so welcome all of you to the fifth lecture on gravitation

So so far what we have done is to analyze how to determine large distances how to determine the periods we also discussed at great length the three laws that were formulated by kepler based on observations by astronomers especially those of copenhagen tycho brahe the great thing that kepler realized was that a very simple description of planetary orbits could be obtained by shifting the center of the orbit from the sun to that from the earth to that of the sun of course these orbits are not completely circular they were elliptical which we will ignore to a large extent in this course but you should know that newton's formulation of law of gravitation naturally accommodates the keplerian orbits the so-called elliptical orbits in fact the most general keplerian orbits are elliptical if the orbit is bound otherwise it can be elliptical and also parabolic or hyperbolic the

So called conic sections which you are either studying or which you will study in your standard geometry

So all these orbits are admitted by newtonian law but here we are interested more in the law rather than the details of gravitational law how to calculate how to compute that you will do at a much later stage in your life we will restrict ourselves to circular orbits obviously bound states we also discussed at length the galilean law of falling bodies which is extraordinarily important which said that the acceleration of a falling body was the same for all materials irrespective of their mass and we formulated that principle by calling it as the equivalent principle although this word was coined by einstein only in the beginning of the 20th century around 1910 or 1912 or

So and we argued that a combination of newton's laws that is newton's laws of motion plus the concept of centripetal force plus the capillary in orbit plus the galilean law of falling bodies

So let me write them down

So first we have newton's laws of motion

So when we are making this statement we are interested in the second and third law we made use of both of them the second law describes how the acceleration takes place the third law describes the symmetry or the reciprocity between the force between a and b they are equal to each other but in opposite directions and we should remember that this statement is essentially a restatement of the conservation of momentum that is very important for us

So we made use of these two then we made the concept of made use of the concept of centripetal force

So what do we say irrespective of the origin of the physical force

So long as an object is moving in a circular orbit with a constant angular velocity then it can always be written as m v squared by r this is a kinematical statement which is specific which is peculiar and which is applicable only to circular orbits that is something that we should remember we made use of this centripetal force the third concept that we used or rather the fact that we used was the galilean law of freely falling bodies ok and finally we combine them with the loss of kepler

So how many laws were there three laws

So the first law was of course the planetary orbit which we replaced by the circular orbit then we looked at equal areas traversing equal intervals of time and the third one which is very very important was a relation between the period and the radius of a planetary orbit which is essentially t squared by r cubed is a constant this is a constant in the sense that it is not a universal constant but it is a constant only for planetary motion around the sun that is something that we have to remember

So this is the fourth one

So we combined all these and asked what would be the law that would be consistent with all these observed facts all of them are facts and we came to the conclusion that all of it could be consistently understood if i postulated that my gravitational force is nothing but minus g m m by r square into unit vector

So let me not write that let me not write the minus sign either this is a gravitational force

So what are we saying we are saying that this is an inverse square law and there are two masses which is denoted by capital m and small m at this point we should not assume that capital m stands for a larger mass and small m stands for a smaller mass in fact it is better to formulate the law without any such prejudice therefore i will write if g is equal to g m 1 m 2 by r square

So what do i mean by that i will place a mass m one here i will place a mass m two here m one m two this is the distance r that is what i have now m one pulls m 2 towards itself m 2 pulls m1 towards itself and you accommodate that by putting the right sign that is what we did

So in other words this great law please remember we did not derive this expression we've only arrived at this expression as a very logical and a very simple way of expressing known facts a law can never be proved it can only be verified

So this great law gm 1 m2 by r squared is a culmination of all the observations due to copernicus tycho brahe astronomers earlier haley because there was also halis comet and galileo and kepler however we should remember that this does not establish the law per se because it is only a consistent description and we need independent verification of this law an independent verification of the law entails that we should be able to deal with bodies with arbitrary masses and we should be able to determine this constant g which is unknown to us whereas all this time all that we made use of was that we know the distances and we know the periods

