so we have talked about the elastic properties of bodies of solid materials one thing that we haven't touched upon and are quite important and in fact is very important in the context of this discussion is the effects of temperature

so the temperature has been put out of the discussion so far and now we are going to talk about temperature effects on these on these properties that we have just discussed such as elongation such as the stresses etc and think of a metal such as a steel bar being subjected to a change in temperature say the temperature increases from 100 degree centigrade to 200 degree centigrade

so what's going to happen to the material and what effects would it have on the thermal stresses and their corresponding applications etc

so let's um talk about uh or rather make the discussion a little more quantitative by saying that ah let us see this

so there is a there is a rod metal rod which has an initial length l zero and when it's subjected to a temperature difference say for example delta t it could actually um increase by an amount delta l okay and there are very familiar examples such as you might have seen that there are small gaps in the railway tracks those gaps are kept to accommodate the the change in length or the elongation of the the rail track the material of the rail track that it is made of

so in this particular case say the temperature goes from an initial ti to a final temperature t f and the length of this bar which is constrained to actually increase in the direction where it is hinged

so this is hinged say in against a wall which we at this moment we ignore the expansion of the wall due to this the heat that it's subjected to and this has caused some stress in the material and it's in the line of the discussion that we had that either you apply a force in order to cause an extension or you can also apply a change in temperature for the material to undergo an elongation or if you actually lower the temperature of the body it could actually undergo a compression and in any case there will be a stress that is going to be developed and this stress will call as thermal stress for reasons that you would see just in a while

so let us take this temperature difference t f minus t i to be called as delta t and not only that it should be small its not large and in this case the the change in length delta l is given by l0 and you have t f minus t i and there'll be a proportionality constant let's call it as alpha and just to make sure that this elongation happens due to the change in temperature we put a sub t subscript t here and this is called as the linear expansion of solids where the the material under the application of heat the temperature increases from ti to tf and the coefficient linear coefficient of expansion is given by alpha and l zero is the initial length before the um the temperature difference being applied is given uh as l0

so this can be written as alpha l 0 and delta t

so my change in length is given by alpha l zero and delta t and um

so if we want to understand that

so what is alpha alpha is called as the linear coefficient of expansion coefficient of expansion um its a linear coefficient of if you like it is a thermal expansion and which actually appears as a proportionality constant in this equation and it has uh let's check the units and dimensions of alpha so alpha

so delta l will have a have a unit of say for example length um and ah alpha is something that we want to find l 0 again has the unit of the dimension of length and delta t has say the dimension of temperature which could be in

either in degree centigrade or in degree kelvin or in kelvin i am
so sorry ah in kelvin
so alpha ah is one over temperature which we usually say in per degree
centigrade if you're talking about centigrade so
so alpha is expressed in per degree centigrade   ah l zero is the initial
length which is known ah delta is the t is the temperature difference   and
that causes an elongation of amount delta l t now this has to be compared
with the quantities that we know and remember this delta l that we have talked
about earlier   under the application of tensile or compressive strength and
this is given by compressive stresses
so this is given by f l 0 divided by a y just to remind you f is the applied
force l 0 is the the   initial length a is the area of cross section of this
rod y is the young's modulus and if we equate delta l to delta l t then we can
equate the right hand sides as well and we can write   alpha l zero delta t
which is equal to f by a and l 0 y clearly l 0 will cancel from both   sides
and we can write f over a as sigma and so this becomes equal to
so sigma can be written as alpha y delta t and sigma is known as thermal
stress just a few minutes back we were discussing that why it's called as a
thermal stress   because now this depends upon temperature sigma which is a
thermal stress depends on temperature   and rather it depends on the change in
temperature which is a temperature difference between the   final and the
initial values it has to be kept in mind that alpha and y both are taken to
be   temperature independent which is true at for delta t to be small that is
the change in temperature is   not large ah if the change in temperature
becomes large then we can have temperature dependencies   coming in y or alpha
which we don't want to discuss that's beyond the linear regime and this   will
be treated as the thermal stress which is which is now takes the place for
mechanical stress   which had been generated by an application of a force and
now this sigma is generated because   of the application of heat and thereby
changing the temperature
so let's do an example problem   which will make things clear for you
so we'll write down the problem here
so it's about these   familiar examples of rail tracks and the small gaps that
they have in between
so let us take this
so pieces of rail track each 10 meter long are laid are laid with a
clearance of five millimeters at a temperature 30 degree centigrade
so the first question is at what temperature do the do the pieces just start
touching and the second question is what is the thermal stress generated or
developed if there were no if there were no clearance it is given that alpha
it's equal to 18 into 10 to the power minus 6 per degree centigrade   and the
young's modulus of the uh the rail tracks that's a material forming the rails
tracks is 200 into 10 to the power 6 newton per meter square okay
so ah i hope the problem is clear that you   have these pieces of rail tracks
which are to be laid down in order for the trains to you know run   on them
but as you know that they cannot be put right next to each other without having
any gap   because in summer where in many places in india the temperature goes
up to 45 or even near 50 the   material will expand and when it expands you
don't want the tracks to be actually putting   a lot of stress on each other in
which case they may crack and if they crack that could   cause accidents which
is what should be avoided completely and in order to do that they have   kept
small gaps in between
so that even if in summer they expand and come closer to each other   they
shouldn't cause stress and that's how they are designed and again in winter

