

so welcome all of you to the third lecture in the series of lectures on gravitation

so what we have done

so far is to revise the kinematical aspects and also the three laws of motion as initiated by newton and after that we also discussed conservation and two laws in fact three laws which are of at most important to us they are conservation of momentum conservation of energy and conservation of angular momentum we shall see that when we discuss planetary motion the so called keplerian orbits and gravitation all the three of them have a role to play in fact the famous second law of planetary motion is nothing but a statement of the conservation of angular momentum which was realized once the gravitational law was formulated by newton we also discussed very qualitatively as to how large distances can be measured by making use of trigonometry

so it is not that one needs a scale or an inch step or any other measuring instrument and that we have to physically travel to determine the distances we can also infer what the large distances are such as the radius of the earth assuming that it is a perfect sphere or the distance between the earth and the moon or the earth and the sun

so on and so forth if we can invoke additional principles both mathematical and physical

so for example we can make use of trigonometry and then measure one side and an angle and try to get either the distance or the ratios of the distances so i gave two examples one of the measurement or the estimation of the radius of the earth by looking at the angle formed by the sun's rays at the same time at two different points i also sort of hinted at how the other distances can be measured through parallax probably

so today what i will do is i will illustrate that in a slightly greater detail as to how distances and ratios can be estimated because this is fundamental otherwise kepler would not have been able to formulate his laws we should remember that when we look at the celestial sphere our eye cannot make a distinction between distances all of them appear to be at the same distance because it is all on the sky the sphere is defined there and therefore we need these indirect techniques and that is what i was going to discuss

so basically today's lecture we will cover a little bit more discussion on how distances can be estimated more and more careful observations will give you better and better approximations better and better values for distance so in the passing i will mention about the accuracy of the observations after that i will discuss galilean law galilean law is the law of freely falling body then we go to kepler's laws which refers to the heavenly bodies we are going to combine the galilean law and kepler's laws with centripetal forces and then argue that it is a very rational thing to formulate the law of gravitation as newton did

so when i say it is rational i do not mean that it is an easy thing or a trivial thing to do because we should remember that when newton worked on universal law of gravitation he did not have the concept of force he did not have the concept of momentum

so he had to introduce momentum he had to introduce force he had to introduce the law and then he had to apply to the law of gravitation even more importantly newton did not have the concept of action at a distance all the forces that people knew were contact forces like for example mass spring system the spring is in contact with the mass

so all forces were assumed because of the contact between two bodies

so here was the first situation where newton was trying to formulate a law where a force can be exerted even when the two bodies are not touching each other they are not in contact with each other so if you read an authenticated biography of newton you will find fascinating discussions on these matters so what we are going to discuss is rational by hindsight natural by hindsight but it was quite revolutionary when newton formulated so when we come to astronomical observations we should remember that the human kind has always been fascinated by the stars i think it was kant who said that there are two things that fill his heart with all one was the order of the celestial spheres in the sky and another was the moral law within him and this goes back to thousands of years to babylonians egyptians greek roman observatory observations indians and the chinese probably the mayans also did a large number of observations we do not have much detail so when we speak of the so called astronomical observations and the formulation of laws we should remember the data collected by all these civilizations spread over many many centuries have had an important role to play we should not forget that so what i will now do is to illustrate to you how the ratio of the distance between the earth and the moon and the radius of the earth can be estimated by looking at lunar eclipse so this is one of the fascinating examples and i think this example is used to aristocrats and let us see how we get about and let me illustrate that in any easy fashion so what we shall do is to assume that we have the earth which i am showing by the small sphere and that the motion moon is going in a circular orbit there is no conflict in this whether it is the aristotelian viewpoint or galilean viewpoint all controversy was about whether the earth is going around the sun or the sun is going around the earth nobody doubted whether the moon is going around the earth or not so what we shall do is we shall denote the distance between the moon and the earth by capital  $r$  so this is moon's orbit so at this point we are making an assumption that the orbit of the moon is circular that is an important assumption this assumption can be tested through various careful observations in fact if you look at the astronomical data observations and also calculations both by the roman greek school and also the indian school arabata or bhaskara they always give the mean distance because they were aware of the fact that the distance changes because by looking at the transit time between any two different angles on the celestial sphere they knew that but here we are not asking for a precise determination of the distance between the earth and the moon but we are interested only in the what estimating the distance between the earth and the moon so we are allowed to make a large number of approximations now in order to illustrate i will magnify the size of the earth so i am going to bring it here and illustrate it here and i will show the radius of the earth to be  $r_e$  i will represent it by  $r_e$  therefore we have two scales here one is the distance of the moon from the earth which i am calling as  $r$  and another is the radius of the earth so earth moon distance are radius of the earth  $r_e$  remember we already discussed how the radius of the earth can be determined very accurately it is about 6000 kilometers maybe about 20 kilometers less than that we will never bother about that