So we played around in fact that is the reason why i am emphasizing that this law is appealing but it is not necessarily true it need not be always valid but we want to claim that newtonian law is indeed a universal law it's called the universal law of gravitation therefore we have to do a little bit more of work

So let me go to the slide and show you what i intend to tell you

So the first slide summarizes whatever i wrote on the sheets of paper

So there is a body a with a mass a at a position vector r a a mod b body b with a mass b located at a point r b

So what we do is to define the vector separating a and b r a minus r b and then we wrote that the force exerted by a on b that is very very important that is why i put the arrow a go on b is minus g m a m b by r a b cubed into vector r a b which is what i denoted pictorially for you on the sheet of paper okay when we made use of the galilean law i made use of only one fact there are two aspects of galilean laws one is the acceleration is independent of the mass of the falling body and the second one is that the acceleration is independent of the height at which the body is from the earth okay that is the statement that i want to make

So let us get back to the drawing board

So what do we want to say what we want to say is if i look at the galilean law there are two aspects acceleration independent of mass and the second one is that acceleration independent of the height

So what do you mean by saying that

So you have the earth here and if the body is falling down at all heights h1 h2

So on and

So forth the acceleration is the same which is given by g that is what you call as the acceleration due to gravity which is roughly 9.8 meter per second squared

So in deriving all these laws i made only use of these facts but newtonian law says that acceleration cannot be independent of the distance from the attracting body there is a one over r square term if there is a one over r square term

So let me show that in your next sheet of paper

So i have my earth here let us say a body is falling

So as the body is falling strictly speaking as it keeps coming down acceleration should increase

So this is a statement in principle statement in principle in fact there are many many ways of formulating that the other thing that you notice that if you drop two bodies let us say separated by a certain distance d galileo would say that that would also fall down like this

So the distance d would be a constant if you made an observation let us say from the top of a tower whereas according to newtonian law they should be both falling towards the center therefore they cannot exactly be parallel with respect to each other

So newtonian gravity gives a correction to the galilean law and what is it the first one is that the acceleration should increase as the body falls down and if two bodies start with a separation d then the distance between them should keep on decreasing as they fall towards that because both of them fall towards the center this is a highly exaggerated figure never mind about that

So what we have to do is to see what kind of a correction newtonian gravity gives

So that it is not

So large that galilean law will be violated in that case newton's formulation of universal of gravitation would not hold on the surface of the earth that is very very important

So let us fix that this fixing is not a difficult thing at all

So what we have to do is to remember that i am going to write mass into acceleration this is a freely falling body on the left hand side and this will be g m mass of the earth divided by now i will be a little bit more careful i will write it as radius of the earth plus h whole square

So remember when we speak of the distance between two bodies we are actually having in our mind the center of the earth it is not from the surface of the earth and this height is from the surface of the earth

So to repeat again

So this is my h and my r is somewhere here

So let me draw a line to indicate that that radius is very very large remember i am speaking of height of the order of 10 meters 20 meters 100 meters maybe but the radius of the earth is 6 400 kilometers roughly

So we are speaking of a very very large ratio i will come to that number exactly

So this is what we have the mass terms cancel the gravitational mass is the same as the inertial mass

So therefore now we are left with a which actually depends on h

So what is the trick that we have to do the trick that we have to employ is to note that this h is very very small compared to r

So i am assuming that it is falling radially inverse

So that i do not have to worry about the

So called dot products and the vectors and things like that is a linear motion

So what i have to do is to remember that h by r_e is much much less than 1 that is what i have to remember

So what we do what we do is to write it as a equal to g_{me} over h square into 1 over 1 plus h by r_e whole square since h by r_e is a very very small quantity this is nothing but a perturbation on 1.

