

good afternoon students we are going to talk about the mechanical properties of solids um

so by mechanical properties what we mean is that um we are going to talk about the the deformations of solids or and we are going to talk about the stretching and bending of solids so in the previous chapters you have learnt rigid bodies and rigid bodies are defined as the ones which have their inter-particle distances which remain constant throughout the course of motion however that does not stop bending or stretching or extension or even other kinds of deformation of the body now uh these deformations can be brought about by the application of forces and you know even the steel rod which is known to be quite strong can be deformed if the applied force is very long large and we are going to talk about two kinds of deformation mainly one in which the body actually regains its normal configuration once the force is removed and one in which they do not regain the normal configuration mainly we are going to consider our constraint our discussion on uh the forces which after which it's if it's removed the body is going to have will regain its normal configuration

so these are temporary deformations and mostly we are going to talk about these temporary deformations all right

so let's take an example of bungee jumping uh have you heard of bungee jumping if you haven't then you might as well want to look at a video in youtube about bungee jumping in which what happens is that a diver or a jumper he or she binds himself or herself along with a extensible cord and then take a dive from a very high distance or large distance um and this is typically of the order of maybe even a few hundred feet to even more than that this could be a dangerous act

so please do not try without adequate supervision by professionals we are not going to talk about the problems that it could bring ah if a body suddenly you know jumps from a few thousand feet to the ground because of the pressure difference rather we are going to concentrate on the amount of um or rather the the material which the bungee cord is made of and the bungee cord is made up of a material which has a great deal of elasticity in this video the boys seem to uh give a bungee jump uh so he falls off from a large height he is tied with a rope so this

so what happens is that uh this as the diver or the jumper jumps from a height the chord bungee cord which is elastic it keeps getting elongated more and more till it reaches a point which is its maximum elongation and then the diver comes to a temporary halt and after that he is going to swing um in an oscillatory manner and this is what makes all these bungee jumping very spectacular

so ah

so after this initial fault the jumper actually comes to a momentary halt and then this chord as i told that the chord is stretched to its maximum ah and then there is an oscillatory motion that is going to take over but this oscillatory motion does not carry on for ever and viscous drag due to air and other you know wind etc would take over and then finally the jumper comes to a complete halt um

so this is uh

so this bungee cord is made up of a material which has got at this intrinsic property of getting extended because of this uh force or which is basically here the weight of the jumper and then the cord is going to regain its original shape and that's why the oscillatory motion occurs

so it's from this discussion of this bungee jumping it's clear that it's a

property of the body by which it tries to regain its original shape and size after the applied force is removed and this is called as an elastic material so what could be the examples of elastic material such as we have a rubber band there could be a spring which is an elastic material however there are examples of materials which are not elastic materials which means that they do not come back to its original to their original shape and size after the force is removed such as a clay or a dough of wheat and they are called as plastic materials

so what actually goes on inside the material which for which some of these materials actually regain their original shape and some of them do not to understand that let us understand the intrinsic building block of these materials and by building block what we mean are the atoms and the molecules and the way they are bound to each other it will be very a preliminary to say that we could think of these elastic materials to be made of springs okay where the two balls at the end that you see actually represent maybe atoms or molecules and the spring like thing that i have drawn here are the connection between them and when a pressure is ah exerted on these materials either this way or this way or even these ways ah they train to regain their original configuration so

so having understood the atomic configurations which really give render these elastic properties let us try to understand that the relationship between the forces the elongations and there are terms such as tensile stress and tensile strength that will come to very soon and what are their interrelationships between them and how do we actually quantify some relations by which elasticity can be understood better

so let us take about two an example of two rods ok and ah consider three different cases in which they have same initial length but different extension Δl and to make matters even clearer lets call l_0 as the initial length Δl extension

so case number 2 where we have same initial length which is l_0 but different cross sectional area and in the third case we have different initial length but same extension

so let us draw this three cases one after another let us consider these two rods which have

so these are the unstretched lengths and the areas of cross section are also same same area of cross section ah same extension here same area of cross section again for the third case