so we know this we know this and what we want to do is to try to get a handle on the value of capital  $r$  that is what we want to do

so now i am going to draw another picture and what i am going to do is to look at the eclipse there are two eclipses that take place one is the lunar eclipse when the earth comes between the sun and moon and we have the solar eclipse when the moon comes between the earth and the sun here we are interested in lunar eclipse

so all of us know that lunar eclipse takes place on a full moon day and the solar eclipse takes place on a new moon day purinima and ahmadiyya that is when it takes place

so now let us ask what happens

so you have the earth here and what we shall do is to assume that the sun is at a very very large distance compared to the distance between the earth and the moon that is what we shall assume and we say that an eclipse is forming we are observing an eclipse simply because the moon enters the earth's shadow that is what is happening

so if the sun is very very far away then you know an object at infinity will produce parallel rays

so what is happening is that i am going to illustrate it schematically

so the sun is at infinity

so the two rays are coming here

so if you forget about the finite distance between the earth and the sun we will come to that later if you forget about the size of the sun itself the angle or size of the sun because that can cause umbra and penumbra we will forget about that also if you forget about both of them then what you have is two parallel rays and the shadow is cast here basically the shadow is of an infinite extent and the shadow is not going to diverge if you have a finite size the shadow is going to diverge as you go farther and farther away so irrespective of where the moon is the moon is going to be in this shadow region this is the time for the shadow region and what is this distance this distance is nothing but the diameter of the earth which is  $2r_e$  this distance is nothing but the diameter of the earth which is  $2r_e$

so all that i need now is to know how long it takes for an eclipse to start and then eclipse to end

so we have to be a little bit careful about that because the moon has a fairly large angular size the full moon appears quite big actually especially when it is rising so we can fix the center of the moon for instance or the periphery of the moon that is what we are going to fix is that ok

so find out the time taken

so this is nothing but the duration of the eclipse

so what are we saying we are going to approximate this true orbit

so the moon's orbit is something like this we will not worry about the difference between the circular orbit in this perpendicular line

so basically we are saying that the moon takes a certain transit time

so tree transit through the shadow and the distance covered is  $2r_e$

so this is the data that we have very simple now let me write down the figure again now here is the earth here is the moon's orbit and here is the distance  $r$  now if i assume that the moon is moving with a constant angular velocity now what is the total distance covered the total distance covered is  $2\pi r$  and i know the period the period of the moon which is close to 30 days

so the moon requires 30 days to cover a distance of  $2\pi r$

so for  $2\pi r$  you have 30 days and for a distance of  $2r_e$  you have the transit time you have the transit time now if you assume that the moon is

moving with a uniform angular velocity and therefore with a uniform speed you get the speed from this because you know the radius of the earth plug it back and you get the distance between the earth and the moon or if you think that ok i don't know the radius of the earth very well then you can look at the ratio  $r$  by  $r_e$  that can be uniquely determined because the two periods are known it is nothing but proportional to the ratio of the two periods so this is nothing but  $t$  over the transit time whatever i wrote as  $t_{\text{transit}}$  this is the duration of the eclipse this was the genius of aristarcus now if you work out this distance it turns out to be roughly 60. today there are extraordinarily precise measurements observing actually measurements not just observations of the distance between the earth and the moon through lasers you send a laser beam and you ask for the time for the laser beam to go hit the moon surface get reflected and come back and you know light travels with an enormous speed three hundred thousand kilometers per second so we can estimate the distance very precisely and this number sixty is remarkably close to whatever we have from modern day observations now all that you have to do is to plug this re equal to 6400 kilometers and this gives you an idea of what this gives you an idea of the distance between the earth and the moon now if you are a little bit smarter and more careful you can see that this observation can also give you an idea of the size of the moon because you can ask how long does it take for the front edge of the moon and the right edge of the moon to come the eclipse starts you know and then it yeah