So what do we have to do we have to simply perform a binomial expansion or a taylor expansion if you feel like

So let me do that calculation

So what we do is to make a binomial expansion h by r_e much much less than one let me call it as x that is what i am calling it remember h is the height from the surface of the earth re is the radius of the earth and i am interested in the term 1 over r_e plus h whole square this quantity is nothing but 1 over r_e squared into 1 plus h by r_e square all that i have done is to pull the r_e out and this quantity is 1 over r_e squared into 1 plus x whole square this is the expression that i wanted

So let me open that up expression and see what it is

So i have 1 over 1 plus x whole square is equal to 1 plus 2 x plus x square that is what i have as i told you i have x square much much less than x much much less than 1 therefore if i am interested in the leading order contribution i can drop this term

So once i drop this term this will be approximately equal to 1 over 1 plus 2 x and your binomial expression essentially give me this is approximately equal to 1 minus 2x that is what we are going to get therefore we are now in a position to compute the correction coming to the gravitational force whatever the constant

So what do we have know my f of g is essentially g m e m by r_e squared into 1 minus 2 h by r_e i can write this particular expression now that is the correction that i am going to get now if i ignore this particular term this is nothing but m a

So ignore this particular term this m cancels and you see that my acceleration is nothing but f g by m which is g m v by r square which is your acceleration due to gravity that is what we have

So essentially what we are saying is that for all practical purposes the acceleration because of the earth is constant because the height at which the body is falling from is a very very small correction or a perturbation to the radius of the earth

So it is not difficult to just estimate what that value of x is

So even if i put h is equal to 100 meters which is quite a large number my radius of the earth is 6400 kilometers

So that will be 6.4 into 10 to the power of 6 therefore you see x is really small

So what have we done we have reconciled all aspects freely falling law freely falling

body law law of freely falling body with the newtonian law

So in other words we have established consistency with known facts

So that should give us the belief and the faith that probably newtonian law of gravitation is the correct law in which case we should be able to go beyond whatever we have done

So far and find more experimental verifications and in fact we should be able to predict new phenomena which we had not seen earlier and that is exactly what newton did

So in order to appreciate that let us go back to the slide that i have made

So this slide shows a number which is roughly 3 into 10 to the power of minus 4 therefore it is a very very small number you can see that means the correction to the constant is roughly given by whatever we treat as a constant 10 meters per second squared or 9.8 meters per second squared the correction will be one part in 10 000 that is the statement that we are making three parts in ten thousand actually the correction coming because of the non-spherical nature of the earth and because of the rotational nature of the earth earth is not entirely an inertial frame because it is rotating about its axis is much larger than this

So it is not a bad approximation at all

So now we have to take stock of situation what have we done we have formulated a law based on known observations but what is it that we knew we knew the distances we knew the periods but we do not know the masses when galileo dropped masses he knew the mass of the falling body but for sure galileo did not know the mass of the earth g value is not determined

So it is an unknown constant

So what galilean law is essentially doing is to determine mass of the earth into gravitational constant actually what galilean law determines is mass of the earth into gravitational gravitational constant divided by the radius of the earth square but i am assuming that we know the radius of the earth after all people could navigate around the earth or the great experiment of aristothen is sort of fix the radius of the earth

So now if you want to go beyond this and if you want to claim look here i have a universal law of gravitation with me i better determine either the mass or g or both independently and verify that the law really holds this was a task which was undertaken by cavendish and what he did was to directly determine the gravitational constant this is one of the classic experiments in the history of physics which he performed over a period of one year between 97 and 98 he did a number of careful observations and he arrived at this value of j which is a tiny tiny number compared to for example the strength of the electrostatic force which is given by e squared over four pi epsilon naught

So on and

So forth and we should better spend some time understanding how cavendish determined the value of g historically cavendish did not claim that he was determining the value of the gravitational constant but he made a more interesting claim he claimed that he was weighing the earth actually he did not even make that statement he claimed that he was finding the density of the earth because in the 18th century when cavendish was doing these experiments people were very very seriously interested in the density of the earth