so ah

so the unstretched lengths are same now if i cause an extension and apply a larger force f_1 here and a smaller force f_2 here i will have more extension in this rod than a smaller extension in this rod okay

so where my f_1 is greater than f_2

so this is case 1 let me draw the same picture to understand the area of cross section

so i have a rod like this and i have a rod like this which have same l_0 however the cross sectional area is A_1 for this and a cross sectional area is A_2 for this

so to create the same extension as it's pointed out in point number 2 ah i am going to apply a much

so the force required to create the same extension in the bigger rod is is more in the the rod with a bigger cross sectional area

so here we have f_2 is greater than f_1 to create the same extension going to the third case we have different initial lengths now to have same extension i need bigger force in this ah the rod with smaller length then smaller force

for the rod with bigger length
 so this is your l_0 and this is your $l_0 + \Delta l$
 so my f_1 is greater than f_2
 so from these three figures it's apparent that my force required to create an extension Δl in a rod of length initial length l_0 of cross sectional area a is f is proportional to Δl f is proportional to $1/l_0$ and f is proportional to the area of cross section
 so if we combine all three of them we can write that f is proportional to $\Delta l / l_0$ and a
 so i can write it a little little more compact by taking this and
 so my f is proportional to now $\Delta l / l_0$ multiplied by the area of cross section now $\Delta l / l_0$ can be called as a fractional change in length let's write $\Delta l / l_0$ as the fractional change in length and hence
 so i can write or change this a constant a i can change this proportionality using a proportionality constant which can now be told as Y which is equal to $\Delta l / l_0$ into a and this is a very important equation in this study of mechanical properties of solid or rather the elastic properties of solid in which it says that the force required to create an extension Δl for a material of length l_0 and cross sectional area a is written as this where Y is known as young's modulus a this young's modulus is named after thomas young 1773 to 1829 and this formula is going to be revisited a number of time in our discussion or some variant of this formula will be revisited a number of times in our discussion of elastic properties of solids
 so we shall make this ongoing discussion more understandable in the sense that let us take the example of a spring
 so the spring is tied at one end and it has an unstretched length l_0 and it's acted upon by a force f and it undergoes an extension which we denote by Δl
 so this is still l_0 and this is under the application of the force it undergoes an extension Δl if we double the force the extension will be doubled which means that the extension Δl is proportional to the force applied on the spring a if we triple the force the extension is going to be tripled now take another example in which there are two springs which are acted upon by a force f each will have assuming that their unstretched lengths are same they extend by the same amount for each one of them which is Δl which we can show it in a different
 so this is Δl and again
 so this is Δl in the application under the application of this force f
 so which means that a your Δl the extension is proportional to the original length l_0 so as we double the length of the spring a under the application of the same force the extension or the elongation becomes twice as much
 so Δl is proportional to l_0 and we have also got Δl is proportional to f now to get the third dependence that is the dependence on the area of cross-section we need to talk about a wire
 so let's take a wire of a finite cross-section and now take an identical wire of an identical cross section and length and suppose you apply a force here this is equivalent to the fact that there is a larger uh area of cross-section same length and is acted upon by a force f
 so this has an area of cross section a this has an area of cross section $2a$
 this has an area of cross section $2a$
 so under the application of this force f the extension will be $\Delta l / 2$

which means as we double the area of cross section the elongation goes down by a factor of 2. similarly if you take three such wires put them together and apply a force F and then consider an equivalent wire with a radius or rather an area of cross section to be $3a^2$ then the extension is going to be $\frac{\Delta l}{3}$ which tells that Δl is inversely proportional to a^2 and hence all these three relations put together we can claim that Δl is proportional to F is proportional to l_0 is inversely proportional to a^2 and again as said earlier the proportionality constant is called as the young's modulus and we can write down this very familiar form which is $Y = \frac{F}{A} \frac{l_0}{\Delta l}$

so with the help of this spring we can understand the dependencies of this elongation on the force the unstretched length l_0 and the area of cross section of the wire and we can again rederive this same formula that we have derived earlier Y is called as a young's modulus