so if there's a eclipse that is going to start and it is going to end i said that we have to look at the center you can also look at the distance between the two edges

so that will give an idea of the radius of the moon and that in fact will tell us that the moon is much smaller than the earth otherwise it is extraordinarily difficult to decide whether moon is bigger than the earth or the earth is bigger than the moon but this measurement should be able to tell us

so we say if we assume and that is a perfectly reasonable assumption to make that whatever we find at our terrestrial scale sum of three angles of a triangle is 180 degrees two parallel lines shall never meet etcetera etcetera all the theorems of geometry they also hold at the large scale over lakhs of kilometers if you assume that trigonometry and these observations will tell us how to estimate such large distances

so now we can safely say that we know the distance between the earth and the moon because somebody used another principle to determine the radius of the earth the important point that you have to know here is that not only can we estimate in fact we can make it more and more precise for example around the 4th century of the 5th century ad the period of the moon was known to a precession of 1 minute we are speaking of one minute here i wrote approximately 30 days it's actually 29 point something something

so you can bring it down to hours you can in fact bring it down to minutes by repeated observations you can

so once you know the period more and more accurately and once you are able to measure the radius of the earth more and more accurately these distance estimates also become better and better

so astronomical observations are of great interest now i will illustrate to you how we can actually estimate the distance between the earth and the sun in fact distance between the earth and any planet the idea is to look at transit times

so there are things called occultation moon for example you ask venus for

example you ask how long does it take for venus to go from one edge of the sun to the other edge of the sun it's a circular disk and if you know the distance between the earth and the venus then you can immediately estimate what the radius or the diameter of the sun is

so this is how we intelligently estimate

so here what we shall do is not look at either the lunar eclipse of the solar eclipse let us look at the half moon

so this is the eighth day roughly what we call ashtimi which is a very important day for many religious purposes

so when does it happen the new moon happens when the moon is directly in between mean apart from some corrections because it is not exactly in the same plane between the earth and the sun and the full moon happens when the moon is directly in the other side the earth is between the earth and this one but half moon occurs

so to show it schematically you have the sun here you have the earth here and you have the moon here please don't be misled by this diagram because this diagram gives you an impression that moon is farther away than the sun this is not to scale

so this is your ust me the eighth day now what i will do all that i need to do is to measure one of these angles all that i knew is to know one of these angles i know this distance already and if i know the other angle i will leave it a simple simple exercise for you people you can find out are yes the distance between the earth and this that is what you can do

so what are we doing

so let us look at  $\tan \theta$   $\tan \theta$  is  $r_s$  by  $r$  and  $\tan \theta$  can be approximated by  $\theta$  because  $\theta$  is going to be very very small

so in the first step you estimate the ratio of  $r_s$  by  $r$  this is the distance between the earth and the moon this is the distance between the earth and this one and then if you know or if you assume that you know the distance between the earth and the moon you know the distance between the earth and the sun there is nothing very specific about this one you can put venus you can put mercury you can assume that they are all moving in circular orbits and in this particular manner you should be able to estimate all distances and that is what the great astronomers did

so when you are speaking of flapler's law what the astronomers did starting from the time of ptolemy or even earlier was to observe the night sky with great patience with great persistence note the reading create elaborate tables and of course you want a table of trigonometric functions

so they calculated signs and causes sentence to enormous accuracy in that process they in fact even developed series expansions etcetera etcetera even though calculus was not formally invented they were able to do a whole lot of things of course they were doing spherical trigonometry

so called but it doesn't matter to us very much

so by the time of kepler after the great observations of copernicus and tycho brahe we were in possession of an enormously rich table of all astronomical data that is very very important and one more important piece of information is that there was absolutely no evidence that the fixed stars which are far far away were moving with respect to the earth

so the general assumption was that the earth is at the center of the universe and then there is this largest sphere where the fixed stars are which i say trust with respect to the earth and then there are all these other spheres in which the planets and the sun was going it was a completely rational picture

so that is the picture that the astronomers had and now we have to see as to

what the observations had to say

so now let me come back to this slide for a few minutes because i have to show you a few more things and then let us get back to a little bit more of calculation

so in this slide what i have done is to put in some numbers i told you already earth's radius is 6400 kilometers period of the moon is about 30 days okay this approximate sign should have been everywhere so please read it as approximate saying duration of the eclipse is about three hours we know that