where many of the places go up to a temperature of say four to five degrees or even lower than that the contraction should go should take them apart but shouldn't take them apart more than the desired distance which could cause actually a gap in the rail tracks uh which is inconvenient for the for these uh materials

so uh for these uh rails

so let's just uh write down all these uh uh all these quantities that we have we have l0 equal to 10 m we have delta l t which is 5 mm once again i remind you that this t stands for temperature at delta l is of course the elongation or the compression in this case is the elongation and your the ti that is given is equal to 30 degree centigrade and in the first part of the question which says that at what temperature do these pieces start coming closer that is they just touch each other which means that they fill for this five millimeter of gap that is there ah and

so uh that's uh the first thing and the second thing is to find out what is sigma which is asked in the second part of the question which says what is the thermal stress developed if there were no clearance

so had there been no clearance they would have expanded and caused stress thermal stress and which as i told you that which could lead eventually to breaking of the material depending on the properties the hardness and the toughness of the material that we have talked about and now uh let's just try to do this uh i'm clearing our part of this question

so now all these quantities that are required to calculate are given here the linear coefficient of thermal expansion is given as 18 into 10 to the power minus 6 per degree centigrade and

so is the young's modulus that is given there

so now we have to calculate delta l t which is equal to alpha l 0 delta t um so your delta t that requires for my 5 millimeter of the delta l to be covered which is equal to 5 into 10 to the power minus 3 meters

so delta t which is equal to delta l sub t divided by alpha l 0 and if you put everything 5 into 10 to the power minus 3 ah and this is 18 into 10 to the power minus 6 multiplied by 10 meters and this comes out approximately as 28 degrees centigrade which means that the delta t which is equal to t f minus t i is equal to 28 degree centigrade and which means that t f is equal to uh 30 degree which is the initial temperature plus 28 degree um which becomes equal to 58 degree centigrade that means at a temperature of 58 degree centigrade this 5 millimeter of gaps will get completely covered which means will completely close and this will cause of course a problem

so let's see the next part of the problem which is about calculation of the thermal stress and in which case as we understand that the thermal stress is computed from thermal stress its equal to sigma equal to alpha y and delta t alpha being 18 into 10 to the power minus 6 y is 200 into 10 to the power 6 and delta t is 28

so when you put together everything it comes out as thousand eight newton per meter square

so just a quick uh understanding of this this ah had there been no gap kept while designing the rail track there will be a stress thermal stress which is more than 1000 newtons per meter square that would develop where a temperature difference of 28 degrees is induced

so this is a large stress thermal stress that is going to be developed in the rail track

so let us carry on with some more problems problems on thermal stresses and the effects of temperature

