

so good morning all of you welcome to the second lecture on gravitation you should remember that this is a first physical force and a fundamental physical force that you are studying in your course so far whatever you studied was essentially modelling whether it is collision or friction or any other force but here we are discussing a force whose physical origin is known coming from the masses and as a preliminary to that what i did was to briefly revise the basic concepts of dynamics so the basic concepts are summarized in the three laws of motion formulated by newton

so if i am to repeat them the first law essentially defines an inertial frame of reference assuming that you know whether force is acting on a body or not

so it says that if a body is not acted upon by any force then there are special frames of reference where the body will move with uniform motion it will not have any acceleration the second law quantifies force and the quantification the way you express it mathematically is done in the inertial frame of reference and much later you will study that when you go to non-inertial frames like a rotating frame or a uniformly accelerating frame there will be unphysical forces where newton's law will have to be modified the second law

so the second law essentially states that in an inertial frame the acceleration or even better the rate of momentum change suffered by the body is proportional to the applied force the most important thing here is that it is understood that applied force is known we do not try to determine what the applied force is by looking at the acceleration it is assumed that the applied force is known and as i told you for example in the case of simple harmonic motion we write f is equal to minus kx that is what we write in the case of electrostatic interaction we write f is equal to $\frac{e^2}{r^2}$ that is what we write or even better we write $\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$ and probably you may like to put a factor $\frac{1}{4\pi\epsilon_0}$ naught etcetera

so basically what we are saying is that when i am speaking of the applied force what i am doing is to model it based on a large number of observations and our own intuition and then we plug it into the newton's equation $\frac{dp}{dt} = f$ applied and check whether our modeling is correct or not this is something that we have to remember the third law as i told you is essentially a statement of conservation of total momentum which is a very very fundamental principle and we should do well to remember that because later when i am going to work out problems involving gravitation and later when others will teach you electrostatic interaction or even other forces mean many many problems conservation of momentum will be used extensively to find out what is the state of a particle after scattering after collision etcetera etcetera conservation of momentum is but one of the conservation laws

so what i wish to do today is to state the other conservation law which is very very important for us namely the conservation of energy this is a very fundamental principle and in physics both conservation of momentum and conservation of energy have a very exalted place very high position and all theories that we propose should be consistent with them however at this point we should add one statement of caution or one statement that is of the nature of a clarification and that is whereas conservation of momentum is straightforward conservation of energy is not a very straightforward concept because energy can occur in many many many forms in fact the precise formulation of the law of conservation of energy involves thermodynamics

where you take it into account all kinds of energies and not necessarily mechanical energy whereas in your mechanics course you are only going to deal with mechanical energy but in thermodynamics or any other any other discipline including you know nutrition you worry about how many calories you are consuming that involves chemical energy heat etc etc it is that total energy that is conserved

so in that sense a proper enunciation of the statement of conservation of energy really comes not from mechanics dynamics that we learn here or in your higher studies actually in the much broader area of thermodynamics that is something that we have to remember

so now let me start with the statement of the conservation of energy so the most important concept for us is that of the work done and this is a technical definition

so what do I do when I look at the concept of the work done what I am going to do is to assume that there is a certain force f and this force is acting on a particle

so force acts on a particle body and the body moves so let us say at time t equal to t initial time the body was here at a later time the body is here

so this is t and this is my

so sorry this is t_1 this is t_2 and we are let us imagine that there is a planar motion this is my x coordinate and this is my y coordinate

so for intermediate times let us imagine the body executed this motion that is what we are looking at now what I can do is at every point I can find out what the displacement is I can find out what the displacement is and what I do is to evaluate the integral $f \cdot ds$ the component of the force along the direction of the displacement between t_1 to t_2 along this path that is what I am going to do you will study this at great depth in your higher classes but essentially the statement is that if this is defined to be the work done we say that a force is conservative if the work done is independent of the path so let me repeat that we say that a force is conservative if the work done is independent of the path what does it mean in particular suppose your body started at a point it went all around the place and came back because of the action of a force then irrespective of what this path is the total work done is zero total work done is zero

