r \sin(\omega t)$ how about the acceleration acceleration at point one is you can see the change is very large so is going to be negative and then it becomes 0 at point 2 so 1 2 becomes positive large at this point largest at this point and then goes down like this okay and this becomes the time period this is the time period acceleration $a(t)$ is given as d^2v/dt^2 which comes out to be minus $\omega^2 r \cos(\omega t)$ which is nothing but minus $\omega^2 x(t)$ so acceleration goes up comes down so it is exactly negative of this is negative multiplied by $\omega^2 r$ of course because this largest value is going to be $\omega^2 r$ so its multiplied by sorry ω^2 times $x(t)$ and change of sign that is the acceleration so let us see what it means in simple harmonic motion $x(t)$ has been given as $r \cos(\omega t)$ the corresponding speed of velocity is nothing but dx/dt which is minus

ω

$r \sin \omega t$ and the acceleration a_t which is nothing but $\frac{dv}{dt}$ which is same thing as $\frac{d^2x}{dt^2}$ is nothing but $-\omega^2 x$

i could also have written this whole thing as a motion along the y axis

so i could have

written $y(t)$ is equal to $r \sin \omega t$ corresponding velocity $v(t)$ is $\frac{dy}{dt}$

which would be $\omega r \cos \omega t$ interesting thing is that acceleration which

is $\frac{dv}{dt}$ which is nothing but $\frac{d^2y}{dt^2}$ is still comes out to

be $-\omega^2 y$ that is its again $-\omega^2$ times the displacement

or in general i can write a motion $x(t)$ given as a constant $a \cos \omega t$

plus some other constant $b \sin \omega t$ then the velocity $v(t)$ would be equal to

$\frac{dx}{dt}$ which is $-\omega a \sin \omega t + \omega b \cos \omega t$

and the acceleration a_t which is $\frac{dv}{dt}$ which will be $-\omega^2 a \cos \omega t$

$-\omega^2 b \sin \omega t$ which is again $-\omega^2 x(t)$

so what

you see is that whether i take motion to be purely cosine purely sine or a combination

of this the acceleration comes out to be $-\omega^2 x(t)$ and that is a typical

sign of simple harmonic motion in other words if i have an equation x'' which

is nothing but $\frac{d^2x}{dt^2}$ if this is equal to $-\omega^2 x$

times x where ω^2 is a positive number then i am inverting the whole argument

now i came from x derive the acceleration now i am going to go backwards then x' is

going to be of the form that was given and $x(t)$ is going to be if i integrate this of the

form $a \cos \omega t + b \sin \omega t$ i can write x' as some coefficient

$a \cos \omega t + b \sin \omega t$

so earlier i showed you that if

a particle is moving around in a circle right uniformly that means its angular speed or

v is constant then the motion comes out to be its x or y component motion x component of the

motion y component of the motion comes out to be a combination of either pure cosine ωt or sine ωt

or in general when i combine the two its combination of the two the important thing is in this motion is that the acceleration

comes out to be exactly $-\omega^2$ times the displacement let us understand that

so when a particle is moving around in a circle if it is at this point here the

only acceleration

it has is centripetal acceleration which you know is v^2/r or $\omega^2 r$ that's the

magnitude and if i take its components its x component is going to be this in this direction

if this is ωt this is ωt you can see this is nothing but $-\omega^2 r \cos \omega t$

ωt which is nothing but $-\omega^2 r \sin \omega t$ similarly the y component of the acceleration

is going to be $-\omega^2 r \sin \omega t$ which is $-\omega^2 r \cos \omega t$

t

so the acceleration in this case the x and y component which show the simple harmonic

motion the corresponding acceleration is nothing but $-\omega^2$ times that displacement and that is simple harmonic motion

so simple harmonic motion is

nothing but a very specific case of periodic motions which of which i gave you several examples

earlier

so let us now solve a couple of problems ah one involving a periodic function and

the other one where we calculate the period of a particle performing ah periodic

motion

so in first problem a function given as a function of time is equal to $\cos 2\pi t + \cos 3\pi t$ plot the function and find its period

so let us see how this function looks if i were to look at $\cos 2\pi t$ right this has a period of $t = 1$ how do i know that because \cos

of $2\pi t$ at $t = 0$ is one and next becomes one is at $t = 1$ because then it becomes \cos

of 2π right and \cos of $3\pi t$ at $t = 0$ is one and

\cos of $3\pi t$ at $t = 2/3$ is again equals \cos of 2π

so

its time period is $2/3$ what is the time period of the two functions

together

when they are added together i could plot the two functions

so $\cos 2\pi t$ is going to look like this

where the period is one

so this is $t = 1$ and $\cos 3\pi t$ is going to look like its period is $2/3$

so this is

point five this is here

so it is going to look like this and the net result is the sum of two its not very obvious what the