so let us say a bronze bar five meter long and a cross sectional area area of

200 meter square is placed between two rigid walls as shown
so there are two rigid walls there is a bronze bar which has an initial length
of 5 meter and there is a gap of a twenty millimeter
so there is a gap of twenty millimeter with the right wall ah and this
happens
so at a temperature minus 10 degree centigrade the gap between the bar and
the right wall is 20 millimeter the question is ah find the temperature find
the temperature at which the compressive strengths the compressive stress in
the bar will be 30 into 10 cube newton per meter square and given alpha which
is the coefficient of thermal expansion expansion equal to 12 into 10 to the
power minus 6 per degree centigrade and y the young's modulus equal to 80 into
10 to the power 6 newton per meter square
so just to summarize the problem uh there are two rigid walls and a bronze
bar is tied with the left wall with an initial length of 5 meters there is a
small gap of 20 millimeters with the right wall and this is the story at
minus 10 degree centigrade the question is at what temperature there will be
a compressive stress that will develop in the bar of this magnitude 30 into
10 cube newton per meter square and the the coefficient of thermal expansion
and the young's modulus are given you have to understand one thing in this
problem that the compressive stress of 30 into 10 cube newton per meter
square will come into the picture only when the rod will grow by 20
millimeter and will try to expand beyond that because of thermal effects and
then this stress will come into picture
so uh at this situation there is no compressive stress because the
compressive stress will come from the when the the bar will touch the right
wall and will try to extend farther
so we have to find that extension uh because of uh because of which the the
compressive stresses developed and to find that we can uh take a note that
this must be the ah the stress versus strain graph um i mean we are assuming
that hooke's law is valid and we are not going beyond the elastic limit in
which case a strain of delta x will be generated ah which is strain into l 0
because uh the definition of strain is uh delta x by l zero
so a delta x is equal to strain into l zero and that from the linearity
relation of the stress versus strain graph so we know that y which is the
young's modulus is the stress versus strain
so this is equal to ah stress divided by y into l0
so this stress is the compressive stress that the question has talked about
so if i put all these values 30 into 10 cube newton per meter square divided
by the young's modulus which is 80 into 10 to the power 6 newton per meter
square into a 5 meter this will give me a 1.875 into 10 to the power minus 3
meter understand that this is the extension which will happen when the rod
has expanded by this 20 millimeter and because of the thermal effects it
will try to ah expand even more and there will be a compressive stress that
will be developed
so now i can put it into this formula which is equal to l equal to l0 1 plus
alpha and tf minus ti where tf is the final temperature which is asked for
and t i is the initial temperature which is equal to minus 10 degree centigrade
so
this ah will be um
so it's l minus l 0 divided by l 0 is alpha and a t f plus 10 degree centigrade
um
so putting um all these
so l minus l 0 is
so l is equal to 5 meter plus 20 millimeter plus the delta x that we have

calculated l 0 is equal to 5 meter plus 20 millimeter thus l minus l 0 is simply delta x

so delta x divided by l 0 which will now have to be taken into account and we now have 1.875 into 10 to the power minus 3 divided by 5 this is equal to 12 into 10 to the power minus 6 and a t f plus 10 degree centigrade so if one solves for t f from here it will come as 21.25 degree centigrade

so its at this temperature the bar will not only will touch the right wall but it will also start putting pressure on it because of which the compressive stress will be developed which is given by this and and this ah will happen ah at this temperature at 21.25 degree centigrade ah and the initial temperature was given as minus 10 degree centigrade

so let us do another problem

so a cylindrical specimen of a certain alloy ah having youngs modulus with a diameter three point nine millimeter experiences and elastic deformation when a tensile load load of 2000 newton is applied calculate the maximum length length of the specimen before deformation if the maximum allowable if the maximum allowable or rather maximum allowed elongation is 0.42 mm

so just to repeat ah the entire question once again a cylindrical specimen of a certain alloy which is a mixture of different substances uh having an young's modulus to be 108 into 10 to the power 6 newton per meter square with a diameter of 3.9 millimeter experiences an elastic deformation