so my force is acting in this direction my displacement is in this direction this is my f this is my ds

so basically I sum over all possible things if it is θ then we say that such a force is a conservative force it is important to remember that here work the concept of work is a technical concept and it is defined to be integral $f \cdot ds$ now what does it mean if you guarantee for me that the work done is indeed independent of the path then again you will learn in your higher classes that this f can be written as let us assume that it depends only on the distance the derivative with respect to r of a potential derivative of a derivative of a potential with respect to r that is what we are going to write so in doing this I am assuming that the force depends only on the mutual separation I will come to that later so in which case we get the condition that $\frac{1}{2}mv^2$ square plus v of r equal to constant and this is the statement of conservation of energy so we identify $\frac{1}{2}mv^2$ square with kinetic energy and this v of r is called the potential energy

so if a particle starts with a certain kinetic energy it will keep on decreasing because total energy is a conserved quantity until all its kinetic energy is converted into a potential energy or if a particle starts with a potential energy at rest then it keeps moving in such a manner that the

potential energy keeps decreasing and the kinetic energy keeps increasing so what is the example that we have in our mind i take a body and throw it up with an initial velocity it was all kinetic energy but then when it reaches the top most point it is all potential energy there its speed is equal to zero the kinetic energy is zero and when it starts falling down then it was all potential energy and when it reached back the surface of the earth or your hand then it is all kinetic energy

so that is what we have and i am sure you people know that total energy is not a measurable quantity only energy differences are measurable therefore you can add any constant to v or r it really does not matter so this is a preliminary that we should be familiar with

so now i am going to state another conservation law i am not going to give you any argument but i will just state

so i have demonstrated the conservation of momentum through the third law of motion of course the third law of motion does not prove that momentum is conserved it only states that momentum is a conserved quantity i have also told you that there is a special class of forces called the conservative forces for which energy is conserved by energy i do not mean any energy by that i mean kinetic energy plus potential energy and that is all what matters to us in the course on gravitation we do not have to worry about all other kinds of energy and the third conservation law is the conservation of total angular momentum at this point it is not very useful for us to study how the conservation of total angular momentum comes from of course we can make a statement that the total torque is zero then the angular momentum is conserved but there are deeper issues let us not get into that we will take that up or whoever teaches you will take it up at the right time now i told you that we are for the first time looking at a fundamental force we are not simply modeling

so let me recall what i told you in the last lecture i say that there is friction i say that there is viscosity but friction is a very weak statement for example friction can be independent of the speed the frictional force can be proportional to the speed frictional force can be proportional to the square of the speed all that we mean by friction is that it is always opposes the motion and whatever energy you lose is dissipated as some heat or some such thing the same thing happens in viscosity

so it is a question of modeling we do not ask for what the origin of the frictional forces

so a good question that we can ask when we embark when we begin on a study like gravitational field is to ask what are the forces which are the most fundamental from which every other force can come

so i have my adhesive tape i have my gum then there are bodies that stick to each other you have your velcro which binds the bodies together

so there are many many forces the geckos climb back the walls for instance the molecules are bound to each other the sun and the earth are bound to each other our whole milky way is a single galaxy where the stars are bound to each other therefore there are any number of forces and a good question to ask is is it possible that there are there is a small number of so-called fundamental forces from which everything emerges there is a question that we can ask and it turns out that the answer is yes and with about let us say 400 or 500 years of physics

so called modern day physics we know that all forces can be brought down to one of the four and that is what i have listed in this slide the first one is the gravitational force which binds us to the earth which binds the moon to the earth which binds the moon earth system to the sun etc etc the next