so let us just come to this uh discussion of tensile stress that we had briefly spoken about but hasn't elaborated on it if i simply divide both sides by A the area of cross section of the material the rod in our case then we will get it as $Y = \frac{F}{A} \frac{l_0}{\Delta l}$ and Y can simply be written as F over A divided by Δl over l_0 and as we have said that Y is called as a young's modulus so the young's modulus is defined as the tensile stress divided by the tensile strain

so the tensile stress is defined as the force per unit area that is given to the rod of cross sectional area A and $\frac{\Delta l}{l_0}$ is a fractional change in length that has occurred because of this tensile stress being given and Y simply appears as the ratio of the tensile stress and tensile strain and

so in we may want to know that what are the values of this young's modulus for most materials for solids it's very high to quote a few representative values we can write that we can prepare a table which you will get it in your book the value of Y

so for a material such as aluminium this is 6.9×10^{10} and just to point out what could be the dimension of this F over A which is newton per meter square in SI unit whereas $\frac{\Delta l}{l_0}$ will be dimensionless

so this will simply have the dimension of the tensile stress which is simply a newton per meter square and for copper uh it is slightly over 10^{11} newton per meter square ah similarly for steel which is known to be quite strong this is ah of the order

so this has a value which is 2×10^{11} newton per meter square whereas nylon which is known to be elastic is 3.7×10^9 which has a value which is less than any of these other metallic items that are coated here and a brass has something intermediate which is 9×10^9 newton per meter square

so this shows an interesting fact that steel which has among these has the maximum value for the young's modulus which is 2×10^{11} newton per meter square simply tells us that this the force required of this the tensile stress required to create the same elongation is maximum in steel as compared to any of the other materials that are presented to you here

so let us take a very practical example of how do we where do we get the stress and what are their relevances in our daily life

so let us consider a circus in which in a particular act act that are usually shown in circuses that a particular performer he supports the weights of six other performers his co-performers in the group and each of his

co-performers are say so there are six co-performers in the group are each weighing 50 kgs and one particular person is going to support their weight in in a certain act and consider g to be equal to 10 meter per second square which you have seen that in many cases they are approximated because 9.8 is a slightly odd number to calculate so the total weight of all those 6 performers put together is equal to total weight equal to 300 kgs so with g equal to 10 ah we get the total force that this particular performer will support is equal to 300 into 10 which is equal to 3000 newtons all right ah so now this particular performer has got his femur bone the bone in his in his legs it's approximately say for example 0.5 meter long so femur bone of this equal to 0.5 meter long and its also equal to cross-sectional area is 10 to the power minus 3 meter square so this is cross sectional area we want to know how much of compression in his femur bones would cause for supporting six performers on top of him so consider the young's modulus of the bone is approximately 10 to the power 10 newton per meter square thus what will happen is that ah it will cause an extension in his femur bone which will go like which is equal to because he has two legs so this will be like 3000 divided by 2 which will get divided into two of his feet and this will be like 0.5 meters divided by ah 10 to the power 10 newton per meter square this will be newton here and this will be like 10 to the power minus 3 meter square and this will give us approximately a 10 to the power minus 6 meter so thus for supporting the weight of 6 performers ah the extension or other compression in his femur bone is of the order of ten to the power six meters which means like one micrometer which is fairly small so we have talked about the compression on the bones of a performer in the circus now let us take the same example not with the bones of a human being but rather with the with the steel wire say for example so a steel wire ah just for our convenience the length ah which is same as the unstretched length lets call that as simple as 1 which is equal to 1 meter take that to be a thin wire say the area of cross section to be 10 to the power minus 5 meter square is subjected to a weight ah which is equal to say for example 500 kg do we really want 500 kg yes maybe we want 500 kg so the force acting on the wire so this mass is hung from a steel wire whose unstretched length l is equal to 1 meter the area of cross section is 10 to the power minus 5 meter square and there is a weight which is m is equal to 500 kg let us denote that by m and hence the with g equal to 10 meters per second square which is an approximate value which we often use we can take this to be 5000 newton so let's just try to calculate the extension of the steel wire due to this mass being hung at the bottom let us write down this formula that y is equal to f over a n divided by Δl over l so you have a Δl over l uh equal to f by a y and Δl becomes equal to f into l divided by a y all the other quantities are given and for a steel wire ah y is equal to 2 into 10 to the power 11 newton per meter square and one can easily find out that this is equal to 2.5 millimeter 2.5 millimeter of extension is actually detectable by the naked eye and so you can see this extension of the rod now we understand that why in this particular example we have taken a mass which is as large as 500 kg now just to let you know that if it's a nylon wire nylon has a y which is approximately 50 times young's modulus 50 times lower than the value of steel