so three hours is the time taken to cover a distance of twice the radius of the earth diameter of the earth that is the statement that we are making that immediately tells us that the distance between the earth and the moon is about 60 times the radius of the earth  $16 \times 10^4$

so  $10^4$  cubed is about  $10^6$  to the power of 4 kilometers with a number of the order of 2 or 3 or whatever that is the kind of number that we are giving at this point it is good to ask how these observations were made

so for example if you go to jaipur or to jantar mantar in delhi or just behind our iit in delhi there is this lal bahadur shah ji sanskrit university if you go there they have built observatories in order to observe the sky they are all concrete structures and you will see sun dials then you will see parabolic surfaces with markings and you basically and there are gnomes gnomes are essentially sticks and you measure the length of the shadows cast by the sun etcetera etcetera to find out for example exactly when the noon is when the moon rises what is the speed at which angle or speed at which the heavenly objects are traversing and all that

so this is how people measure all angles and all periods period measurement is a little bit more tricky because they did not have accurate watches so they had probably hourglasses

so you put some sand or you put like take a water with pot with water and the water keeps trickling down drop by drop into count the number of drops

so on and

so forth

so one may think that it is impossible to get something like a period of the moon with such a great accuracy with such observations because they certainly did not have a clock which measured time with a resolution of one minute although people use words like nimish and all that the answer to that again is in a principle so remember when you perform an experiment with your simple pendulum your least count may be a minute but then you are not going to measure the period of a pendulum with one oscillation you are going to do a large number of oscillations let us say 10 15 etcetera etcetera

so if you take the initial reading and if you take the final reading and let us say that your pendulum is pivoted very beautifully

so that there is very little friction and you were able to do 50 oscillations let us say 100 oscillations then the least count does not add up with every oscillation because you are going to make an initial time and the final time and then you are going to calculate the period but the uncertainty is only given by the least count at most by twice the least count but that least count gets divided by the number of oscillations

so that way you are invoking a new principle in fact that was the great contribution of galileo when he looked at the oscillating i think one of the lanterns in saint peter's church in rome and he thought it was executing a periodic motion that is what you do therefore you are able to beat a so-called physical hurdle by invoking a principle what is that that it is oscillating with a perfect period

so even here given if your clock itself does not have a very great accuracy let us say it has an accuracy of one minute or even two minutes but if you observe the period over a large number so you don't ask what is the time taken by the moon to complete one full revolution but 10 full revolutions 100 full revolutions then this least count does not become a limitation that is exactly what they did and they were able to fix it as one minute up to about a minute so there are many people who get confused and say oh they did not have any accurate clock they simply made all kinds of statements that is not correct without realizing or perhaps with realization they were actually more likely with the realization they were able to beat the so-called physical hurdle that is something that we should always remember and that is how physics always grows there is something called a diffraction limit which people are able to overcome through some other physical principle but the most important point in all these observations was that they were all naked eye measurements they were all naked eye measurements so it could not have been able to satisfy the demands of precision and accuracy that we make today but it is remarkable that they were able to make so many observations

so then comes galileo galileo not only gave us the first law and the law of falling body galileo did something remarkable in the field of observation he made the first telescope he observed people you know using these lenses are double lenses to read or magnify and he thought instead of daring it to directing it towards the earth or some distinct object on the earth let me direct towards the sky and when he did that he discovered many many remarkable things one of the most remarkable things was the moons of jupiter which are not visible to the naked eye so he was able to discover them he was able to decide decisively that the markings the beauty spots on the moon or because of the craters so here is a original piece of galileo it is in museo galileo somewhere in italy musus museum with a magnification of ten and that completely revolutionized astronomical observation after that there was no going back they were reflecting telescopes based on reflection people built parabolic mirrors if i remember correctly newton actually built the first reflecting telescope now you make the aperture larger and larger the gathering area larger and larger and you are able to make measurements so that is something that we have to remember

so what we have at our disposal is very very accurate tables now what i will do is i will digress what i will do is i will not continue to discuss keplerian law but i will discuss something remarkable called the equivalence principle now the word equivalence principle was coined by einstein almost 500 years after galileo made the observation galileo himself did not call it equivalence principle newton implicitly used it everywhere and after that every physicist every mathematical physicist who worked on planetary motion from newton to poincare everyone used it but nobody was able to realize that it is a fundamental principle and this equivalence principle is what is embedded in galilean law of a freely falling body i want to discuss that then i want to go back to kepler's laws which were formulated based on the planetary observations especially of copenhagen tycho brahe and then i will combine the two and i will try to show you how gravitation is a something very causes a very pleasing description unified description of both terrestrial and celestial phenomena okay actually i have gone to planetary motion so what i should do is i should go back here and start discussing equivalence principle so let us get into that all of you have used this without paying much

