

having studied equation of continuity ah let us look at bernoulli's equation bernoulli's equation is for a steady flow of fluid so this was written down by daniel bernoulli ah so this is and from seventeen zero eight seventeen hundred to seventeen eighty two so he wrote down bernoulli's equation which is simply a statement of conservation of energy let us see how we can derive this equation or the equation that he has written down what applications do they have with physical systems or how it is of help in measuring the flow of fluid and as i said that this is for a steady or a streamline flow steady or streamline flow of a liquid ah of an incompressible liquid so in order to derive bernoulli's equation ah let us take this picture so this is the fluid flow through a pipe so there is an upstream flow call this region as so i am considering an elemental fluid in this region and also considering an elemental fluid in this region let us call this as region 1 and call this as region 2 this is at a height h_2 this is at a height h_1 let us write h_1 here and ah the pressure that is recorded here is say p_1 its recorded by a pressure gauge i am not drawing the pressure gauge but the pressure is p_1 which is determined by a pressure gauge ah the pressure is p_2 here and so it is moving upstream so the velocity flow is in this direction here and its in this direction here which are respectively v_2 in region two and v_1 in region 1 and we also have the areas of cross sections are different and let the area of cross section be a_2 here and the area of cross section be a_1 here so this is the situation so this is only element the fluid is drawn here which is flowing through a pipe pipe is not of importance to us we are only considered we are considering the mass so this fluid has mass m and this fluid is flowing upstream the heights from some where the height equal to 0 some reference line the heights are h_2 and h_1 and the velocities are shown there so now this fluid is of course under gravity so the energy is constant and the total energy so e is given by the kinetic energy of the fluid plus the potential energy so this total energy is conserved and so this is equal to $\frac{1}{2} m v^2$ plus mgh where v and v is the velocity at any arbitrary point along the flow and h is the height corresponding height measured from this reference level so the energy difference between the region one and region two so energy difference between region one i will just in short write it as e_1 and e_2 that is region two is ah e_1 minus e_2 this is equal to $\frac{1}{2} m v_1^2$ plus $mg h_1$ minus $\frac{1}{2} m v_2^2$ plus $mg h_2$ so this is the energy difference between the two regions region 2 here and region 1 here so they are given by e_1 minus e_2 and these are the kinetic energy and the potential energy at region 1 kinetic energy and potential energy of the liquid or the fluid in region 2. now this energy difference should be doing some work and we can calculate that work by it should be spent in doing some work rather so this energy should be spent in doing some work and by the work energy theorem this work done that we just talked about is equal to e_1 minus e_2

which is equal to $\frac{1}{2} m v_1^2 + m g h_1 - \frac{1}{2} m v_2^2 + m g h_2$ now we can also find the expression for this work an alternate expression for w which is the work done in moving the fluid from a point to the neighboring point for that let us take a small element of fluid very small it's only overemphasized this has an area of cross section a and the length to be l right

so i am only taking a small elemental fluid which has an area Δa and the length of this Δl

so this much of fluid that i am considering and now there is some pressure here in this end let us call that pressure as p now there is a bigger pressure at the bottom point let's call that as $p + \Delta p$

so this is the pressure here and this p is the pressure here

so we have we can erase some of these Δl things here and we will have a pressure at let us call this point as o and this point as o'

so pressure at o is equal to $p + \Delta p$ and a pressure at o' equal to p ok

so now the force that is acting in the upward direction at the point o is equal to

so force at o equal to $p + \Delta p$ into Δl let's just call it as a instead of Δa

so we have we know that the cross section the area of cross section here in this small portion which is assumed to be constant even though the whole liquid flow the Δl the filament does not have a constant cross section but since we have taken this area to be small enough or rather this region to be small enough we can take that to be constant and let us call that as some a

so this is the force acting and the direction is upward as its shown here

so this is the direction of force at o sorry and there is also a force that is acting here which let's call that as $f_{o'}$

so force at o' is equal to $p a$

so the work done or rather the difference in force

so difference in force between o and o' o and o' or you can call it as o' and o its equal to $p + \Delta p$ a minus $p a$ which is equal to $\Delta p a$ now this force will be spent in doing the work that we have mentioned here and for this force to push this liquid column we would just need the force the

so the work done work done Δl is w is equal to $\Delta p a$ into a into l this is the the distance in Δl moving the liquid film Δl go from here to here and this will be Δl that which is equal to $\Delta p a$ and v where v is the volume of this small portion that is drawn here

so this has a volume v and

so the work done will be Δp into v

so now will have to equate the work done that is written here on the right hand side to the work done that is there Δl you might wonder that this v Δl is not