force is the electromagnetic force which is operating everywhere in fact everything that is happening in our human body is at some sense an electromagnetic force it binds the atoms atoms to molecules molecules to molecules in some sense all of chemistry is an approximation to or a applied branch of electrodynamics then we know that an atom is made up of an electron and a nuclei the nuclei are themselves made of protons and neutrons they are very very powerful forces because it is very difficult to break an atom and if you break an atom in an uncontrolled way actually it becomes a bomb an atomic bomb

so that enormous energy is contained in the nuclear force and you people also study beta decay in your radioactivity chapter probably in your 12 standard and the interaction responsible for that is the weak interaction so i have listed all of them it is a matter of interest to know what the difference between these forces are

so i have listed that in this table

so what i do is i will take the nuclear force strength to be of the order one that is what i will take and i will start comparing all the other forces with respect to them

so gravitation is of great interest to us you see that gravitation is almost zero compared to the nuclear force because its magnitude is relative strength is 10 to the power of minus 37 it is an enormously enormously small number

so if you just went by this strength we should have forgotten all about gravitational force you should not have any role at all in the dynamics of our universe or in human life but where gravitation loses in strength it gains in range it has an infinite range and mother nature has given us very very massive objects and because of that when you go to very far off distances gravitation plays a very important role and that is the reason why you do not worry about the gravitational force about when i am looking at what is it that holds this pen but i do worry about gravitational force when i ask myself what is it that doesn't allow me to jump away from the earth and escape to outer space that is what is happening what about nuclear forces nuclear forces are 10 to the power of 37 times stronger but then their range their empire is in a very very minuscule region

so we have the empire of gravitation which is the whole universe it is weak but the whole universe but the empire of nuclear forces is very strong but it is in a tiny tiny region of about 10 to the power of minus 15 meters okay and there it operates but beyond that it is very difficult to see it now comes the electromagnetic force electromagnetic forces are only about 100 times weaker than the nuclear forces its range is also infinite

so if you people remember all of you know even from your ninth and ten standard my gravitational force goes like one over r squared inverse square law my coulomb also goes like 1 over r squared

so if this is infinite range my coulomb force is also of an infinite range therefore a good question again that we should ask ourselves yes why is it that gravitation is able to dominate over electromagnetic forces why is it that such a thing is happening the answer to that comes a very very peculiar reason namely we have mass which is responsible for gravitational force and the mass occurs of only one kind all masses are positive and they all attract each other irrespective of what the mass is

so it is a very happy situation where there is no concept of a repulsion at all whereas when it comes to electromagnetic we know there are two kinds of charges the positive charges and the negative charges like charges the positive charges repel each other negative charges triple each other and

positive and negative charges attract each other and of course if a body does not have any charge it does not have electromagnetic interaction so what we have is because there are two kinds of charges if i bring a system of positive charges and negative charges then what will happen the system will try to get into your state in fact it will get into a state where positive charges and negative charges combine to form neutral objects now the interaction between these neutral objects is very very small because they are electrically neutral there will be some some small residual interaction because positive charge is distributed in some way negative charge is distributed in some other way so because of that there will be very very negligible interaction and that is the negligible interaction which you study for example in van der waals forces and that is what changes for example your equation of state in your van der waals equation of state you try to put in volume effect triples and effect and things like that and this becomes effectively short ranged what is a $1/r^2$ force will become a $1/r^6$ or $1/r^7$ so this phenomenon is what is called as screening every positive charge likes to get surrounded by negative charges every negative charge tries to get surrounded by positive charges the screening takes place and because of this screening the effective interaction between objects which are separated by large distances is very weak compared to the gravitational force so gravitational force beats the range of electromagnetic forces is only to the extent of you know holding objects together that is what is going to happen weak interaction of course loses on all counts it is about 10^{-17} times weaker that is what i am showing in this particular slide the last line in this slide and its range is less than 10^{-17} meter one even wonders why weak interaction is there at all well it has a very important role to play which you will study probably much much later in your life so that is responsible for weak interactions and we need not be bothered about that but by the time you complete your 12th standard you would have learnt gravitation electromagnetic and to a certain extent nuclear forces when you study fission fusion how much energy is liberated in the nuclear reactor etc etc using einstein's mass energy equivalence so this is a good occasion for us to actually record all the forces so what i am essentially trying to tell you is that of all these four fundamental forces gravitation is the one which is important for us the minute you know the distance between objects become of the order of a let us say even a centimeter okay only when the distance is of the order of a micrometer electromagnetic interactions become important and this is what we are going to study now i want to make some very important statements these things are not there in your textbook but then if i did not pay attention to that all of gravitation will appear mysterious therefore it is worthwhile spending some time on this so let me start writing down a few things and please pay attention now all of you know the form of gravitational force so let me write in big letters $F = -G \frac{m_1 m_2}{r^2}$ ok let me not bother about the sign at this particular point so since i am not being very precise in my mathematics i will try to be precise with my words and i will use the word attractive so i will also put a g here to remind myself that there is a gravitational force now i told you that applied force is something that i know and i am