which means the extension for a nylon rod for a nylon wire rather is going to be 50 times more than 2.5 millimeter
 so this is a large extension which of course can be recorded
 so this distinguishes different materials undergoing different extensions under the application of a given force
 so long we have talked about either compression of an object or extension of an object along one dimension or along one direction
 so to say now we can consider a different kind of deformation of an object which is other than this simple extension or compression
 so we just for that let's take this example of a thick book which is kept on so this is a side view of a book which is kept on a rough table and if we want to consider that
 so
 if we apply a force on the top of the book the book has a area of cross section to be a
 so we apply a force on the top surface of the book which has a area of cross section a there is a force that acts in the opposite direction which is of the same magnitude and let's denote it by a negative sign and this will cause an extension let us call it as Δx here and this is the original length l_0
 so a this resulting deformation of the book from its initial configuration as shown here is called as a shear or this force a per unit area is called as shearing stress as you see in this picture there is a shearing stress being developed between two parts of the cube they're sliding against each other so there's a shearing stress that is developed between the two parts okay and just a similar line of arguments will show that my
 so your f will still be proportional to Δx which where Δx is indicated by this extension a is proportional to the original length here and it's also proportional to the area
 so combining just like before if all these quantities we can write it is Δx by l_0 into a and similarly we can write that f equal to or f over a is equal to g r by Δx by l_0 just like the young's modulus g will be called as the shear modulus and we are and f over a as we said will be called as a shearing stress Δx over l_0 is called as the shearing strain we can understand this shearing strain in a better manner if we draw a cylinder so this is my cylinder this is my extension which is there and there is a force which is acting this way and equal force is acting on the other direction and my this length is l
 so the shearing strain is given by Δx over l a or let's just put it as l naught as we have been doing and this is given by \tan of the angle of the deformation okay and a for this θ to be small we can approximate \sin $\tan \theta$ as $\sin \theta$ a because just to point out that $\tan \theta$ equal to $\sin \theta$ over $\cos \theta$
 so as θ is small $\sin \theta$ will increase linearly as θ whereas $\cos \theta$ will become equal to one
 so $\tan \theta$ becomes equal to a $\sin \theta$ which becomes equal to i am
 so sorry this θ and $\sin \theta$ will be written as θ
 so a
 so these are the shearing stress and the shearing strain relation and this is very similar in not only in the in looks but also in form accepting that you have not caused any extension or compression but has simply given a shearing force uh because of which this upright cylinder has become a cylinder which is inclined at an angle θ
 so the next thing that we are going to talk about is volume deformation a

this is to say that its not only ah in length or in area we can also create a deformation to a particular object in all three possible directions and by applying uh stress or applying a force which acts uniformly in all directions a familiar example of this can be given for a body submerged in a liquid

so this is a liquid and let us take a body just a cube in our case and the forces acting are from all sides and this will definitely clear compression ah create a compression in the in the system and this uh since the force is acting perpendicular to this uh we can replace the force by the area and which is by the pressure due to the liquid

so that tells you that in this particular case it will cause a deformation to the body where this pressure let us call it a change in pressure compared to the original pressure which is equal to ah which should be proportional to $\Delta v / \theta$ and hence this Δp which is again the volume stress is can be written as $b \Delta v / v \theta$ where b is called as the bulk modulus of the system

so this is and we have to put a negative sign here in accordance with the fact that as pressure increases volume decreases