So he wanted to measure the density of the earth i don't know if i said radius

So kevin disclaimed that he was determining the density of the earth as if earth is a homogeneous uniform mass distribution which is not true but never mind but we can make some kind of a contact with all these concepts quite easily but as far as we are concerned what cavendish really determined was the gravitational constant a historical aside which is of interest at this point is that cavendish did many many experiments but this experiment is called the cavendish experiment like we say michaelson marley experiment because obviously this was the most celebrated

So when you say the cavendish experiment we always refer to the determination of capital g the constant of gravitational force

So let us see what is it that cavendish did

So the basic idea of cavendish experiment is a null torque experiment

So this is a standard technique which you employ whenever you want to measure some delicate forces there are two ways of measuring a force

So let me indicate that to you

So how to determine forces one way is measure the acceleration measure the body's acceleration now this is a tedious process because you should be able to measure different positions at different times with very very great accuracy then you have to

smoothly join them that is essentially calculate the tangent you get the velocity plot the velocity and again smoothly join them and then you calculate the tangent again and you will get the acceleration which is a tedious process and it is prone to errors the second procedure is to actually arrest the motion by known force and this is a very useful method arrest the motion by known force

So for example suppose there is a capacitor plate here carrying a surface charge density sigma there is a charge particle q

So this is minus and this is plus

So that it gets attracted in this direction

So what i will do is i will attach a spring here and i ask what is the new equilibrium position where will it get rest i know that f is equal to minus kx coming from the spring and if the charge particle is at rest because of this then i know that the force exerted by this plate which is carrying a surface charge into the sigma is cancelled exactly by this and i should be able to determine the electric field that is the trick

So this arresting the motion is called null force the net force is null

So this is a technique which you employ essentially by applying a counteracting force

So that the acceleration is zero and hopefully the velocity is also equal to zero and then you determine the unknown force when it comes to electrostatic forces coulomb realized that an even better way of determining the coulomb law the so-called one over r squared force in fact that was also known to cave in dish is through a torsion balance and what does a torsion balance do your torsion balance does not create a null force but it creates a null torque

So what you do is you have a let us come back to this slide here nl torque experiment

So this picture is from wikipedia and this is exactly the apparatus which was used by cavendish which i am going to use again and again

So what you have is a thin wire on which you suspend two rods and then you make it get attracted by two other objects which will apply equal and opposite force

So that there is a torque now because of the torsion the whole thing swings and comes into equilibrium position

So the torques are countermanded because of the torsion of the spring that would be a null torque experiment and then you find the angle of deflection this is the technique which was employed by coulomb and what cavendish did was to employ the same technique

So you can see the two masses here the other two masses are not seen i am going to describe this experiment in great great detail but this is the basic idea now i am going to amplify this concept through another figure which completely illustrates whatever i want to show

So you see what's happening here basically what cavendish did was to go to the rooftop of his laboratory i will tell you what his laboratory was in a few minutes then there is a thin wire that you see that is coming down which is indicated as the torsion wire and to that thin wire is attached to slender rod okay that is the darkened black line bolt black line there is a slender rod and it is exactly attached at the center of the slender rod this is a uniform cylinder rod and the two ends you have two small masses m which are sitting there

So here small m refers to lighter object big m refers to the heavier object that is what you have

So the pink color that you see or the peach color that you see is the undisturbed position now in that undisturbed position what happens is that there are two very heavy masses in fact the smaller masses and heavy masses were both made of lead all of them were completely spherical they are placed such that they are at equal distances from these two objects

So what happens the heavier mass moves this lead towards itself

So let me go back the heavier mass one heavier mass moves the one of the objects in this direction the other heavier mass applies a force in the opposite direction therefore there is a net torque but this net torque is countermounted by the torsion which opposes by producing a counter mounting torque therefore there should be a net angle theta