going to apply newton's law to solve this problem i will look at circular orbits straight line orbits rectilinear motion i will look at the motion of satellites etcetera i will look at escape velocity you people will solve all those problems but there is a problem the problem is on the right hand side we have one two three four unknowns as a mathematical problem it is very easy to solve any problem in gravitation at your level because you will be asked a particle of mass m is moving in the field of the sun's gravitational force and then in your examination in parenthesis in brackets your examiner will say mass of the sun is so many so many kilos the distance between the earth and the sun is so many so many kilometers and you will also be given the gravitational constant but now the big question for us is that when you are introducing the concept of gravitation we have to ask ourselves how do i determine the so called universal constant the gravitational constant newton's gravitational constant how do i measure the mass m how do i measure this mass m and how do i measure this distance this is extraordinarily important we should remember that the great successes of newtonian gravitation came from astronomy and during the time of newton there was no way of knowing what the mass of the sun is in fact it is very difficult even to know what the mass of the earth is you can only make a crude estimate and we all know that it is also equally difficult to estimate what the distances are so what i advise you is go out and look up at the night sky tonight and you see so we have this vision i think we are able to distinguish separation between objects up to about seven kilometers or whatever so that is where you see the sky and that is where you find the zenith or the horizon from the zenith to the horizon beyond that you can't make out the distance scale so all the stars all the planets everything appears to be at equal distance from us well the moon appears to be bigger than stars so we don't know whether moon is intrinsically bigger or moon is closer to us therefore we need a number of methods indirect methods to be able to estimate the masses and the distances let me make this concept a little bit sharper so how do i make that let us start with newton's equation so i have a mass body of mass m and it is going to move with a certain acceleration and i want to write $g = \frac{GM}{r^2}$ let us assume that this equation is correct for the time being now one of the curious things that you find and that is something that i am going to spend quite some time at a later time is that the acceleration of the this is the test mass because i am only worried about the so let me write that a has a mass m_a b has a mass capital M so i am going to assume that this capital M is much larger than this small m so let us say very very large i am not going to worry about how the motion of the body a is going to affect the motion of the body b now you see irrespective of what the mass of a is the two masses cancel each other therefore the acceleration of a test mass in a gravitational field is independent of your own mass now that is a real tragedy because this is the only thing that i can measure i take a ball and i throw it up i know the mass of the ball but that is not going to give me any information it is not going to tell me anything about the gravitational constant it is not going to tell me anything about the mass of the earth it is not going to tell me

anything about the radius of the earth i can write $r + h$ whole square whatever that height is i only know the height but it tells me one important thing that whether you throw a pebble or a block of lead or any other object so long as frictional forces do not play a role all of them suffer the same acceleration and this principle is called the equivalence principle i will not spend too much time on that but this is a very very important principle i will come to that after i state gravitational law again after a while but at this stage i want to make this statement this is called equivalence principle so the big question is how do i know capital g how do i know capital m and how do i know these large distances i am speaking of distances like what the distance between the earth and the moon the earth and the sun the moon and this and so on and