so as you submerge this body deeper and deeper into the liquid ah there will be increased pressure and that will cause more um the volume to compress more and this is what the underwater divers do experience in their course of journey into the deep into the sea and

so we have understood now there are three constants which describe adequately describe the mechanical properties of the body which are why as young's modulus γ as shear modulus and b as bulk modulus

so as you can see in this picture a cube is completely submerged in a liquid so it's been acted upon by the force from all directions and

so there is a on the right there is an underwater swimmer and the forces are again exerted on him on all from all directions so the bodies are undergoing volume deformation

so now uh we are going to look at hooke's law ah this is named after robert hook from 1635 to 1703

so this is summarizing the findings that we have

so far and we can write for the first case of this tensile stress and the tensile strain we had $\Delta l / l_0$ connecting to f / a and similarly the f / a is connected to $\Delta x / l_0$ and finally this uh Δp which is connected to the bulk modulus by Δv and just in accordance with this l_0 will write a v_0 which represents the original volume of the system now as you can see that in this left hand side all these are stresses and in the right hand side all these are strains

so so there is a stress and this is strain and in each one of them the stress is actually proportional to the strain where the proportionality constants γ and b are called as young's modulus shear modulus and the bulk modulus which depend upon the properties of the material and this is known as this fact that stress is proportional to strain is known as hooke's law and

so just to revise that the stress has an unit of pascal ah which is equal to 1 newton per meter square

so 1 pascal equal to 1 newton per meter square and strain is of course dimensionless and this brings us to the plot where we can plot stress versus strain in a generic manner and will get a straight line till certain values of stress ah and after that this is going to have a non-linear behavior and till this point till which the straight line behavior is obeyed and that will be called as the so this we will term as the elastic limit

so the significance of elastic limit is that that the body will regain its original configuration once the force is removed and beyond this the body is not going to regain its original configuration because stress is no longer proportional to strain and you will have a deformation here which is usually referred to as the plastic deformation. Ah the stress-strain graph that we have talked about needs a little more closer look. If you look at the stress versus strain relationship let's draw that again. Stress, which has the unit of pressure, which is like f/a whereas strain is dimensionless and this behaves like this initially which is a straight line which means the stress is proportional to strain and in this regime all the way from this point o to the point a Hooke's law is valid.

so stress is proportional to strain and we know that the proportionality constant is called as the young's modulus. Once you surpass this point a which means the strain becomes larger than the value that is here and you keep applying the stress via the weight or wire the mass that was that was discussed earlier this graph no longer remains a straight line as before and it becomes like this and it goes and it becomes like this. Ah let's discuss this region now it is not a straight line but it is fairly flat here between a and b and which tells you that for a small increase in stress the increase in strain is enormous. In other words this region in this particular region the material has a plastic flow so it almost behaves like a fluid and it flows and it reaches a point b so this region from b to c is quite interesting it's interesting in the following sense that there is a non-monotonicity at this point c. Uh what I mean by non-monotonicity is the following that as σ here from here to here in this region σ even though the stress is decreasing the strain is increasing which hasn't been the case so far.

so which means that even if it could happen that in a particular circumstances the stress could actually decrease however the strain will continue increasing there are specialized equipments where these kind of behavior can be detected and this detection of this behavior can only happen close to the breaking point and this d is called as a breaking point where the material breaks.

so the this part between o and a is of importance to us because this is where all our discussions are mostly concentrated to because this is called as the elastic limit or as I told earlier Hooke's law is strictly valid and beyond that Hooke's law begins to fail.

so before we end this class let us take a look at what's going to be done in the next few classes at least a couple of classes.

so we'll take a look at things such as different materials.

so different materials will be talked about and maximum allowable loads.

so that when construction takes place we know exactly how much of load to put such as the tensile stresses or the compressive stresses.

so these have to be known for different materials especially the ones that are used in construction then we'll take a look at experimental determination of young's modulus. Ah third we'll take a look at some of the technical definitions of certain terms which are used very often to discuss the elastic properties of matter and they are

so so technical definition of of say quantities such as called as toughness, brittleness, hardness, resilience and stiffness and finally we will solve some problems these are the plans for the next class you.