attention perhaps to the mystery of the whole thing  
so in order to formulate equivalence principle let us start with newton's  
equation second law of motion and let me illustrate what is happening  
so mr newton tells us that the rate of change of momentum of a body is equal  
to the applied force we had a fairly long discussion on the nature of this  
applied force

so let me write a few examples again redundancy is extraordinarily good when  
you are learning a subject there is no harm in repeating for example i will  
write  $-kx$  this is hook i will write  $q \mathbf{v} \times \mathbf{b}$  this is lorentz  
motion of a particle in a magnetic field i will write  $q\mathbf{e}$  and this is coulomb  
in an electric field this is the force which is experienced i can write a whole  
lot of thing for example there is a frictional force which is proportional  
to velocity minus  $k \text{ mod } v$

so when i am reading all these things if you look at the right hand side there  
are different applied forces which are characterized by different properties  
and there are two properties essentially the first property is the  
sensitivity to distance or velocity for example hooke's law is sensitive to  
the distance from the equilibrium position but it is not sensitive to the  
velocity whereas if you have a uniform magnetic field for example the  
lorentz force is sensitive to the velocity and not where you are located of  
course if your magnetic field also changes with a position if it is an  
inhomogeneous magnetic field then this force is sensitive to both velocity and  
the position same thing with electric field it is sensitive to where you are  
located the electric field in general takes with this one here you are  
sensitive to velocity here you are not sensitive to position that is one  
property the other properties are various strengths

so strictly speaking i should put a  $k$  here and i should put a  $k'$  here  
because they are two different things one is the sensitivity how does it  
change with the distance how does it change with velocity does it decrease  
does it increase if it decreases at what rate does it change that is the  
question that we are asking the second one is what is this strength now this  
strength is characterized by what i call  $q$   $k$   $k'$  and these are  
parameters these are the strengths for the same magnetic field or for the same  
electric field if you put different bodies of different charges they respond  
differently

so increase the charge by a factor of two the force increases by a factor of  
two decrease the charge by fifty percent the force decreases by fifty  
percent

so on and

so forth

so distance is a property of the field the whatever the thing is this is a  
property of the object how do you respond to that field  $k$   $k'$  charge  
etcetera etcetera it's a property of the test body therefore the response to  
an applied force depends on your own characteristic like your charge or like  
your resistance to motion etcetera etcetera like the spring constant

so on and

so forth

so this is very very important now what we shall do is to look at the left hand  
side of the thing left hand side is universal

so my momentum is nothing but mass into velocity that is what i have

so there are two things if i therefore write  $m$  equal to  $dv$  by  $dt$  into let us

say some  $k$  this is also a property of the test body into some function of  $r$

$v$  i can you can make it  $t$  that is what i am doing

so if it is an electric field this will be the electric field then this will

be the charge if there is a magnetic field this will be  $v \times b$  and this will again be the charge

so on and  
so forth

so that is what I have now you see there is a competition my particle has a charge my body has a charge which says oh please interact with the force let your response be greater and greater then it has a mass which says no no no resist the action of the force

so this is inertia this is resistance to motion and this is the charge so when a person gets very emotional we say the person was charged because the person speaks with a great force that is the statement that we make and it is not accidental whereas if there is no great enthusiasm we say the person has a great inertia

so every object comes with an inertia which is characterized by its mass and then you have the charge corresponding to the interaction your spring constant is the charge for hooke's law your electric charge is the charge for your for instance for your courage

so on and

so forth and the final response depends on this charge by inertia  $k$  by  $m$  and that is the reason why newton call this as the inertia greater the mass

greater the inertia and that is the reason why many times you say if you have a very very massive body take it to be infinite and don't worry about its motion that is what you do when you look at the motion of the earth around the sun because the sun is

so very heavy compared to the earth you do not have to worry about its motion it has momentum but very little velocity that is what we are going to do