period is going to look like what the net function is going to look like but if you just play around what you are going to get is going to look like

something like this and you can

see this is going to the period it is not very obvious what

the period is going to be

so for that i am going to play a

trick and write $f(t) = \cos 2\pi t + \cos 3\pi t$ as

$\cos 2\pi t + \cos 3\pi t = 2 \cos \left(\frac{2\pi + 3\pi}{2} t \right) \cos \left(\frac{3\pi - 2\pi}{2} t \right)$ divided by

$2 \cos\left(\frac{3\pi}{2t}\right) + \cos\left(\frac{2\pi}{t}\right) - 3 \cos\left(\frac{\pi}{2t}\right)$

that's $2 \cos\left(\frac{3\pi}{2t}\right) + \cos\left(\frac{2\pi}{t}\right) - 3 \cos\left(\frac{\pi}{2t}\right)$

and that's $2 \cos\left(\frac{3\pi}{2t}\right) + \cos\left(\frac{2\pi}{t}\right) - 3 \cos\left(\frac{\pi}{2t}\right)$

so this is $2 \cos\left(\frac{3\pi}{2t}\right)$ and this one is $\cos\left(\frac{2\pi}{t}\right)$

of 3π and therefore this becomes $2 \cos\left(\frac{3\pi}{2t}\right) + \cos\left(\frac{2\pi}{t}\right) - 3 \cos\left(\frac{\pi}{2t}\right)$

and you add them together you are going to get $2 \cos\left(\frac{3\pi}{2t}\right) + \cos\left(\frac{2\pi}{t}\right) - 3 \cos\left(\frac{\pi}{2t}\right)$

of $\frac{\pi}{2t}$ that is your function and now you can find the period easily

so $f(0)$ is $2 \cos(0) + \cos(0) - 3 \cos(0)$ which is $2 + 1 - 3 = 0$ and I want to find at what time t is $f(t) = 0$ again

so $2 \cos\left(\frac{3\pi}{2t}\right) + \cos\left(\frac{2\pi}{t}\right) - 3 \cos\left(\frac{\pi}{2t}\right) = 0$

and you notice $f(t)$ is equal to $2 \cos\left(\frac{3\pi}{2t}\right) + \cos\left(\frac{2\pi}{t}\right) - 3 \cos\left(\frac{\pi}{2t}\right)$

so $t = 2$ gives $f(2) = 2 \cos\left(\frac{3\pi}{4}\right) + \cos\left(\frac{\pi}{2}\right) - 3 \cos\left(\frac{\pi}{2}\right)$

which gives you 2 again

so it is repeating after $t = 2$ notice that is the smallest number t equals to that it repeats itself over

so time period of this function is 2 and that is the answer for the second problem I am going to take a particle moving along the x axis in a potential which is let us say $U(x) = \frac{1}{2} m k x^2$ where m is the mass of the particle k as a constant and on this side it is $U(x) = \frac{1}{2} m k x^2$

zero

so potential it is moving in $U(x) = \frac{1}{2} m k x^2$ for $x > 0$

and $U(x) = -\frac{1}{2} m k x^2$ for $x < 0$

short the potential can also be written as $U(x) = \frac{1}{2} m k |x|^2$ you can see if I take a particle and leave it from one side it's going to come down go up again come down go up again and it's going to perform a periodic motion

so a particle with energy E is going to perform a periodic motion notice that the motion is going to be periodic but not simple harmonic motion because for simple harmonic motion you need the potential to be of the kind $\frac{1}{2} k x^2$ because the force is linear

so for simplicity I am going to take $E = \frac{1}{2} m v_0^2$

so this immediately tells you this immediately tells you that v_0 is the speed when $x = 0$ by energy conservation $E = \frac{1}{2} m v^2 + \frac{1}{2} m k x^2$ that is the kinetic energy plus one half $m k x^2$ when $x = 0$ potential is zero the one half $m v_0^2$ is going to be total energy

so v_0 is the speed when the potential is zero and all the energy is kinetic

so what we have now is that $\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + \frac{1}{2} m k x^2$ I can cancel half m all throughout and I have $v_0^2 = v^2 + k x^2$ now as this particle performs