so so this will be an elemental work

so that is Δp into v now in order to transport the liquid film from region 2 to region 1 we have to sum over all these elemental work done and that will simply be equal to

so the entire work done will be p_2 minus p_1 into v where p_2

so we are talking about the work done to take the liquid or the transport the liquid from region 2 to region 1 where the pressure difference is equal to p_2 and p_1 p_1 p_2 is at a larger pressure because the liquid is flowing upstream

so $p_2 - p_1$ multiplied by v should be equal to $\rho v_1^2 - \rho v_2^2$ which is equal to $\frac{1}{2} \rho v_1^2 + \rho g h_1 - \frac{1}{2} \rho v_2^2 + \rho g h_2$
 we can divide this entire equation by v
 so that $\frac{p_2 - p_1}{\rho v}$ becomes $\frac{p_2}{\rho} + \frac{1}{2} v_1^2 + g h_1 = \frac{p_1}{\rho} + \frac{1}{2} v_2^2 + g h_2$ and this is known as Bernoulli's equation
 so in a generic sense we can write that
 so this is called as a pressure head this is called as a kinetic head this is called as a potential head
 so we can write that in a stream line flow the pressure plus $\frac{1}{2} \rho v^2 + \rho g h$ will remain constant at all points throughout the flow of the fluid
 so this is written down by Bernoulli somewhere in the 18th century between 1700 to 1782 and this has a lot of applications
 so let us see one application to begin with and that application is called as venturi meter i will tell you what it is
 so we study the application of Bernoulli's equation and this is applied to a device called as venturi meter and what does venturi meter do venturi meter measures the speed of the flow of a certain liquid through a pipe
 so let's just write that
 so a venturi meter is a device which measures the speed of a fluid within a pipe
 so there is a pipe here let's draw the pipe and let us just say that this has the area of cross sections uniform pipe the area of cross section of the pipe being A_1 the velocity of water through this is v_1 and suppose that is known v_1 is known that is you have connected it to a water supply and you know that what is the speed of water that is entering through this area of cross section and would leave eventually through the other end and say we know v_1 now we need to know the what we can do is that
 so venturi meter is a device which is like this um its not drawn exactly to scale but what i meant to do is that this is my A_2 this is my A_1 same as the the cross section of this initial pipe and i have put a constriction here and this is the reason which is called as venturi meter
 so this is A_2 as well and this has an area of cross section which is A_1 and i want to find the speed or the velocity of the fluid through this constriction and this part of the device is known as
 so i introduce a venturi meter inside the pipe
 so this is in in do introduced inside the pipe and we need to know the flow of the liquid through this constricted part
 so let us call this
 so this is A_2 and this is v_2 and again this is v_1 now it is clear that um we can measure pressure at this place by introducing a pressure gauge ok
 so this will measure pressure here and this will also say measure the pressure here by introducing another pressure gauge
 so this measures the pressure p_2 and this measures the pressure p_1 now because of equation of continuity we know the equation of continuity says that the flux or the area of cross section into the velocity of the fluid is constant for an incompressible fluid
 so $A_1 v_1 = A_2 v_2$ is constant where A is the area of cross section A_1 and v is the velocity
 so we have for these two points $A_2 v_2 = A_1 v_1$
 since A_2 is greater than A_1 i have to have v_2 to be greater than v_1 .

so the speed of the fluid as it enters the constriction will be larger than the speed of the fluid at the other parts now because of this the pressure actually falls the pressure falls here

so my p_1 will be less than p_2 and we can apply Bernoulli's theorem and we can write that $p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2$ note that we have neglected the potential head here because we can think that the fluid is resting on a table and the table marks the potential energy where the potential energy is zero so we have not written the potential term here or the potential head but however from Bernoulli's equation the pressure plus the kinetic head at region 1 should be equal to the pressure and the kinetic head at region 2 and if we have that then we can simply if we know v_2 as I said at the beginning if we know v_2 and we have measured p_1 and p_2 so v_1 can be determined

so these are direct application of Bernoulli's principle

so far we have talked about what happens to a point that lies completely submerged in a liquid that is for points which are well within the liquid we have seen the calculation of pressure and various things that we have talked about Pascal's law and other things now we want to know that what happens at the surface of the water and in fact the surface of the water or a liquid also has very interesting properties

so what we now want to consider is let us take a vessel of water containing water and there is this surface and the properties of the surface is what we are going to see now and this comes under the heading as surface tension

so before we start surface tension you might have noticed that let us discuss something which you have seen in real life say you have closed the tap but the tap is before it stops issuing water completely the last drop of water which almost hangs on to the end of the tap assumes a spherical shape