so by the same token now let me look at the freely falling body

so there is this very famous experiment

so this is your leaning tower of pisa

so this is where galileo was standing let us say and it is some height  $h$

so he dropped objects of course when you do an experiment like this you should minimize the contribution coming from air's viscosity etcetera etcetera

so you have to take fairly heavy objects

so let us assume that we take balls of lead iron heavy metals stone etcetera etcetera of different masses

so imagine that you have a graduating scale here and you start measuring the rate at which they fall now what is it that galileo observed when you say freely falling body what do we mean by that all of them are released from rest all of them are released from rest

so you can imagine three of you four of you you climb up the tower of pisa or you climb up a high building in your apartment complex or whatever find a time when there is nobody around each of you holds different blocks iron stone whatever whatever heavy objects on which the resistance from the air is negligible and you drop them and

if you were to carefully observe what is it that you would find that they will all move together at any given time if they are released simultaneously at any given time both of them will be together and eventually they will reach the earth at the same time

so with respect to lead iron is at rest with respect to iron lead is at rest and of course with respect to us both of them are moving together they have

been released from rest there is one more law it is not just that they are moving together and what is that let me write it in the next sheet of paper

so all bodies are at rest with respect to each other

so we should remember freely falling bodies released together there should be

no confusion number one number two they have uniform acceleration  
irrespective of their mass that is what we have  
so the first one is a qualitative statement that they are at rest  
with respect to each other the second one is more quantitative  
so  $a$  is independent of the body size shape mass it is independent of  
everything and we know what this is  $g$  acceleration due to gravity that is  
what we have now please try to combine this observation with what mr newton is  
saying of course when galileo discovered his law of really falling body  
there was no formulation of newton's law but we are not under any obligation to  
follow the historical sequence in understanding forces newton says that  $m$   
into an acceleration is applied force that is what i have therefore my  
acceleration should be force applied divided by the mass that is what i  
should have now i will become a little bit fussy and i will remind myself that  
this mass is not any ordinary mass it is inertia  
so there are two concepts of mass we have to understand that one concept is  
the quantity of matter contained in an object  
so imagine that it is made up of identical atoms  
so i will simply count the number of atoms and i will simply say that is the  
mass and if there is another body of different atoms i will ask you know how  
many atoms of the other body should be there in order to make one atom of  
this mass wise the quantity  $y$   
so i can do that  
so the point is that the acceleration should be inversely proportional to the  
mass that is very important and this law cannot be violated  
so what is the word that we use in indian languages for example for gravitation  
gurudwa there is an attraction there is a force that binds us all to the  
earth  
so just as a body should have charge in order to respond to electric field you  
should have some other property in order to respond to it should be  
connected to the spring in order to respond to the hook slot  
so on and  
so forth the body should have a property which responds to the gravitational  
field for example if a body does not have a charge it will not respond to the  
electrostatic field a neutral object take a neutron and put it in an electric  
field nothing will happen take a neutral object put it in an electric field  
nothing will happen it is not going to accelerate that is what is going to  
happen therefore we should ask what is the charge quote unquote charge that  
is strength what is that property of the body charge is called gravitational  
mass it could be it should actually be called gravitational charge like  
electric charge some other charge but people call it gravitational mass that  
is what it is called now the beauty of freely falling body is that the force i  
don't know what it is exerted by the earth on the bodies is independent of  
their mass  
so what do i mean by that i am going to write  $ma$  is equal to  $m$  into some  
function that is what i am writing earlier i wrote  $q$  into some function  $k$   
into some function  $k'$  into some function that is what i am going to  
write i cancel them and this is extraordinarily remarkable it is remarkable  
in two senses one is that the inertia is counter balanced by its charge  
so it has two hats as an inertia it opposes as a gravitational charge it  
supports and they perfectly balance each other number one number two we can  
say that a particle may have a charge it may not have a charge a particle may  
have a magnetic moment it may not have a magnetic moment but at least in  
newtonian physics you cannot think of a body without mass in fact a body is  
characterized by its mass without mass nothing is going to happen that means

the earth our mother earth is going to act on everybody because everybody has mass

so there is a universality of earth's pull on the bodies newton noticed this universality in his formulation of gravitation but einstein not only noticed the universality he found that this is a very profound principle in fact this is what allowed him to formulate the famous general theory of relativity