So this is the idea and the distance between the heavier mass and the smaller mass is denoted by r i will probably use the notation d and basically i want to find out the new equilibrium position because of the balance of these torques this is the experiment now comes the experimental details which we should pay great attention to we should also pay great attention to the experimental numbers because unfortunately your ncrt textbook is singularly lacking there are no numbers associated with it but we shall rectify that here

So what kaventish did was he he established his lab in a big shed in his estate cavendish was an aristocratic person a rich person a man of means

So he was a landed person

So he had a very very big shed

So what he did was to convert the big shed somewhere in his fields as his laboratory and what he did was this apparatus which i showed earlier he placed the whole apparatus in a thick wooden box

So if you look up references for example even wikipedia they will give you the dimension of the box they were it was a very very huge box and the thick box the thickness itself was several feet it was made of a wooden box and this wooden box was itself enclosed in a shed because cavendish did not want any disturbance coming either from the wind or from the vibrations which are because of the moving cards or chariots or wagons of those days

So he wanted essentially some kind of insulated environment today of course we perform very very sophisticated experiments on

So called vibration free tables they were not available at that particular time but kamentesh did a remarkable improvisation and that is how we proceeded in order to observe and in order not to disturb the operators even your own approach may actually affect it you will see that when i write down the numbers and if you convert it into for example compare it with our own weight

So he never went anywhere near the even the wooden box he made through small two small holes both in the box and the shed and he put two telescopes and he pipped and he looked at the whatever the angle or the oscillation of the torsion pendulum was at a far distance that is something that we have to remember you see the

So called null experiment is an idealization because if there is a null force that simply means that the object has no force acting on it it does not ensure for us that it is at rest it can move with a uniform velocity the slightest perturbation can give it a uniform velocity in a similar manner when you have a null torque experiment a slightest perturbation can actually giving it a small angular acceleration

So although in our analysis we will treat the experiment to be null actually there was a motion but that period was enormously large it was 20 minutes

So to complete one full oscillation it was taking 20 minutes which for all practical purposes you can ignore because the period of observation you know when you measure the angle is over a period of let us say a second or a few seconds or even half a minute compared to that the motion over 20 minutes was very very small correction that means cavendish had taken enormous care to actually make the pivot completely frictionless

So on and

So forth that is what he had done and the important point that we have to notice is that he exploited the most accurate possible measurement means that was available at his time because his vernier scale you know it was like a travelling microscope

So to say whatever microscope he had fixed had a least count of 0.1 millimeter

So that is something that we have to remember of course when i say that the vernier scale had a least count of 0.1 millimeter i i should compare it with the other dimensions and that is what i am going to come down to and here you have the details i do not know whether it will be visible or not

So let me write it down here

So that you can see what it is

So experimental details

So you have the large lead balls they were themselves suspended and they are not supposed to move

So the lead ball is playing the role of what the earth and the small balls the small lead balls are playing the role of a falling body that is something that we have to remember

So they had a mass of 1 58.04 kg

So he got a massive lid sphere of 158.04 kg made if you look at the original paper the number is given in the

So called grains and it is 24 lakh 39 000 grains

So you should remember that when cavendish did the experiment or for that matter when even newtonian formulated is loss si units or the cgs units were not used in great britain what the british used this is

So called f p s unit foot pound and second that is what they used

So they had inches and since cavendish wanted a very very great accuracy he chose a very very small fraction of the pound and that is what was called as a grain

So it was 24 lakh 39 000 grains

So grain was presumably you know the weight of a small seed or some such thing okay that is what they use that is why if the grain is used okay

So there was a remarkable accuracy that was there and what about this small lead balls the small lead balls had a mass of 0.73 kg

So we have a fraction of more than 300 between

So the heavier mass is at least 148 158.73 something like 300 times let us say heavier than the lighter mass and that is what they did we are going to calculate moment of inertia it is very very important

So we have to worry about the mass of the rod

So the lead balls were suspended from very fine metallic wires in fact the radius of the metallic wire was probably about a few centimeters now the mass of the rod was actually wooden rod