so we are still in the elastic limit uh when a tensile load which is a force of 2000 newton 2000 newton is applied so calculate the maximum length of the specimen before the deformation which means that the original length is asked for if the maximum allowed elongation is 0.42 mm so delta l is given to be 0.42 mm

so uh for this cylindrical specimen your initial area of cross section is equal to pi and a d 0 by 2 whole square which is like a pi d 0 square over 4 where d 0 is the original diameter by original i mean that before the extension and a0 is the original area of cross section ah

so we have to calculate l 0 the original length that is the length before deformation

so l 0 is equal to delta l into y ah divided by sigma where sigma of course is equal to f over a 0 f is given to be 2000 newton

so if we put in all these things 0.42 into 10 to the power minus 3 ah into 1 0 8 into 10 to the power 6 divided by 2000 into 4 into pi and a 3.9 into 3.9 into 10 to the power minus 3 whole square ah this comes out to be ah 0.257 meter which is equal to 257 ah sorry this is millimeter this is 0.257 meter which is equal to 257 millimeter

so that's the original length of the sample

so let us now ah talk about um the energy stored in an elastic solid see we compress the solid under some uh tensile force and we elongate the solid under some compressive force etcetera and in order to do that the some work is done and the work that is done against this elastic forces that we have discussed earlier that is there inside the material for deforming it is a measure of the elastic potential energy that is stored in the sample okay and upon again recovering back the original shape ah the elastic potential energy is recovered okay

so we let's try to calculate this potential energy that is stored ah and the best example to do that is let's talk about a spring and also let us constrain ourselves to the elastic limit that means hooke's law is valid and where we have uh the force to be proportional to the elongation let's write it for the moment as x

so this is the the force that is applied and this is the elongation or the

compression and we can write this as uh kx and we have to also put a negative sign because just to make sure that the ah the applied force and the displacement are taking place in the opposite direction

so this is like a restoring force so it restores the normal configuration of the body now if we try to calculate the work done which is also the potential energy stored

so this will be given by f dx and at this moment let me take the magnitude of the force because we are just going to calculate the work done ah magnitude of the work done and this is going to be kx dx by between some 0 to some maximum displacement or elongation x and this is half k s square

so this is the elastic energy that is stored in the deformed body

so if you look at the stress strain curve

so or rather let us look at the force versus the displacement curve so this is the linear line which talks about this f equal to k x we are ignoring the negative sign for the moment and the work done or equivalently the potential energy stored let us write it with a u is half k s square similarly if you are talking about a shearing force suppose a cylinder an upright cylinder of like this is given a shear and there is a deformation angular deformation that is caused to be to be say theta then the force is equal to g a theta where g is the shear modulus as we have discussed earlier is the area of cross section and theta is the angle of shear

so again ah your d x will be equal to l d theta where l is the length of the cylinder or height of the cylinder

so again ah the work done or the potential energy stored is given by g a theta and the l d theta which will be written as half g a l theta square

so these are the energy expressions for energy stored for a shear as well as in a spring for a linear extension of the amount s let us look at applications of elasticity on different components of human body which is an interesting uh thing to look at because even inside our body there are a lot of materials or rather lot of components which display elastic behavior

so let's start with the bones um the bones as you know are the more weight bearing structures i mean the beer lot of body weight that we have and a lot of activities that we do you have seen those examples of a circus in which a person who does stunts he can support a weight of six people on top of him and his femur bones only get compressed by 10 to the power minus 6 meters which is quite negligible and these shocks or these pressures or these weights are actually supported by cartilages that are there in between the bones

so let's talk about the femur bone the femur bone is designed to bear a lot of weight but even these bones do fracture ah and the fracture is mainly due to applying of stress in a direction in which they are not supposed to ah bear the stress

so it's in a wrong direction if a stress is given or a stress is generated that can actually break the bones and

so no matter how well designed the human body is if there are stresses given at wrong postures then they will sort of break or they'll rupture um

so let's talk about other materials where the strength is not all that important but what is important is about their stretchability or their elastic properties such materials are the arteries and the veins