so forth and this is where the great ingenuity of the astronomers comes in and we should remember that the groundwork for this great law newton's law of gravitation universal law of gravitation was laid not over a period of a few hundred years but over a period of thousands of years astronomers all over the world made observations astronomical observations they developed the mathematical tools for the understanding these observations and they came up with great physical insights of course they had to make a number of assumptions fortunately all of which were very very rational and therefore it was substantiated by observations later observations in nature so what i am going to do now is to systematically discuss how these constants can be determined this is not going to be a very difficult thing because all of you are studying enough mathematics for that the mathematics at the level of 11th and 12th standard is more than enough for whatever we are going to discuss

so if you know come back to this slide let me list out what are all the concepts that i am going to discuss probably i would have time only for discussing a few of them today the first that i am going to discuss is the size of the earth so please look at this light carefully i have made five bullets so the first bullet mentions that we should first of all know what the size of the earth is what do i mean by the size of the earth i should first of all know that the earth is spherical i need an evidence for that then i should be able to estimate the radius of the earth then i should know what the size of the moon is i should know what the earth moon distance is then i should know what the earth sun distances in other words my primary preoccupation first will be in estimating distances

so we are going to take a hierarchical approach first get a robust method of getting distances as accurately as possible then try to get a robust method of estimating either mass or gravitation together that is a little bit tricky thing you need both of them

so let us do that historically what happens is that you should know the distances very precisely then you take two objects of known masses you don't have to look at the mass of the moon or the sun or the earth which is difficult known masses like i said dumbbell to heavy metal masses then find use them use newtonian law of gravitation to estimate the law of g the constant of gravitation with the gravitational constant then combine that with either the galilean law of motion or some other law and try to get the mass of the earth mass of the sun and then combine with it whatever knowledge of the masses that we are known

so essentially what i am trying to convey you is that studying gravitation or for that matter any physical force is not a mathematical exercise physics is not applied mathematics physics involves very careful observation and a

very careful interpretation and since you are getting exposed to this subject for the first time this is the time and this is the place for us to actually put in an effort to understand what is happening so i will start with the concept of the radius of the earth now there is a certain friction which is very popular namely that people believed that the earth was flat and there were heroic people sometime in 15th century 16th century 17th century who believed that the earth was actually spherical in shape and they defied they were great heroes they were willing to give up their life so we have the lord of columbus who decided to reach india by traveling westwards instead of traveling eastwards but these are really essentially fiction they may be inspiring but they are not correct because astronomers all over the world over more than thousand years knew that the earth is a spherical object and so is the moon and the evidence for that came from eclipses so there is a certain mythological literature in every civilization whether it is sumerian or babylonian or greek or roman or indian or chinese where you say you know the earth is flat the earth is supported by so many elephants or in the greek mythology i think it is at class or somebody who holds the earth so you could be anybody okay we should not confuse that these mythological stories mean they have their own very very important role to play because it speaks of the human psych not about the nature of the external world but astronomers always knew their earth is spherical so to give you an example from our own country for example the great astronomer mathematician aryabhatta he lived in the 5th century eid not too early he argued that the earth is spherical in shape and when the student asks him if the earth is spherical in shape why is it that i do not fall off when i am on the other side audi but i correctly answer central same look here when i look at earth as a sphere there is nothing called up and down which is absolute up is when you move away from the surface of the earth and down when you move towards the earth so in a similar manner if i am coming to the diametrically opposite point okay when i am standing here i may say i am going down under the earth that is what we say down under but once i come here up is away from the surface of the earth down is towards the earth in fact aruba also argues that there is a mysterious force that holds everything together of course at that time they did not know anything about gravitational force so he says that it is in the very nature of the things so astronomers knew that we are not saying that aryabhatta was the first person to make such a statement the greek astronomers or probably even other astronomers in egypt or babylonia these people knew that essentially all heavenly bodies including the earth were all spherical in nature and they were all in space they were all moving as to who was moving with respect to who is a different question altogether for example the indian astronomical school seriously believed that there are very very powerful winds which direct the heavenly bodies to move in prescribed orbits so that was their model for the force they can be sophisticated versions of that but the point is that i want to be able to determine the radius of the earth there are many very crude ways of determining the radius of the earth so you might study this so if the earth is a spherical object you give a certain height and you ask how far i can see so this is my radius let us say this is an exaggerated figure