So wood is much much lighter than lead we all know that and that was 0.03 kilo it its very important for us

So if i look at this rod and i put two masses and there is this string that is coming here almost all the mass is at the edges the mass of the rod itself can be ignored because we are speaking of 0.03 to 0.73

So we are speaking of something like 200 times or

So we can forget all about the mass of the rod when we are going to make an estimate

So that is fine then we have to worry about the distance between the balls how long was the rod

So this distance is this what i want yeah this distance was 1.860 meters actually it was a six foot rod which is 1.860 meters

So you can imagine how big that wooden box was and how big that shed was we are speaking of 1.860 meters that is the distance that was placed

So probably i should not call it as d let me call it as l because that is the notation that i am going to employ the distance between the larger masses was also the same is that right except that one mass was here and another mass was also there is that okay

So if i bring it down here and this distance is also the same that is what i mean they were on the either side of the two rods okay we don't have to worry about the radius we have to worry about the distance between this small rod in the big rod

So let me see what the distance between the small rod and the big rod is if i have it at my disposal

So that was of the order of 0.225 millimeter remember i want a very large gravitational force therefore the distance must be as small as possible otherwise it will die very very rapidly therefore the distance was of the order of point 0.225 meters it cannot be millimeter because he certainly did not have the least count

So 0.225 meters that is the distance that was there this was 1.860 meters and he was measuring the angle that was the distance that was being looked at now what we have to do is to make an analysis and find out how we are going to determine the value of g this is what we have to do and i told you the least count of the vernier was 0.254 millimeter

So let us start making the analysis

So it is a very very simple way of doing that what i will do is i will write the torque my torque is nothing but the length of the rod into the force because what is it that i have i have a rod of length l the string is at the midpoint there are two masses and they are attracting is that okay

So there is a perfect couple acting on the two bodies that is the length into force therefore this is nothing but l into g into the mass of the heavier leg ball the mass of the smaller lead ball divided by d square that is what i have

So if you assume that it is exactly counteracted by the torsion then what we are going to do is to write it as the torsion constant or whatever k into theta

So the stars write f is equal to minus k x where k is the spring constant this is my torsion constant and this theta is the angle of deflection this is the angle of deflection you know by how much it has moved and that is what cavendish determined

So i know how to determine g it's a simple algebraic exercise

So what is happening is that i am going to equate this therefore i am going to get g is equal to

So i should not be committing any mistake

So what am i going to get i'm going to get a k theta d square over m m l

So this is my primary expression for the gravitational constant k theta d square m m l

So i know the angle of deflection i know the distance i know the heavier mass i know the smaller mass i know the length of the mass but how do i know this constant

So everything boils down to determine the constant

So the question is how to determine k the answer is simple remove everything remove the heavy mass remove everything and look at the natural oscillation

So what is the solution look at the natural oscillation of the system

So what do i have i have this rod there are two balls and i give it a twist theta and ask how it oscillates

So if you give a small angle theta is small you know it executes a simple harmonic motion and the period is essentially given by t equal to 2 pi root i by the unknown torsional constant k that is what we have which is a generalization of a point mass which is going to oscillate and your i is essentially the moment of inertia what is the moment of inertia of the system of two masses with a separation l now comes the important observation that i made that the mass of the wooden rod was a very very small fraction compared to the mass of this red lord these two

So the moment of inertia can be very easily computed and that is nothing but m l by 2 whole square because it is about this particular point of this plus m into l by 2 whole square of this

So that makes it 2 m into l by 2 whole square this is my moment of inertia

So what i will do is i will measure the period i know my moment of inertia and that immediately gives me the spring constant

So if i did all these calculations i get a very elegant expression for g which i have exhibited in my slide here i invite all of you people to please substitute all the expressions and convince yourselves and what is your g given by two pi square l d square by m t square please remember this small d is the separation between the big rod the big ball and the small ball capital l is the length of the rod this capital m is the mass of the big lead ball the mass of the small ball gets cancelled and then t is of course the period