so please remember that galilean law of freely falling bodies has given this principle equivalence principle that earth attracts all bodies towards it with the same acceleration the forces differ actually but then the inertia and the quote unquote gravitational charge cancel each other that is very very important for us to know and what about a piece of paper for example even that obeys the same law you can create something like a torically in vacuum evacuate all the air everything and if you drop a piece of paper that will also fall with the same acceleration there is nothing to stop it so this is the one of the most important inputs and this is what i showed in the slide as the equivalence principle now let me come back to astronomical observation

so basically i am going from i am toggling between terrestrial to celestial in plain english earth sky that is what we mean so now let me go back to astronomical observations and let us write down what kepler did and here we have many lessons to learn and let us proceed slowly

so before i proceed let us imagine that somebody has a long thread so here is a person who is standing there is a long thread and then this is moving in a circular motion of radius  $r$  so it is executing a motion all of us know how it is  $x$  equal to  $r \cos \omega t$   $y$  equal to  $r \sin \omega t$  i'll write it

so this is the kind of example you know that you write and you immediately discovered this is nothing but an equation of a circle

so what do i do i will observe that  $x^2 + y^2 = r^2$

so i declare that it is a circular motion now what happens this equation can become a little bit more complicated simply because i did not choose a simple coordinate system i do not choose this center but i will choose this setup this has the origin now what will happen now what will happen is i will write right some  $x - x_0 = r \cos \omega t$  because i have simply shifted my origin  $y - y_0 = r \sin \omega t$  now if i open it up i am going to get a very complicated equation between  $x$  and this as a function of time

so let me open it up let us not be lazy

so what will it be for example i will write  $(x - x_0)^2 + (y - y_0)^2 = r^2$  that is what i am going to get plus  $x_0^2 - 2x_0x + x^2 + y_0^2 - 2y_0y + y^2 = r^2$  similarly  $2y_0y - y_0^2 + y^2 = r^2 - x_0^2 + 2x_0x - x^2$

so if you choose a wrong coordinate system you will plot  $x$  as a function of time  $y$  as a function of time

so i will put a square root here it does not look like an equation of motion for a circle it looks very complicated you see that right but it is a circle

so in your geometry course in your 11th standard 12 standard you actually devise method of finding out whether the trajectory is a circle or not independent of the coordinate system is it a parabola it is an ellipse

so these are the exercises that you are going to do and this is what we have now i will complicate your life a little bit more imagine that there is a merry-go-round and it is going round and round with uniform velocity and i am not observing that first of all from the center of that central pole where the merry-go-round is i am standing somewhere outside and standing somewhere outside corresponds to this  $x$  naught and  $y$  naught not only that i am myself running i am on a bicycle which is moving along a certain direction know what will happen with respect to me this will be something like  $x$  minus  $x$  naught minus  $v t$  because i am moving with a constant velocity now if i open up now you can easily see that not only i will have a  $\cos^2$  omega  $t$  term there will be  $v t v$  square  $t$  squared so on and

so forth now i have got a very very complicated equation but the fact is although the kid the child who is sitting on a horse in the merry-go-round is going in a circular orbit for us it will look very complicated in fact the orbit does not even appear to be open because with respect to me this student this child who is sitting in a merry-go-round is executing a very complicated motion what i am doing is to superpose my own motion of course we don't know whether the whole merry-go-round system is moving with respect to the person outside or the person outside is moving with respect to the merry-go-round that's a different matter altogether luckily here the earth is the standard for us but the big question that we can ask is whenever we make an observation of positions and angles with respect to time what is the simplest description and this is a very very important question

so when we speak of a simple description we are asking what is the simplest frame of reference how do i choose the origin of my coordinate system that may require my moving myself to a new origin reorienting myself or perhaps even giving myself a certain velocity these are the three things that i have to do and that is what i have illustrated now it was the genius of kepler that he could make sense out of the planetary observations through a trick the trick was that kepler discovered that the simplest description is when we make a shift to rest frame of the sun and this is what normally people call the heliocentric model of the planetary system but we should remember this is not going to justify heliocentric model because we are only making a kinematical transformation but this strongly suggests that it is more likely or certainly it is a better description if you assume that the planets are going around the sun rather than around the earth so we have almost run out of time

so what i will do is i will stop at this particular point and in the next class i will start with a careful enunciation of the keplerian laws which are some of the most important laws which are written in the history of humanity and then we will show how newton was able to exploit that and formulate his law of universal gravitation the word that you should remember is universal so let me stop here bye you