so i will make use of trigonometry as the height keeps on increasing i can see farther and farther i know this height i know the angle i know this distance therefore from that i should be able to estimate the radius of the earth so this is rather crude in fact the simplest version that i can state is that i am a person let us say ok let us take a person who is about six feet six or six plus something feet tall about two meters let us say so there is a person two meters tall how far can the person see because of the spherical nature of the earth because that is going to get curved if the earth were flat your range of vision would be infinite you would be able to see beyond infinity of course you don't see buildings or human beings or for that matter the tallest buildings which exist in dubai or whatever place we are not able to see them because of this very city so go home make a simple model and estimate what the radius of the earth is so this is something that we can do so this is a simple example these things do not prove that the earth is spherical you will have to do a navigation which is what people did and people know fairly lot for example ptolemy's book on geography contains a large number of sites in india both along the western coast and the eastern coast so people sort of fairly knew that earth is spherical in nature what i want to show for you is an extraordinarily beautiful estimate of the radius of the earth by arastothinis erastor dennis lived in the 4th century bc so please look at this slide i hope this is visible to all of you otherwise anyway i will write on the sheet of paper and i will show it to you erastratinis was indeed a great mathematician and he was also he was not an armchair mathematician he was a person who also did observations by travelling long distances and he lived in the fourth century bc okay so i was speaking of aryabhatta in the fourth century a.d so we are now speaking of the fourth century bc so we are speaking of a span of 800 years eight hundred years before arya mata made his statements erastotinis already had found good methods of estimating the radius of the earth so let me explain that to you so what you have is the surface of the earth and what you have to do is to pick up a particular day in fact for us a good particular day should be something like summer solstice or the winter stalls days he probably did not do that because you know on that particular day the sunlight falls directly either on tropic of cancer or tropic of capricorn 23.5 degrees either north or the south never mind about that but the whole idea is that there were two points so that is what i have to worry about this point corresponds to alexandria and this point corresponds to os1 in fact this was a well this was a well in as1 and well contains water so that is what we have now the distance between them is known and that was given to me i am reading straight away in this 50 stadia so this distance is 50 stadia so the unit of distance is a stadium stadium is a place where we go and sit in order to watch in india a distance was called a unit of distance when people travel was called yojana in greek it was called as stadium the only problem with the concept of stadium or yoga is they keep on changing with time the measures keep on changing with time but the name persists so we have to be a little bit careful now the point is i know the distance between them we will convert this stedia into ordinary units in a while now i am going to draw another picture because i have already used data

so this is one point this is another point on that particular day this is a highly exaggerated figure there is a well here the sun's rays were falling perfectly normally it was making 90 degrees now obviously if i am going to look at this point it should be sufficiently far away it makes a certain angle that is why you know the length of the day and the length of the night keeps changing correct

so if you go to the poles you have 6 months of day and 6 months of night because of that particular reason because the sun's rays become more and more tangential and after a point to stop reaching that in this particular for example region that is what you have and we should know what the angle is and this angle is about 7 degrees this angle is about 7 degrees and now i am going to extrapolate this