So this is the great expression

So this is exactly what cavendish did over a period of one year he must have taken a whole lot of readings and if you go to the internet and if you google you will find a reference to philosophical transactions of royal society of london where you will find the detailed paper of cabin and cavendish arrived at the remarkable number which is given by six point seven four into ten to the power of minus eleven

So indeed he performed an extraordinarily delicate experiment determining a very very small number in writing this of course i have been completely slip short and careless i mean that is deliberately done

So that we spend some time on what we are writing this is of course meaningless unless we specify the units and the units are in si units

So i leave it as an exercise for you people to compute what the si units are my force will be in terms of newtons and newton itself is what kg into meter divided by second square on the right hand side you have the mass square divided by length square

So if you substitute the whole thing you will get the value of g in newtons

So this is the experiment that he did and you can easily see there were a large number of approximations if cavendish had gone too near the apparatus assuming you know hitti was a person of roughly let us say six feet tall and he weighed well 70 or 80 kilos that could have caused quite a lot of perturbation on the gravitational forces

So he was wise enough to stay probably meters and meters away and he was looking at the force of attraction which are separated by a fraction of a meter it is very very important to know that in fact an experiment of a very similar nature but of enormously greater accuracy is the classic experiment of decay which was performed in 1964 who actually tried to verify that the mass term cancels in the gravitational force expression m a equal to gm and we cancel the tm equivalence of inertial and gravitational mass there actually um decay ensured that even the mass of the earth will have no effect on the experiment that is very very important because for example in the cavendish experiment it is assumed that the length of the strings which supported that the heavier masses and the smaller masses were exactly the same otherwise there will be a slight mismatch and the torque will not be in the plane okay and then the gravitational field of the earth would play a role decay was able to actually eliminate that we don't have to get into that but this is a really interesting experiment

So what i have to ask ourselves at this particular point is how good was this

experimental determination by comparing it with the known values and here in my slide i have picked up the known value the most recent value this was probably sometime in 2014 or

So and the number is displayed as six point six seven four zero eight and three one are in parenthesis into ten to the power of minus eleven

So what do we find we find that the first significant digit is completely agreeing there are no order of magnitude errors the absolute error is of the order of about seven percent actually a more careful experiment which was done by two boy or somebody i don't remember almost 100 years later increase the accuracy only by two percent okay there was a five percent error compared to the real you know today's experiment and you can see the relative error between the modern value today's current value and cavendish value is about one percent

So given the fact that the apparatus that was used was quite crude compared to what we use by modern standards and compared with the fact that the least cone was not

So great compared to what we used today compared with the fact that they were not completely vibration free in spite of enclosing it there might be a gust of wind or a small whiff of wind you see that the cavendish experiment was indeed a great success and very justifiably cavendish is credited with the determination of g of course there is a fantasy that we want to weigh the earth it goes back to archimedes who said that when he discovered it is you know principle of liver you remember which you studied in your eighth ornament standard he said that give me a place to stand and give me a sufficiently long rod i can even weigh the earth for you i can weigh anything for you that was the archimedes claim okay

So probably people were influenced by that and for various reasons people were interested in the mass of the earth and the density of the earth now if we know the mass of the earth the gravitational constant we should be able to find the mass of the earth also which i have shown in these slides but i will write them down

So let us write it down what i would like you people to do is to actually substitute the real values and convince yourselves okay

So now i go back to galileo that is what galilean law of falling bodies i will write m j equal to g mass of the earth mass of the object divided by radius of the earth square that is what i will write but now i am not as powerless as i was 15 minutes earlier not as helpless because thanks to cabin dish i know the gravitational constant this m will go away and i know the radius of the earth thanks to observations therefore i can immediately determine the mass of the earth to be g r e squared divided by capital g that is what i have as i told you cavendish was not entirely interested in the mass of the earth but he was interested in the mean density after all when i am writing the radius of the earth that is also the mean radius because after all we know that the earth is not a perfect sphere but it is a geoid it is flattened at the poles and slightly bulging at the equator therefore this radius is only a mean but anyway if you ignore that we will write this to be 4 pi by 3 r e cubed into rho and i will put a bar on all of them