so let's just list them

so we have talked about bones now let us talk about arteries and veins

so the arteries and veins are supposed to carry blood and the fact that blood is carried smoothly through the arteries or the veins because the inner walls of the arteries and the veins are elastic in nature and because they

are elastic when the blood flows   they accommodate that extra pressure that is generated by the pumping of blood say   by the heart and similarly with the veins the internal walls of the veins are also have   elasticity because of which the flow of blood is smooth there are other materials where there are which also has components which have elasticity such as the lungs and the tissues they are   the elasticity of lungs as we know plays a major part in our existence that is the lungs has to   pump air and that pumping efficient pumping of air ah is crucially dependent on the elastic   properties of of the lungs and as we age the walls of the arteries or the surface of the lungs they   lose elasticity and the walls get hardened which uh create trouble into their normal functioning   as we know that when a person grows old all these problems are likely to occur um also the other   stretchable components that we have are the muscles and the skin and if one gets hurt   there is a swelling and that swelling is because the skin has some elastic property and over a period of time the swelling goes down and the skin comes back to its original configuration   you must have seen really old people the skin loses a lot of elasticity as people grow old   and

so by and large what we are trying to say is that there are the components of human body   also has a lot to offer regarding the elastic properties of matter   however the stress versus strain character that we have seen earlier in case of solid materials   differ a great deal with the components that we have just discussed of the human body

so the   stress versus strain curve can actually differ significantly from what we have learned and   each of the crystalline solids or the solids that we have learned they have a generic behavior   of the stress versus strain while each of the physical components that we have talked about such   as bones such as about lungs about the arteries veins skin etcetera they can have very different   stress versus strain relationship as compared to each other so let's see a   typical stress versus strain relationship of a wool fiber okay the sweaters are made of wool   and let's see that what kind of stress versus strain relationship they could have

so this is the familiar graph that you have

so this is stress and this is strain and it goes like this

so this part is somewhat known to us up till this and then of course it becomes   uh flat for quite a while that is uh application of uh stress uh is not there is no there's no   application of stress however the strain keeps changing and suddenly after a certain point the   stress becomes large without significant increase in strain and this is very different than what   we have learnt

so far before we conclude this chapter on the elastic properties of matter let   us recapitulate the things that we have learnt and list out some of the things that are important for   our discussion and

so we have ah in the context of the elastic properties we have learnt hooke's law ah we have learnt about different kinds of of elastic modulus moduli ah such as we have learnt about young's modulus bulk modulus and shear modulus um stress versus strain curves in order to show that how the elastic limit is defined   and when do we actually talk about going beyond the elastic limit and talk about deformation   in that context we have also discussed the difference between elastic ah inelastic and plastic deformations we have listed down a number of properties that are related to elasticity of a body such as properties such as toughness brittleness etcetera um elastic properties of human body where we not only discussed about the strength   of a particular component but also we have talked about the stretchability of different components of human body which are essential for normal functioning

of the body

so a few important points to mention and to ponder about are the following
before we end this list we should also talk about a number of example problems
so we have looked at ah all these things ah one after another in the last
three classes that we have studied elastic properties of matter
so write points to ponder
so in order to list out a few things which you should remember and which may go
against the   common sense sometimes and you should keep it in mind one one of
them is that
so a material having large y wise young's modulus it requires a large force
requires a large force to produce a small elongation or compression in
fact the second point is quite interesting and it is often thought that the
material which stretches more is known to be more elastic and this is clearly a
misnomer the   actual technical definition is that the material which stresses
 material which stretches or of course compresses to a lesser extent extent
due to a due to a given load is termed as termed as more elastic
so this second point has very important consequences because it tells that a
steel   is more elastic than rubber because under the application of a given
load are still   stretches or compresses by definitely a smaller extent than a
rubber specimen third   which is ah important and subtle is that stress is
not a vector quantity unlike force even though its force divided by the area
its not referred to as a   vector quantity in fact because to talk about
stresses either it's if it's compressing   then we call it as a compressive
strength or if it's expanding we call it as a tensile   strength all these
terms are coined to show the stresses being developed in materials   which are
either going outward for a tensile or going inward for a compressive stress
you