so let us say that this is my center of the earth so this is what i have and this is what i have and this is 90 degrees and this is 7 degrees and this is my radius of the earth so what should i do all of you have studied the formula $s = r\theta$ when i speak of the distance i do not mean boring a tunnel and finding out what the distance between these two points is it is a distance traveled by these two points when i move on the surface of the earth so i am really looking at yes it is the total distance covered not the euclidean distance okay not the shortest distance now you know theta because this was 7 degrees you know what this angle is i will leave that as an exercise for you people

so your r is essentially given by s by theta yes is a fairly small quantity namely 50 stadia i told you but theta is very very small because 7 degrees is very very small and theta should be written in units of radians

so 2π radians you have to divide it by whatever the 2π is if you work out you should be able to estimate the radius at this point i should point out to you people that erasos otherness was a either a lucky person or a very smart person because this argument works out only if both of them are at the same longitude by hindsight if you move away to some other longitude then you cannot estimate that but it turns out they are roughly at the same longitude go to the globe and look at that and there are strongest was able to get that okay no it was not 50 stadia i am very sorry it was 5000 stadia the distance was 5000 stadia and it translates to 800 kilometers yes translates to 800 kilometers now if you plug in and if you try to find out what the radius is you get did you get the radius or the circumference you estimated the circumference multiplied it by $2\pi r$ you get 40 000 kilometers so circumference is equal to two pi r is equal to forty thousand kilometers is what you are going to get i think the current radius of the earth i forgot to write that down is about 6 400 kilometers

so take 6 400 kilometers multiplied by 2π whatever

so that is 6 6 into 6 is 36 and there is another 4 into six twenty four thirty eight thousand four hundred and there are some corrections because i simply used pi is equal to six it is six point something and therefore you see it is remarkably very very close to the current value

so what we are able to do is to make use of known trigonometry whatever you study by drawing triangles and circles okay in fact all of trigonometry was developed in order to understand the heavenly motion okay and make of course trigonometry was also required for sculpture for architecture for buildings for demarcating areas

so on and

so forth but it was used primarily and developed primarily for astronomy and this is one of the great great achievements of human ingenuity you may

wonder how he measured this to be 5000 stadiums stadia actually it is a very interesting thing he knew the circumference of his wheel and he tried a small stick and he actually sat on a cart let us call it a chariot so this chariot is being driven their horse and every time the wheel of the chariot completes one circle that stick hits the ground so what does he do he keeps on counting the number of time the stick hits the ground so you know how many circles how many revolutions the wheel has completed and you know the circumference of the wheel is 2π into the radius of the wheel multiplied by the total number and that gives you the total distance that is covered so this is how people actually devised simple effective and ingenious methods in order to measure large things and these are the things that we should imbibe it is not necessarily solving a problem in a smart way it is not a question of you know i integrate by substitution or by integration by parts these are not the only skills that those technical skills are also required so this is done and we have a fairly good idea of what the radius of the earth is actually if you give me the radius of the earth it should not be too difficult to estimate the mass of the earth also if i know what should be the mean density but that is a different story altogether however at this point we should remember that this involves a large number of assumptions namely in fact when i look at other situations it will involve even larger number of assumptions and that is whatever law of mathematics whatever results in mathematics i have obtained from my daily observation they are also valid at large distances so let me make that statement so i draw a triangle and i measure the sum of the angles of a triangle to be 180 degrees now that of course is a theorem because i am saying that two parallel lines wont meet each other that is an axiom correct that is what i am assuming in other words when i am using these quote unquote mathematical results for deriving conclusions from physics how do i know that these results are correct i know them because i found by observation it doesn't help a physicist to say oh no they are absolute results because they are mathematical results unfortunately that is not correct because the so-called axioms of mathematics have to be verified by observation there is no reason why two parallel lines should not meet there is no reason why pythagoras theorem should hold they are all equivalent to each other there is no reason why the sum of three angles of a triangle must be 180 maybe it is greater than 180 maybe it is less than 180 this is something to be verified but then this realization came much much later only sometime in the 17th or the 18th century not even during the time of newton people assumed that there can be no other properties of nature that are possible other than what euclid wrote in his geometry for example that was the universal assumption and it turns out when we are measuring distances between the earth and the sun the earth and the moon and even the nearby stars these results are robust but if you go to far far distances those results are not robust there are going to be corrections so the message that i am trying to tell you is that physics is different from mathematics in the sense that the so called fundamental exams of mathematics are themselves under constant verification in physical laws we have to know what the correct mathematical principles are that we should use for example we say romer measured the speed of light by looking at the eclipses of moons of jupiter you know the