okay also you might have seen that the dew on the grass as a winter sets in the dew on the grass also assumes a spherical shape

so this tells you that the spherical shape is related to the surface of the liquid we know that liquid itself does not have a shape and it takes the shape of the container

so why are these the last drop hanging on to the water tap and the drop that is the dew that it forms assumes such a spherical shape more examples take a small balloon filled with water

so a balloon filled with water and say it's a small balloon let's not make it too large or even a needle ok

so these are two examples which you can see that they float on the surface of the water even though they are more dense than the water itself

so why does this happen it happens because of surface tension okay

so it is a property of the surface and let's see what kind of property it is

basically the surface of the water it behaves like a membrane you might have seen membranes say a common thing to see is the top membrane of a tabla where you play the tabla that the membrane or there are membranes that are taught in biology

so it acts like a membrane which is under tension

so surface of a liquid acts like a membrane under tension and this tension this tension that we have just talked about here this tension acts parallel to the surface and it acts along any line on the surface as if it is trying to pull open the surface

so that is the definition of surface tension that is the definition of surface

tension it acts parallel to the surface acts along any line which is on the surface and as if it is trying to pull open the surface tending to pull open the surface

so this tension is called a surface tension and it is defined as the force per unit length and this is has unit as newton per meter and this is the definition of surface tension that it is a force divided by force per unit length and the unit as you understand is in newton per meter let us try to understand that why does surface tension arise or what are the consequences of surface tension for that let us consider this u-shaped tube

so this u shape tube has a a movable rod say for example and this movable rod encloses some liquid ok

so this is

so there is a thin film of liquid that is enclosed by this moving rod and you require some force in order to pull this rod to pull this movable rod you need some force

so it is a u-shaped tube encloses a thin film of fluid by a movable rod

so as we pull this movable rod the surface area of this liquid film increases and let us take a small element of fluid and this is lets say a cylindrical element of fluid and the force acting

so all these

so there is a surface tension acting on the surface this is a thin film its not a volume fluid that we are considering its a thin film just a surface and all these the surface tension is acting at all point parallel to the surface so let us take one small column here and consider this force f acting on this which increases the surface area this will increase the length of both these sides of the tube

so that tells you that the surface tension is s here s here

so s is equal to f over $2l$ because we have talked about a film which has the two sides of the the 2 dimensional cylinder that we are talking about so the surface tension is defined as f over $2l$

so $2l$ is the increase in length

so this is uh the surface tension definition of surface tension s is equal to f over $2l$ and apparatus of this kind actually can be used to determine the surface tension see if we know f and of course we know l by the amount it increases as you pull this ah as you apply this force suppose f and l both are known

so you can simply get s using this formula and let us now write down the surface tension due to some common fluids that we come across every day

so

lets just make a table as we have done many times earlier

so this is will write down the substance and will write down the surface tension which in newton per in units of newton per meter

so ah remember ah surface tension is a function of the temperature

so you need to mention the temperature at which you are talking about the surface tension

so this is ah mercury at 20 degree centigrade mostly we will talk about 20 degree centigrade its equal to 0.44 ah then blood again ah this is at not at 20 degree centigrade this is a 37 degree centigrade which is equal to point zero five eight um and then ah ethyl alcohol again at twenty degree centigrade is equal to point zero two three and soap at twenty degree centigrade again its equal to point zero two five zero two five now water ah let me write it separately here

so ah again substance and the surface tension in newton per meter

so water at 0 degree centigrade which is equal to 0.076 water at 20 degree

centigrade 0.072 water at boiling point which is 100 degree centigrade equal to 0.059 it is interesting to see that which we will not discuss at length that the surface tension decreases with increase in temperature so it goes down from 0.076 at 0 degree centigrade to 0.059 at 100 degree centigrade

so let us understand this surface tension little better i just give you one more view of this mercury at 20 degree centigrade is 0.44 which is at least something like one order of magnitude more than the other substances that we have considered blood at 37 degree centigrade is point zero five eight ethyl alcohol at twenty degree centigrade is point zero two three and soap at twenty degree centigrade is point zero two five and the water the values for water are given here all right

so let us take a view of understanding surface tension from the molecular perspective or what happens really at the molecular level and what gives rise to surface tension

so let us take a vessel containing water

so this is the level of water let us take two points a point a here and a point b here on the surface molecule the water molecule here experiences an attractive force due to all other water molecules

so this is uniform in all directions there is an attractive force due to all other water molecules now consider this lets call this point as a point and lets call this point as b point ah there are no counter parts above