motion back and forth at any x the velocity v at x is given as
 $\sqrt{v^2 - kx}$ therefore if it travels a
 distance dx the
 time taken is equal to only for traveling distance dx the time taken is $\frac{dx}{v}$
 which is going to be $\frac{dx}{\sqrt{v^2 - kx}}$ all right let us see now the
 farthest
 the farthest that the particle goes to it travels between these two points the
 highest
 point the right most point the left most point at these points the velocity
 is zero
 so when v is zero x gives you that point
 so $v = 0$ implies
 $\frac{1}{2}mv^2 = \frac{1}{2}kx$ $\frac{1}{2}mv^2 = \frac{1}{2}kx$ $\frac{1}{2}m$ $\frac{1}{2}m$ cancels and
 $v^2 = \frac{k}{m}x$
 $v = \sqrt{\frac{k}{m}x}$ are the two points where it reflects
 so the higher point is $\sqrt{\frac{k}{m}x}$
 square over k and the left most point is $-\sqrt{\frac{k}{m}x}$
 so now it is clear
 that the particle is performing motion between $-\sqrt{\frac{k}{m}x}$ and
 $\sqrt{\frac{k}{m}x}$
 over k and the time taken from $-\sqrt{\frac{k}{m}x}$ to $\sqrt{\frac{k}{m}x}$
 over k is this
 and this is going to be one half the time period because it is time taken from
 one side to the
 other and the total time taken will be exactly the same when it goes back the
 time period divided
 by two is the time it takes from left most point to the rightmost point and
 this is going to
 be $\int_{-\sqrt{\frac{k}{m}x}}^{\sqrt{\frac{k}{m}x}} \frac{dx}{\sqrt{v^2 - kx}}$
 $\int_{-\sqrt{\frac{k}{m}x}}^{\sqrt{\frac{k}{m}x}} \frac{dx}{\sqrt{v^2 - kx}}$
 mod x which for $x < 0$ is plus $\int_0^{\sqrt{\frac{k}{m}x}} \frac{dx}{\sqrt{v^2 - kx}}$
 over $v^2 - kx$ square root
 so we have figured out that the time period t
 by two is equal to $\int_{-\sqrt{\frac{k}{m}x}}^{\sqrt{\frac{k}{m}x}} \frac{dx}{\sqrt{v^2 - kx}}$
 of $v^2 - kx$ plus zero to $v^2 - kx$
 over square root of $v^2 - kx$ you can simplify this further
 in this integral the first integral take y to be $-\sqrt{v^2 - kx}$ or x to be $-\frac{v^2 - y^2}{k}$
 so dx is
 $\frac{2y}{k} dy$ then you get t by two equals integral with a minus sign $\frac{2y}{k} \frac{dy}{\sqrt{v^2 - y^2}}$
 square minus k y and this is $-\frac{2}{k} \int \frac{y}{\sqrt{v^2 - y^2}} dy$
 square by k to
 zero plus the second term remains the same zero to $\int \frac{1}{\sqrt{v^2 - y^2}} dy$
 over
 square root of $v^2 - y^2$ which we can write as $\int \frac{1}{\sqrt{v^2 - y^2}} dy$
 over square root of $v^2 - y^2$ plus zero to $\int \frac{1}{\sqrt{v^2 - y^2}} dy$
 over
 $\frac{1}{k} \int \frac{2y}{\sqrt{v^2 - y^2}} dy$ or $\frac{2}{k} \int \frac{y}{\sqrt{v^2 - y^2}} dy$ does not matter because this is variable over which we are
 integrating
 so this t by two therefore is two times zero to $\int \frac{1}{\sqrt{v^2 - y^2}} dy$
 $\frac{2}{k} \int \frac{y}{\sqrt{v^2 - y^2}} dy$ now the integral is very

simple you take
 y equals $v \sin^2 \theta$ therefore $\frac{dy}{d\theta}$ is $2v \sin \theta \cos \theta$
 $\frac{dy}{d\theta}$ is $2v \sin \theta \cos \theta$ and the limits are from zero to π by two
 so $\int_0^{\pi} 2v \sin \theta \cos \theta d\theta$ which is $2v \int_0^{\pi} \sin \theta \cos \theta d\theta$
 $\int_0^{\pi} \sin \theta \cos \theta d\theta$ divided by $v \cos^2 \theta$
 so you get $\frac{4v}{k}$ this $\cos^2 \theta$ cancels integral zero to π
 by two $\sin \theta d\theta$ which is nothing but $\frac{4v}{k}$ and therefore
 the time period is
 $\frac{4v}{k}$ that is the answer and frequency of motion is going to be $\frac{k}{4v}$
 in the next lecture i am going to be focusing more on simple harmonic motion look
 at this equation
 of motion based on what we have learnt
 so far you