So what does bar refer to bar refers to the mean value of the radius of the earth and row bar refers to the mean density what density is this the mass density of the earth

So if you plug in all the numbers well people those days used to like to give the specific gravity rather than the density and please remember specific gravity is the ratio of the density of a material to the density of water presumably at room temperature

So row of earth mean divided by rho water let us say normal temperature pressure turns out to be 5.448 plus or minus 0.033 this is the number that caventish got interestingly we are told that actually cavendish made a mistake an algebraic numerical error apparently he declared it to be 5.84 or some something or 5.448 it could be a transcription error or a numerical error it is very trivial error substitution error we are not going to penalize anybody for that but the correct number is 5.448 which tells you that the earth is mostly solid and it is heavy even if it is not solid it must be consisting of very very heavy elements which is what you have at the core of the earth

So the water content is not great although the two thirds of the surface of the water is covered earth is covered by water

So for comparison you should remember the density or the specific gravity of iron is about 7 plus 7 point something of led is 11 point something

So we have a mixture of various other elements okay lot of silicon

So on and

So forth

So this is what cavendish discovered or cavendish measured

So for that reason he is famously known for weighing the earth or finding the weight of the earth for the first time and we should also remember those days people did not make a distinction between the mass and the weight

So he said that i have made the earth today when we say weighing we mean the mass but he actually meant that he found mass into gravitation m into g that is what he discovered but that is a slight some kind of deviation or integration which we need not bother about

So we have come a long way now but this is not where our calculation ends because again i don't have any independent means of measuring the mass of the earth and verifying that the law of gravitation is correct strictly speaking it should be possible

So what should i do i should be able to look at the orbit of the moon and again determine the mass of the earth that is what we have to do let us spend some time now if the mass of the earth agrees with the value obtained by cavendish or cavendish like experiment where you looked at two independent masses from the observations coming from the orbit of the moon then our faith in gravitational law increases what is the next thing that i should do i should be able to look at the orbital motion of the earth around the sun and i should be able to estimate the mass of the sun now of course i do not have any means of finding out the mass of the sun but then i should be able to look at the orbital motion of various planets and that should establish the mass of the sun but luckily that is already established by the third law of kepler because that is how we got a constant but then if i am intelligent i should be able to combine the motion of the moon around the earth the motion of the earth around the sun and i should be able to establish a relation between the period of the moon around the earth given the distance between the earth and the moon the period of the earth around the sun given the distance between the earth and the sun

So these are various way of consistency checking if we did that then gravitation would be established but why should i stop at that point i can proceed a little bit ahead i look at i can look at the moons of mars for example i can look at the moons of jupiter jupiter has a very very large number of moons i don't know there are many names i am not getting them right now ok it has 12 moons or some such thing i should be able to look at their periods if they are circular i will be able to use the formula that i have written if they are highly elliptical mr newton anyway tells us that it is possible to determine because newton's laws gives you the correct planetary orbits i should be able to determine various masses and if all of them completely agreed with each other now you see i am making prediction for the mass of uranus or mars or jupiter that would completely establish law of gravity and there is one unexpected bonus that we are going to encounter and that is an explanation of tides people have always observed tides and people have been fascinated by tights because you know the full moon night and the new moon night causes the tides and any number of supernatural explanations were given newton observed tights are nothing but due to the difference of gravitational force of the sun or the moon at two different ends of the earth

So that is the correction that we have to compute all these we will do in the next lecture

So my sincere advice to all of you is please go back work out all these things carefully i did not plugged in plug in the numerical values and we will wind up the study of whatever i told you all these days in the next lecture and then go on to discuss the applications ok have a good day bye you