so the the force lines parallel to the surface are oppositely directed and

also they will be forces which will not be balanced by upward forces and because of these forces downward the liquid surface is trying to compress a little bit and the area will be minimized okay and this is the same reason

this minimization of surface area is the same reason that we see that dew to be spherical in shape or the dripping water the last drop of water to have a spherical shape at the end of the tap because spheres have minimum surface area and that minimum surface area ah is is that of a sphere

so that's why they take spherical shape

so the surface tension is responsible for this spherical shape of the dew that we see on the ground now let us look at a relevant extension of this idea and talk about what is called as surface energy

so there is some amount of energy needed to increase the surface area as we have seen in this youtube experiment that we have mentioned

so this is that movable rod and this film encloses or this region encloses ah thin liquid film

so we need some amount of work to be done and apply a force in order to increase the area

so there is this work done will be stored as some surface energy

so

this work done will be called as or rather stored in the system as the surface energy and the surface energy or rather the work done to begin with is defined as $f \Delta x$ where f is the force given and Δx is the small you know extension that happens because of that application of force and this we know that can be written as $s \Delta l$ because of s equal to f over l

so f equal to s into l and then Δx and this is equal to s into Δa where Δa is the increase in area that has happened because of this small extension

so we can also write that s is equal to w over Δa where w is the work done in the process and Δa is the area net change in area because of this work being done

so this also can be represented in joule per meter square and as we have seen

that it is represented in newton per meter it is also represented as joule per meter square we shall uh do another thing which is again related to this is called as the angle of contact

so you might have seen that that the some insects actually can walk on water or as we have given an example earlier that a small balloon can actually water field balloon it is filled with water it still can float on water and that can happen even though they are denser than the liquid or the water but it can still float and the reason is the surface tension

so we can calculate the angle of contact of say a sphere

so there is a ah liquid film here and there is a sphere there

so there are these ah surface tension acting this is considered to be the that water filled balloon that we have talked about and this is the surface of a liquid

so this is the surface and the liquid is below and this ah water filled balloon is not getting submerged rather its floating because the surface tension is acting which is equal to f over l and the angle of contact is given by θ

so this has a radius r this sphere has or the balloon has a radius r and has a weight that is acting downwards which is due to the gravity

so the surface tension is acting in this direction in order to support the weight the horizontal components of the surface tension will cancel on both sides because the horizontal components along the surface of the liquid will cancel from both the sides and the vertical component will add up and go on to support the weight of this of this balloon

so we have $s \cos \theta$ surviving the vertical component of s if we draw θ to be like this

so this will be the vertical component will be $s \cos \theta$

so $s \cos \theta$ is the vertical component of surface tension and supports weight

so water filled balloon on the surface of a liquid

so this force is acting at all points on a circle of radius r

so this liquid is enclosing that and this is dipping that surface and

so we have ah $2 \pi r s \cos \theta$ is going to support my the weight of the balloon right

so $2 \pi r$ coming from the at each point on the

so where the sphere is dipping

so this circle at all points this surface tension acts and will give rise to a total force upward which balances the downward force due to the gravity and the weight of the of the object or the balloon in this case and this is equal to

so $\cos \theta$ will give me now ah equal to ah w over $2 \pi r s$ and we can calculate θ to be $\cos^{-1} w$ over $2 \pi r s$

so this is the angle of contact of an object which floats ah on the surface of the liquid surface of a liquid and this is the angle that it makes then the this direction makes with the vertically drawn normal

so let us see one example of this ah

so write down a problem in order to explain this

so an insect can walk on the surface of surface of water the base of the food of the foot of the insect is approximately spherical with a radius 10^{-4} meters the insect has 6 legs you have seen a six legged insect ah and a mass point zero zero two gram calculate the angle θ that its legs make with the vertical calculate sorry

so this is a statement

so calculate the angle that the angle θ that the legs make with the

vertical

so we know that $2\pi r \sigma \cos \theta$ equal to w $2\pi r \rho h$ is three point one four will take a sort of simpler value

so 2π becomes six point two eight h r is ten to the power minus four meter h now surface tension of water

so now it is given that assume the temperature to be the temperature to be 20 degree centigrade which means that you have to take the surface value of the surface tension of water at twenty degree centigrade which is what we have said we will write it once more its point zero seven two newton per meter and that is equal to mg 2 into 10 to the power minus 6 that is the mass h and

so this is kg and this is nine point eight h in to nine point eight meter per second square but there are six legs

so they will be supported

so this six legs will support the total weight

so this thing has to be divided by six and we need to know the angle that one of the legs the foot makes on the water

so this h if you calculate it $\cos \theta$ becomes equal to point three three divided by point nine zero h which is equal to point h three seven point