distance you know the time taken actually there is a very very major assumption that we make namely light travels with constant speed between its emission and reaching the earth that is an assumption so the way physics works is that you make an assumption you make a hypothesis you verify you get a conclusion and then you make further prediction so on and so forth and you make a correction so please remember throughout your course whatever you are doing in 11th 12th and beyond we are going to take the viewpoint that physics is not applied mathematics laws of nature are not god given although we call it universal and all that it is our modeling and everything requires very very strenuous and a diligent verification to better and better accuracy that is something that we have to do whether it is electromagnetic theory or gravitation or strong or weak this is the spirit that we are going to take in order to continue our studies

so let me come back to the slides

so please go back and work out the great result of erasureness it is a great thing now what i want to do is to estimate the distance between the moon and the earth i am not going to work out all the details because the fun of it will be taken away as i told you people knew that moon itself the moon itself should be a spherical object because we have the faces of the moon and the phases of the moon are there because a part of the spherical surface gets reflected whatever is other part is in the shadow region

so we also know that we get the new moon when the moon is completely on the opposite side of the earth

so what are we saying to put it very crudely you have the sun you have the earth here

so the full moon is sorry the new moon is when the moon is here this is the sun this is the earth and the full moon is when the moon is here because that is what we have in fact the orbit of the moon is slightly inclined to this plane otherwise every full moon we would have had an eclipse there is no escaping that we would have had an eclipse because the moon would come between the earth and descend there would have been a solar eclipse but such a thing does not happen anyway

so now what we can do is ask what happens at a half moon half moon is the eighth day what we call as the ashtami right now the moon is a perfect semicircle

so we have the full circle on the full moon night absolutely no circle because there is a complete shadow on the new moon

so now you look at what happens when there is a half bone therefore the moon should be here obviously

so this part of the moon let me exaggerate that this part of the moon is getting illuminated

so i see this the other moon is not getting eliminated therefore i only say half of it or whatever the sun's rays is coming no this estimate was done by hipparchus so hipaaka said that this should be 90 degrees here and this is what i have

so all that i need to know is this angle if i know this angle very precisely i should be able to estimate the moon earth distance is that ok or at least the ratio again i am not going to work this out because it will take us far far away except that the very concept of a half moon is a very tricky thing how do you know it is exactly half moon and you know the whole thing is that in all these situations remember we are using a formula like $s = r \theta$ we are going to approximate $\sin \theta$ by θ and various small errors in θ will give rise to very large errors in estimate of the distance we

should be careful about that but this was done and you can get a fairly esteem
good estimate between the distance between the earth and the moon
so we have to worry about the earth and star distances
so probably what i should do at this particular time is to actually stop
because maybe instead of leaving these examples to you people in the next
lecture i will myself for quote how you get the angles on the half moon day
and then i will introduce the concept of a parallax which is extraordinarily
important and they will show how the distance between the earth and the
stars can also be measured in fact measurement of parallax also brings into
the four a very very important question and that is a question which plagued
all the ancient astronomers and that is whether the sun goes around the
earth or the earth goes around the sun with respect to the background stars
and we will take that in the next class
so if you find time please read them up and come
so that you will be well prepared
so let us stop now have a good you