three seven and θ equal to \cos^{-1} point three seven is

approximately sixty eight point two degrees

so this is the angle that the foot of the insect makes on the water and it doesn't drown rather it can walk on the water and

so this is the angle of contact there is some more interesting thing about the angle of contact which is called as capillarity

so we will discuss capillarity you may have noticed that water kept on a in a glass of water in a glass the edges actually bend upward okay

so this is the liquid meniscus when its water

so this is water and if it is not water say if its mercury then the meniscus is not like that in fact it gets lowered

so as the surface of a liquid touches the surface of the container that it kept its kept in either the where the meat they will either rise as it does for water and it will dip for mercury and you can explain it by an angle

let us call it as θ and this angle is acute for this for water and the angle is obtuse for mercury where it is measured from the vertical

so

these are the two kinds of liquid that we have h that is one of them the where the surface comes in contact with the surface of the beaker or the glass

or the container that is kept in either it rises the liquid meniscus rises or the liquid meniscus dips as shown here making either an acute angle or an obtuse angle now why does this happen that's the question it happens because

of two things one is called as the force of cohesion and the competition between two forces one is called the force of cohesion the other is called as

the force of adhesion

so force of cohesion and force of adhesion force of cohesion is the intermolecular force between the liquid itself

so that is the the force that one molecule exerts on another molecule that is called as a force of cohesion and the force of adhesion is the force that

the liquid exerts on the the molecules of the liquid exerts on the molecules of the glass beaker say here or the molecules of the container that is kept in

so in water the force of adhesion is more than the force of cohesion

so we let's call it as f_c as force of cohesion and let us call this as f_a as force of adhesion

so in this case for the case of water we have the force of adhesion to be larger than force of cohesion

so the water molecules are strongly attracted towards the glass molecules and that's why they tend to move up and just the reverse happens here you have f_c is greater than f_a which means the molecules of mercury the force of attraction or the force of the that exists between the molecules of the mercury f_a mercury molecules are more than the cohesive f_c or rather the adhesive forces f_a that is there between the molecules of mercury and the molecules of the container that it is kept in

so these two will make

so one is that it is an acute angle θ that the water makes θ with the container and that the force of adhesion is greater than the force of cohesion and exactly the opposite occurs for mercury where it makes an obtuse angle with the normal and the cohesive force between among the the molecules of mercury is more than the adhesive force between the mercury and the container that is kept in

so we can actually calculate this θ

so how to calculate this θ let us take one example as that of water and let us just accentuate or rather emphasize it more say this is this is the meniscus and take some height h here and we want to calculate the height h here this has two r as the distance so this is s and this is s this also is angle θ we want to calculate the height that h the liquid column rises from the you know the horizontal h level

so the surface tension act at an angle θ all around the circle which is what we have said surface tension acts all around the circle of radius r so the h the magnitude of the vertical force

so magnitude of the vertically upward force force due to the surface tension so these are the surface tension acting there h due to the surface tension is f and $a \cos \theta$ into l that's the definition of h surface tension and l is equal to two πr which is equal to two $\pi r s \cos \theta$

so because l is equal to since l is equal to two πr and this is going to support

so this two $\pi r s \cos \theta$ which is the vertically upward force will balance the vertically downward force due to gravity which is equal to mg which is equal to ρv into g and let us consider volume of a cylindrical column of water or the liquid of density ρ

so this is equal to $\pi r^2 h \rho$ and g where v is replaced by $\pi r^2 h$ which all of you know that the volume of a cylinder is given by volume of a cylinder equal to $\pi r^2 h$

so my h which is what i want to calculate comes out to be $h = \frac{2 s \cos \theta}{\rho g r}$

so r will cancel and this will be $h = \frac{2 s \cos \theta}{\rho g r}$ and

so this is the expression for the height of rise the capillary rise at the towards the edges of the the beaker or the container containing the liquid here h for water θ is almost equal to 0 that is the we have definitely enlarged the picture actually the the angle is very small its close to zero so if θ is close to zero $\cos \theta$ becomes one and in which case i have a simple expression $h = \frac{2 s}{\rho g r}$

so knowing the surface tension of water and of course knowing the temperature the temperature has to be specified at which we would h know this s ρ of will be known g is of course known and the radius of the beaker is h say is given then you can calculate the height that is that is they will be recorded or noted at the edge

so you understand that keeping all these other things same if we take a bigger bigger or bigger bigger glass which has more radius then this actually goes as 1 over r

so as r increases h will go down

so this height will be smaller and smaller as you have the bigger containing water becomes bigger and bigger you

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